Chapter 4

A Decentralized Beam Selection Algorithm for mmWave Beamspace MU-MIMO Systems

4.1 Introduction

RF complexity is the main hurdle of high dimensional mmWave beamspace MU-MIMO system because each beam requires a dedicated RF chain corresponding to each other. However, the power consumption per RF chain is 250mW for mmWave frequencies [50], and it leads to high power consumption. To circumvent this hurdle, beam selection algorithm is deployed at transmitter, which enforces the system to select only equitable beam while other beams remain inactive. Hence, beam selection algorithms having different performance and complexity are proposed in [53, 109, 110, 113, 117].

The algorithm in [113] selects beams of the highest channel magnitude. This algorithm further improved by the beam selection algorithms [53, 109]. "IA" beam selection algorithm in [110] select beams considering multi-user interference at users' side. The algorithm "M-SINR" and "MC" [109], also select beams but holding high complexity. The "QR-based" [53] and "MWM-based" [117] beam selection algorithms are presented in chapter 3, with the new precoder which outperforms [109, 110, 113]. However, the "QR-based" beam selection algorithm [53] is superior to "MWM-based" beam selection algorithm [117]. Thus, the aforementioned algorithms have been deployed with a centralized approach.

In the centralized beam selection algorithms, transmitter requires the channel information of each user to perform the beam selection. Further, the amount of channel information increases as the number of users in a cell increases. Therefore, it is difficult to handle a large amount of channel information for transmitter, additionally, high computations are required in such a beam selection algorithm.

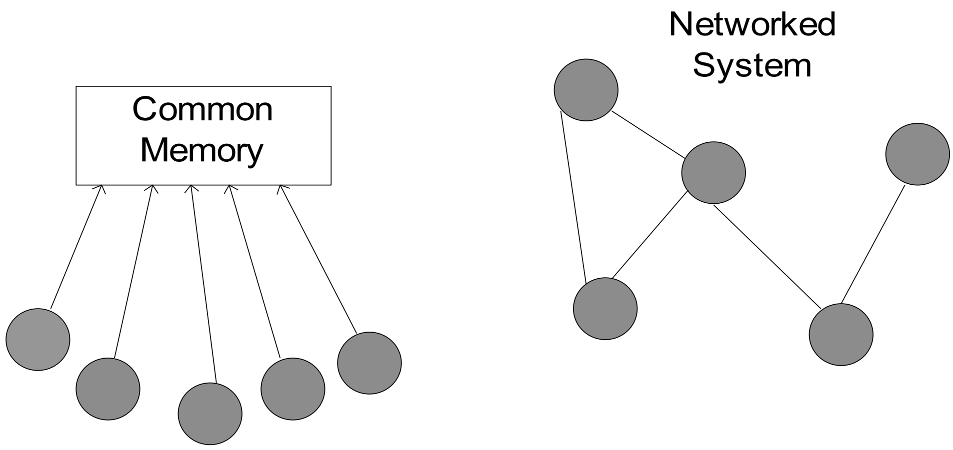

Figure 4.1 Common memory (left) versus networked (right) systems

Keeping this computational problem in mind, one can think of decentralized algorithms to deal with a large amount of channel information [118]. A decentralized algorithm requires distributed system or networked system, and there are also no requirement of common memory, as shown in Fig. 4.1. Unlike centralized algorithms, decentralized algorithm [118] distributes the computation to the multiple processors and these processors are the users themselves. Therefore, the users themselves play an active role in performing the beam selection and all of them form a dynamic network topology, as shown in Fig.4.1. Consequently, beam selection is performed by sharing information between neighbouring users in the network. The information exchange between users takes some time and causes some delay which depends on the distance between connected users in the network [118]. Thus, the performance of the decentralized algorithm also depends on the network cooperation among the users. In this chapter, we are proposing two beam selection algorithms which approach to a concept of real bidding to achieve the following objective:

1. A centralized beam selection algorithm: the central unit assigns beam to users through well-developed auction algorithm.
2. A decentralized beam selection algorithm: beams are assigned to users without a central unit through a distributed auction algorithm with an assumption that one of the nearest users to transmitter in the network will feedback index number of the beams to transmitter, as shown in Fig. 4.2 .

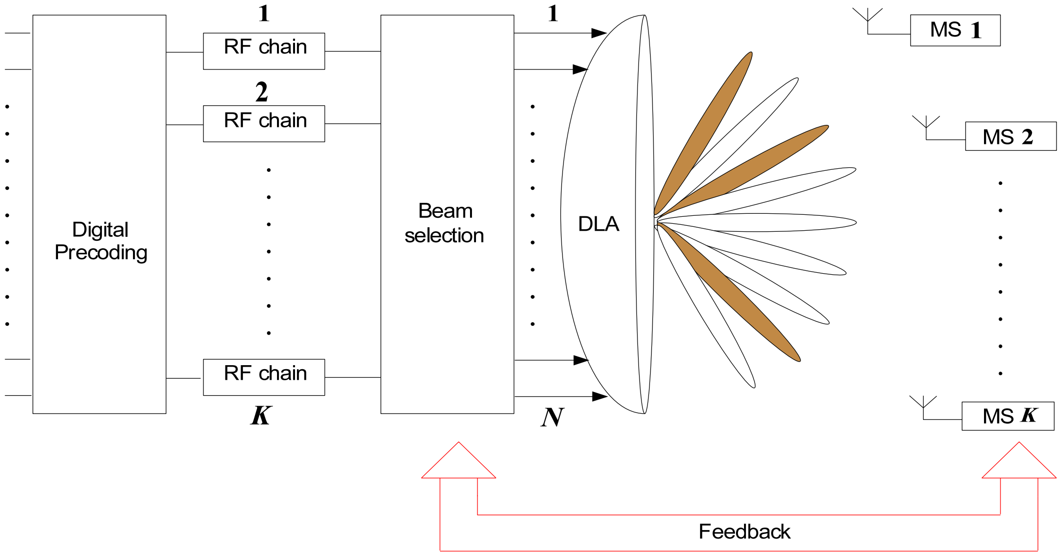

Figure 4.2 mmWave beamspace MU-MIMO System with Feedback

Thus, we are investigating both the centralized and decentralized algorithm to examine the performance and complexity.

4.2 mmWave Beamspace downlink MU-MIMO Communication System Model

We are considering a downlink mmWave beamspace MU-MIMO communication system having a transmitter equipped with N beams, and K users equipped with a single receive antenna, and $N >> K$. Thus, the input-output relation for a mmWave beamspace MU-MIMO system [113] can be expressed as

$$\mathbf{y}_b = \mathbf{H}_b^H \mathbf{P}_b \mathbf{x} + \mathbf{w}_b, \tag{4.1}$$

where $\mathbf{H}_b^H = \mathbf{H}^H \mathbf{U} = [\mathbf{h}_{b,1}^H, \ldots, \mathbf{h}_{b,K}^H]^H \in \mathbb{C}^{K \times N}$ is the beamspace channel matrix, Each $\mathbf{h}_{b,k}^H = \mathbf{h}_k^H \mathbf{U} = [h_{b,1k}^*, h_{b,2k}^*, \ldots, h_{b,Nk}^*]^H \in \mathbb{C}^{N \times 1},\ k = 1, \ldots, K$. $\mathbf{H}^H$ is the spatial channel matrix, and $\mathbf{h}_K^H$ is the spatial channel vector for user k. $\mathbf{x} \in \mathbb{C}^{K \times 1}$ is the transmitted symbol vector. $\mathbf{y}_b \in \mathbb{C}^{K \times 1}$ is the received information vector, and $\mathbf{w}_b \in \mathbb{C}^{K \times K}$ denotes AWGN noise vector with $\mathbf{w}_b \sim \mathcal{CN}(0, N_0 \mathbf{I}_K)$. $\mathbf{P}_b \in \mathbb{C}^{N \times K}$ is, a digital precoder, to remove MUI while satisfying an average power constraint as

$$\mathbb{E}[\| \mathbf{P}_b\mathbf{x} \|^2] \leq \rho.$$

Further, the input-output relation for a mmWave beamspace MU-MIMO system, after beam selection, can be expressed as

$$\tilde{\mathbf{y}}_b = \tilde{\mathbf{H}}_b^H \tilde{\mathbf{P}}_b \mathbf{x} + \tilde{\mathbf{w}}_b, \tag{4.2}$$

where $\tilde{\mathbf{H}}_b^H = [\tilde{\mathbf{h}}_{b,1}^H, \ldots, \tilde{\mathbf{h}}_{b,K}^H]^H \in \mathbb{C}^{K\times K}$ is the beamspace channel matrix corresponding to K selected beams and $\tilde{\mathbf{P}}_b \in \mathbb{C}^{K\times K}$ is a digital precoding matrix. $\tilde{\mathbf{w}}_b$ denotes AWGN noise vector with $\tilde{\mathbf{w}}_b \sim \mathcal{CN}(0, N_0\mathbf{I_K})$.

4.3 Proposed Beam Selection Algorithms

DLA provides N beams out of which single beam is assigned to each user. So, we need only K RF chains. The beams should be selected to achieve the optimum sum rate in the mmWave beamspace MU-MIMO system. Thus, the selection of K beams out of N is the problem which we are addressing here.

In this chapter, beam selection is modelled as a resource allocation problem. As in [119], auction algorithm can be used to allocate resources such as bandwidth, power to the users. Similarly, in the mmWave beamspace MU-MIMO system, beams can be considered as resources and those can be allocated (or assigned) to the users in an optimal manner through a central auctioneer using the auction algorithm. Further, the auction algorithm has a distributed variant that doesn't require a central unit to do the allocation [118]. A distributed algorithm distributes computation to each agent or user in the cell network to alleviate the burden of transmitter. Each user in the formed network must cooperate and exchange the information to each other. Note that, the performance of the system depends how fast and efficiently users exchange their information with each other. Thus, it requires network cooperation. Further, there will be some propagation delay while exchanging information among users. The delay can be reduced by choosing an efficient topology. In this paper, we make use of both the centralized and distributed framework of the auction algorithm. So far, in the open literature, all the existing beam selection algorithms use centralized solutions, namely “MM” [113], “IA” [110], “M-SINR” [109], “QR-based” [53], and “MWM-based” beam selection [117].

4.3.1 **Beam Selection through Auction Algorithm**

In auction algorithm, a central unit or an auctioneer conducts an auction of items among the bidders, and bidders raise the price of items which they desire the most. After getting bids from all the bidders, the auctioneer sells each item to the highest bidder of that item.

Let $\mathcal{K} = \{1, \ldots, K\}$ and $\mathcal{N} = \{1, \ldots, N\}$ be the set of users and beams, respectively. Let ψ_{kn} be the benefit gained by user k from beam n and r_n be the price that a user has to pay for beam n. Thus, the net benefit to user k from beam n is equal to $(\psi_{kn} - r_n)$. Each user wishes an access to a beam that offers a maximal net benefit to him/her. Equivalently, user k wants to get access to beam n_k

$$n_k = \arg\max_{n \in \mathcal{N}} \{\psi_{kn} - r_n\}, \tag{4.3}$$

where $n_k = \arg\max_{n \in \mathcal{N}} \{\psi_{kn} - r_n\}$ is the beam that results in the highest net benefit for user k. If this condition satisfies for all the users then the algorithm terminates otherwise unassigned users (the users who desire the same beam, but which is already assigned to another user) increase the price of beam or bidding amount for which they are benefited the most in such a way

$$\begin{aligned} b_k &= \{\psi_{kn_k} - r_{n_k}\}, \\ b'_k &= \max_{n \in \mathcal{N}, n \neq n_k} \{\psi_{kn} - r_n\}. \end{aligned} \tag{4.4}$$

Note that b_k is the highest net benefit and b'_k is the second highest net benefit for user k and the bidding increment for his/her desired beam is given by δ_k

$$\delta_k = b_k - b'_k + \epsilon, \tag{4.5}$$

where ϵ is a participating price which enforces the algorithm to achieve an equilibrium condition. Now, let n be the beam for which the bidding amount has been incremented by the unassigned users who belong to set $\mathcal{U}(n)$, where $\mathcal{U}(n) \subseteq \mathcal{U}$, where $\mathcal{U}$ is a set of unassigned users, and $\mathcal{U}(n) \neq \emptyset$. The price of beam n is increased by the highest bidding amount $\max_{k\in\mathcal{U}(n)} \delta_k$ and beam n is assigned to the highest bidder $k(n)$, and it is defined as

$$k(n) = \arg \max_{k\in\mathcal{U}(n)} \delta_k. \tag{4.6}$$

Thus, the highest bidder who was unassigned user in $\mathcal{U}(n)$ is now assigned to n^{th} beam and the previously assigned user to n^{th} beam is unassigned. This process is repeated until all the users are assigned a beam. Algorithm 4 represents the procedure of the proposed beam selection based on auction algorithm.

4.3.2 Beam Selection through Distributed Auction Algorithm

For the centralized beam selection algorithms, transmitter requires channel information. But, transmitter gets heavily loaded due to large dimension of channel handling matrix and it becomes a bottleneck to process over a large matrix dimension to select desired beams. Therefore, we propose a decentralized beam selection algorithm which is a better alternative and it is detailed in this section.

As in [118], the distributed auction algorithm distributes the computation among all the users. Unlike the auction algorithm, the price list of beams is not shared through a common memory because only local communication is possible among the neighbouring users, as shown in Fig. 4.1. But, for the correct assignment of beams, the updated price list should be known to each user. Therefore, each user updates their price list using the neighbour user's protocol agreement. However, performance of the distributed auction algorithm may vary with network topology, and the information exchange can occur between the two nearest users in the formed network-topology.

In the beginning, each user bids for the desired beam and store the price list. But, each user is unaware of other highest bidders. So, they set themselves as the highest bidder. Let $\mathcal{K} = \{1, \ldots, K\}$ and $\mathcal{N} = \{1, \ldots, N\}$ be the set of users and beams, respectively. As shown in [118], let $\alpha_k(t)$ be the assignment status of user k

Algorithm 4 Beam Selection through Auction Algorithm

Requirement: the price list r_n and the benefits ψ_{kn} , $k \in \mathcal{K}$, $n \in \mathcal{N}$

Initialize:

$$r_n = 0,\ \forall n \in \mathcal{N}, \mathcal{U} = \emptyset$$

for $k = 1 \to K$ **do**

$$n_k = \arg\max_{n \in \mathcal{N}} \{\psi_{kn} - r_n\}$$

  if Beam n_k is unassigned **then**

    Assign beam n_k to user k

  else

    Add k to the set of unassigned users:

$$\mathcal{U} = \mathcal{U} \cup \{k\}$$

  end if

end for

while $\mathcal{U} \neq \emptyset$ **do**

  Initialize:

$$\mathcal{N}' = \emptyset$$

  for $k \in \mathcal{U}$ **do**

$$n_k = \arg\max_{n \in \mathcal{N}} \{\psi_{kn_k} - r_{n_k}\}$$

$$\mathcal{N}' = \mathcal{N}' \cup n_k$$

$$b_k = \{\psi_{kn_k} - r_{n_k}\}$$

$$b'_k = \max_{n \in \mathcal{N}, n \neq n_k} \{\psi_{kn} - r_n\}$$

    Compute bidding increment for desired beam:

$$\delta_k = b_k - b'_k + \epsilon$$

  end for

  for $n \in \mathcal{N}'$ **do**

    Assign beam n to user $k(n)$, where $k(n) = \arg\max_{k \in \mathcal{U}(n)} \delta_k$

    If beam n was previously assigned to another user, make him/her an *unassigned* user and add to set $\mathcal{U}$.

    Increase the price of beam n by $\max_{k \in \mathcal{U}(n)} \delta_k$.

  end for

end while

at time t, such that $\alpha_k(t) = n$ if user k is assigned to beam n, where $k \in \mathcal{K}, n \in \mathcal{N}$. Furthermore, the price list of user k corresponding to each beam is represented as

$r_{kn}(t) \geq 0$, n $\in \mathcal{N}$ at time t and the highest bidders with the largest index is given as $g_{k\alpha_k(t)} = k$ at time t, and $\alpha_k(t)$ is defined as

$$\alpha_k(t) = \arg\max_{n \in \mathcal{N}} \{\psi_{kn} - r_{kn}\}. \tag{4.7}$$

All users are allowed to update their price list as well as the corresponding highest bidders according to protocol agreement among the neighbouring users ($\in \mathcal{D}_k(t)$, is a set of the neighbour users to user k) with the aid of local information as per communication cycle. Each user inquires the highest bidders and bidding amount via the information exchange with the neighbouring users. If the bidding amount is found to be higher which is made by another user then user updates the highest bidding amount and the corresponding highest bidders. Next, each user increases the bidding amount for the beam that gives him/her maximal net benefit by an amount $\delta_k = b_k - b'_k + \epsilon$, where

$$b_k = \{\psi_{kn_k} - r_{kn_k}(t)\}, \tag{4.8}$$

$$b'_k = \max_{n \neq \alpha_k(t+1), n \in \mathcal{N}} \{\psi_{kn} - r_{kn}(t)\}, \tag{4.9}$$

and $n_k = \alpha_k(t+1) = \arg\max_{n \in \mathcal{N}} \{\psi_{kn} - r_{kn}(t)\}$ is the beam that results in the highest net benefit for user k. Note that b_k is the highest net benefit and b'_k is the second highest net benefits, δ_k is the bidding increment for the desired beam, and ϵ is a participating price which enforces the algorithm to achieve an equilibrium condition. Thus, this algorithm needs to be run separately by all users and the several communication cycles (Δ) are required to achieve the correct assignment. Note that, when the price list doesn't change, the algorithm terminates. Algorithm 5 represents the procedure of beam selection based on distributed auction algorithm.

4.3.3 **Precoding Scheme**

Beam selection algorithm select beams for the users, whereas precoding is employed to eliminate the multi-user interference, evaluated through the reduced channel matrix, i.e., $\tilde{\mathbf{H}}$ after beam selection. ZF is one of the precoding schemes but ZF degrades the performance when $\tilde{\mathbf{H}}_b$ is a ill-conditioned matrix and is not applicable

Algorithm 5 Beam Selection through Distributed Auction Algorithm for User k

1: Requirement: the price list r_{kn} and the benefits ψ_{kn}, $k \in \mathcal{K}$, $n \in \mathcal{N}$

2: Initialize: the price of each beam is 0 except the most desired beam, $r_{k\alpha_k(t)}(t) + \delta_k$, $\alpha_k(t) \in \mathcal{N}$ and the highest bidders $g_{k\alpha_k(t)}(t) = k$

3: User k updates the price list by information exchange from the neighbor users:

$$r_{kn}(t+1) = \max_{l \in \mathcal{D}_k(t)} \left\{ r_{kn(t)}, r_{ln(t)} \right\}$$

4: User k updates the highest bidders by information exchange from the neighbor users:

$$g_{nk}(t+1) = \max_{m \in \arg\max_{l \in \mathcal{D}_k(t)} \left\{ r_{nk(t)}, r_{ln(t)} \right\}} g_{mn}(t)$$

5: **if** **then** $r_{k\alpha_k(t)} \leq r_{k\alpha_k(t+1)}$ and $g_{k\alpha_k(t)} \neq k$

6: User k updates the status of the assignment:

$$\alpha_k(t+1) = \arg \max_{1 \geq n \leq N} \{ \psi_{kn} - r_{kn}(t+1) \}$$

7: User k updates the highest bidder:

$$b_{k\alpha_k(t+1)}(t+1) = k$$

8: User k computes the bidding amount for beam $\alpha_k(t+1)$:

$$\delta_k = b_k - b'_k + \epsilon$$

9: User k increases the price of the beam $\alpha_k(t+1)$:

$$r_{k\alpha_k(t+1)} = r_{k\alpha_k(t+1)} + \delta_k$$

10: **else**

11: The assignment status for user k remains the same:

$$\alpha_k(t+1) = \alpha_k(t)$$

12: **end if**

to rank deficient matrix. A precoder that eliminates multi-user interference, based on QR decomposition of $\tilde{\mathbf{H}}_b$, has been proposed in section 3.3.3, [53]. By choosing $\tilde{\mathbf{P}}_b = \tilde{\mathbf{Q}}_b$, ($\tilde{\mathbf{H}}_b = \tilde{\mathbf{Q}}_b \tilde{\mathbf{R}}_b$), one can obtain the system described by (3.6).

$$\tilde{y}_{b,k} = \tilde{r}_{kk} \tilde{x}_k + I_{b,k} + \tilde{w}_{b,k}, \ k = 1, \ldots, K, \tag{4.10}$$

where $\tilde{r}_{kk}$ is the effective channel gain of k^{th} user and is the k^{th} diagonal element of $\tilde{\mathbf{R}}$, $\tilde{\mathbf{H}}_b = \tilde{\mathbf{Q}} \tilde{\mathbf{R}}$. $I_{b,k}$ is the interference. $\tilde{w}_k$ is the AWGN noise vector with

$\sim \mathcal{CN}(0, N_0\mathbf{I}_k)$.

Interference $I_{b,k},\ k = 1, \ldots, K$, can be eliminated by diagonalizing $\tilde{\mathbf{R}}_b^H$. Thus, diagonalization can be done by having $\tilde{\mathbf{P}}_b = \tilde{\mathbf{Q}}_b\mathbf{L}$. The effective channel gain for user k, and the sum-rate is given by

$$R_s = \sum_k \log_2\left(1 + \frac{1}{N_0}\frac{\rho}{K}\tilde{r}_{b,kk}^2\right) \text{ bits/s/Hz.} \tag{4.11}$$

We implement the same precoder for cancelling the multi-user interference. Note that $\mathbf{L}$ need not be unitary and to satisfy the transmit power constraint, one needs to normalize $\mathbf{L}$ to have unit norm. Thus, $\tilde{\mathbf{P}}_b$ in (4.2) is chosen as $\tilde{\mathbf{P}}_b = \tilde{\mathbf{Q}}_b\tilde{\mathbf{L}}$, where $\tilde{\mathbf{L}} = \frac{\mathbf{L}}{\|\mathbf{L}\|_F}$, $\tilde{\mathbf{Q}}_b$ is obtained from the QR decomposition of $\tilde{\mathbf{H}}_b = \tilde{\mathbf{Q}}_b\tilde{\mathbf{R}}_b$.

4.3.4 **Complexity Analysis**

In this section, we discuss complexity of beam selection of the proposed beam selection algorithm. Complexity of beam selection algorithm through the auction and distributed auction algorithm are order of $\mathcal{O}(N^3)$ and $\mathcal{O}(\Delta N^2 \lceil \frac{\max_{n,k}\{\psi_{nk}\} - \min_{n,k}\{\psi_{nk}\}}{\epsilon} \rceil)$, respectively. But, the complexity of the "QR-based" [53] and "MWM-based" [117] beam selection algorithm is $\sum_{i=0}^{N-K-1} (N-i)\mathcal{O}(2(N-i)K^2)$ and $\mathcal{O}(NK^2)$, respectively. Similarly, complexity of the "IA" beam selection algorithm is $\sum_{i=0}^{K-\text{NIU}-1} (N - \text{NIU} - i)\mathcal{O}(K^3)$, where NIU is the number of non-interfering users. Complexity of beam selection through the auction and distributed auction algorithm is much lower than "QR-based" beam selection [53], and approximates to "MWM-based" beam selection algorithm [117]. Further, complexity of beam selection through a distributed auction algorithm is constant irrespective of changes in K.

4.3.5 **Numerical Results**

In this section, we discuss performance metric, specifically, the spectral efficiency of the proposed beam selection algorithm, beam selection through the auction and distributed auction algorithm. It is evaluated through simulations. We also compare the performance of the proposed beam selection algorithm with the existing beam

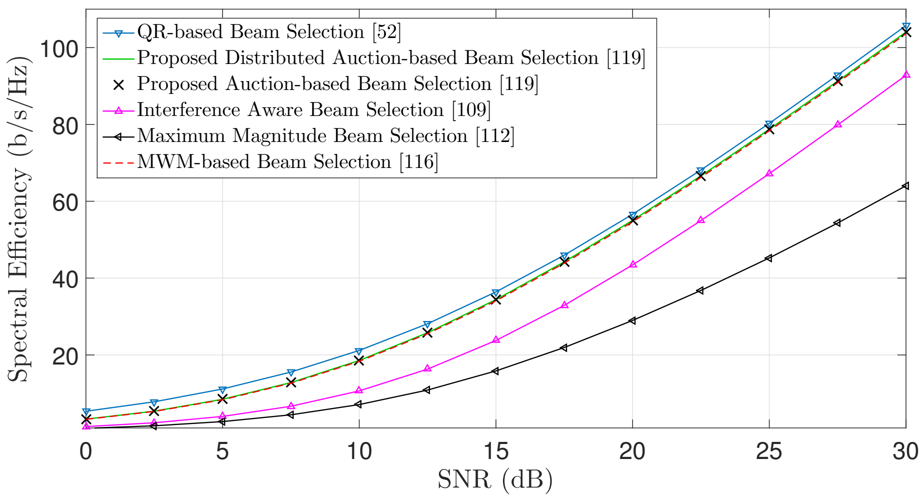

Figure 4.3 The sum rate performance against SNR (dB) for K=16 users

selection algorithms.

The downlink mmWave beamspace MU-MIMO communication system having a transmitter equipped with N=256 beams and varying number of users, i.e., $K = 16,\ 24$ users, where each user has a single receive antenna. The channel between the transmitter and user $k, k \in \{1, \ldots, K\}$, is considered to be having one LoS component with complex-valued path gain $\beta_k^{(0)} \sim \mathcal{CN}(0, 1)$ and two NLoS components with complex-valued path gain $\beta_k^{(1)} \sim \mathcal{CN}(0, 10^{-2})$, $\beta_k^{(2)} \sim \mathcal{CN}(0, 10^{-1})$. The complex-valued path gains β_k^{ℓ}, $\ell = 0, 1, 2$ are considered to be uncorrelated to each other. The spatial frequencies, θ_k^{ℓ}, $k = 0, 1, 2$, of user k, are uniformly distributed in the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ and independent of each other. We are considering line topology for the network formation. Further, for the proposed algorithm, the ψ_{kn} is the benefit gain by user k from beam n, which is captured by beamspace channel vector of user k as $|\mathbf{h}_{b,k}^H| \in \mathbb{R}^{N\times 1} = [|h_{b,1k}^*|, |h_{b,2k}^*|, \ldots, |h_{b,Nk}^*|]^H$, and $|\mathbf{h}_{b,k}^H|$ is comprising of non-complexed N elements.

Fig. 4.3 and 4.4 is a plot of the spectral efficiency for $K = 16$ and $K = 20$ users, respectively of beam selection through the auction and distributed auction algorithm against SNR in dB. We compared the performance with the “MM” [113], “IA” [110], “QR-based” [53], and “MWM-based” [117] beam selection algorithms. In the “MM” beam selection, a single beam can be selected by the multiple users.

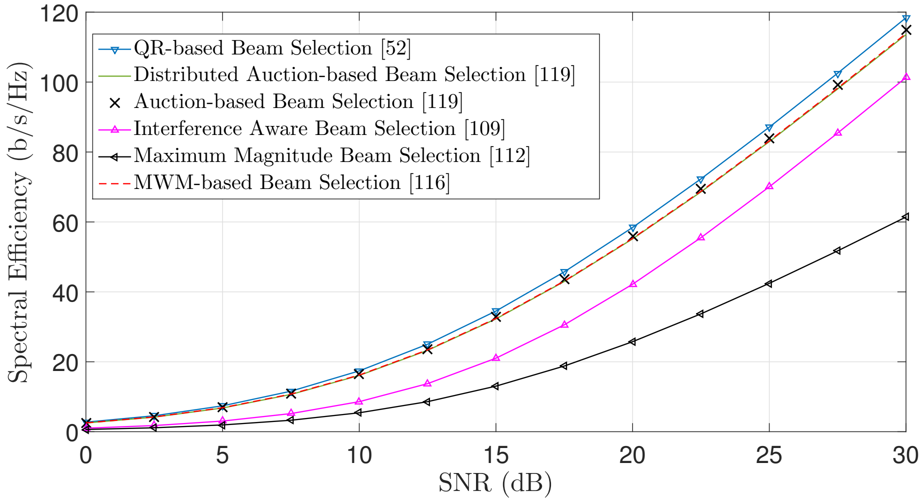

Figure 4.4 The sum rate performance against SNR (dB) for K=20 users

Thus, it degrades the system performance. On the other hand, "IA" beam selection selects the beams which provide the maximum rate while nullifying the interference. But, the interference is eliminated through a ZF precoding. As we discussed in Chapter 3, ZF degrades the performance when the beamspace channel is a ill-conditioned matrix and is not applicable to rank deficient matrix. Finally, the proposed algorithms; beam selection through the auction and the distributed algorithms outperforms existing "MM" [113], "IA" [110] beam selection algorithms. Further, the performance of the proposed algorithms is inferior to the "QR-based" beam selection [53] because it is not considering interference as in "QR-based." However, its performance is very close to "MWM-based" beam selection algorithm [117]. Note that, the performance of beam selection through auction and the distributed algorithm is almost the same, as shown in Fig. 4.3 and 4.4.

Complexity of the plotted beam selection algorithms in Fig. 4.3 and 4.4 is already discussed in sub-section 4.3.4. It is found that complexity of the beam selection through a distributed auction algorithm is very less than "QR-based" beam selection [53], and it is constant irrespective of changes in K. Thus, beam selection through a distributed auction algorithm holds comparably very low complexity with excellent performance.

4.4 **Concluding Remarks**

In this chapter, a mmWave beamspace MU-MIMO downlink communication system is considered. First, the need for decentralized beam selection algorithm is discussed in details. Further, this chapter discusses auction-based beam selection algorithm. Next, a variant of auction algorithm, i.e., distribution auction algorithm is discussed. In this context, a distributed auction-based decentralized beam selection algorithm is discussed. Eventually, the performance metric, i.e., spectral efficiency are discussed and compared with the other existing beam selection algorithms. In addition, complexity of the proposed beam selection algorithms are discussed, and compared with the existing beam selection algorithms. It has been observed that the proposed decentralized beam selection algorithm performs as good as "MWM-based" beam selection algorithm with very low complexity as decentralized beam selection algorithm distributes the computation among all the users and overall complexity is constant irrespective of the increasing number of users.