Appendix A

Gram Schmidt Orthogonormalization

It is possible to convert a linearly independent vectors $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ into an orthonormalization basis $\{\tilde{\mathbf{v}}_1, \tilde{\mathbf{v}}_2, \ldots, \tilde{\mathbf{v}}_k\}$ by performing the following steps of computations [128] [129]:

1. set $\mathbf{v}_1 = \mathbf{u}_1$
2. $\mathbf{v}_2 = \mathbf{u}_2 - \frac{\langle \mathbf{u}_2, \mathbf{v}_1 \rangle}{\|\mathbf{v}_2\|^1}\mathbf{v}_1$
3. $\mathbf{v}_3 = \mathbf{u}_3 - \frac{\langle \mathbf{u}_3, \mathbf{v}_1 \rangle}{\|\mathbf{v}_1\|^2}\mathbf{v}_1 - \frac{\langle \mathbf{u}_3, \mathbf{v}_2 \rangle}{\|\mathbf{v}_2\|^2 \rangle}\mathbf{v}_2$
4. $\ldots$

upto k steps

Then, to convert this orthogonal basis into an orthonormal basis $\{\tilde{\mathbf{v}}_1, \tilde{\mathbf{v}}_2, \ldots, \tilde{\mathbf{v}}_k\}$, normalize the orthogonal basis vectors.

1. $\tilde{\mathbf{v}}_1 = \frac{\mathbf{v}_1}{\|\mathbf{v}_1\|}$
2. $\tilde{\mathbf{v}}_2 = \frac{\mathbf{v}_2}{\|\mathbf{v}_2\|}$
3. $\ldots$

upto k steps

Note: the symbol $\langle , \rangle$ refers to the inner product between two vectors.