Appendix A

Proofs of Theorems

A.1 Proof of Theorems in Section 4.1

Proof. (Theorem 4.1) As discussed earlier, both context-aware diffusion models CLT and CIC are directly inherited from traditional models LT and IC described by Kempe et al. [11]. The propagation strategy in both context-aware models is the same as the traditional diffusion models. These models only take the user's interest (as topics) into consideration for improving the quality of seed. From **Definition 4.1.1**, it is clear that if the number of the topic of interest is considered to one, i.e., $t = 1$ then C2IM problem reduces to original IM problem formulated by [11]. Therefore, monotone increasing and diminishing return properties directly inherited from the traditional diffusion models. Hence, it can be stated that the expected influence spread $\sigma(S|Q)$ remains submodular under CLT and CIC models. □

Proof. (Theorem 4.2) The C2IM problem can be reduced into conventional IM problem proposed by [11] considering the number of the topic of interest is one, i.e., $t = 1$ with the whole network as a single community. As we know conventional IM problem is NP-hard proved by [11]. Therefore, it can be said that C2IM problem is NP-hard under LT and IC. □

A.2 Proof of Theorems in Section 5.1

Proof. (Theorem 5.1) The MIM2 algorithm incorporated traditional diffusion models [11] and utilized the same propagation strategy as these models. MIM2 considers multiple products and multiple networks simultaneously to improve the effectiveness of seed nodes. From problem definition, it is clear that if MIM2 consider $l = 1$ and $m = 1$, then MIM2 problem reduces to original IM problem formulated by [11]. Therefore, diminishing return and monotone increasing properties directly inherited from LT and IC diffusion models. Hence, we can conclude that the expected influence spread $\sigma(S)$ of MIM2 remains sub-modular under traditional diffusion models. □

Proof. (Theorem 5.2) As we discussed earlier, the MIM2 problem can be reduced in conventional IM problem presented in [11] by considering the number of networks $l = 1$ and the number of products $m = 1$. As we know, IM problem is NP-hard under traditional diffusion models proved by [11]. Hence, we can say that MIM2 is NP-hard under traditional diffusion models. □