Appendix A

Conservation Equations for 2D Axisymmetric Swirl model

The Conservation of Mass (Continuity Equation)

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial z}\left(\rho U_z\right)+\frac{\partial}{\partial r}\left(\rho U_r\right)+\frac{\rho U_r}{r}=\dot{S} \tag{A1}$$

The Momentum Conservation Equation

z-component (axial velocity)

$$\begin{aligned}\frac{\partial}{\partial t}\left(\rho U_z\right)+\frac{1}{r}\frac{\partial}{\partial z}\left(r\rho U_z U_z\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho U_r U_z\right)=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial z}\left[r\mu_{eff}\left(2\frac{\partial U_z}{\partial z}-\frac{2}{3}\left(\nabla.\vec{V}\right)\right)\right]\\+\frac{1}{r}\frac{\partial}{\partial r}\left[r\mu_{eff}\left(\frac{\partial U_z}{\partial r}+\frac{\partial U_r}{\partial z}\right)\right]+F_z+S_{M_z}+\dot{S}U_z\end{aligned} \tag{A2}$$

r-component (radial velocity)

$$\begin{aligned}\frac{\partial}{\partial t}\left(\rho U_r\right)+\frac{1}{r}\frac{\partial}{\partial z}\left(r\rho U_z U_r\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho U_r U_r\right)=-\frac{\partial p}{\partial r}+\frac{1}{r}\frac{\partial}{\partial z}\left[r\mu_{eff}\left(\frac{\partial U_r}{\partial z}+\frac{\partial U_z}{\partial r}\right)\right]+\\\frac{1}{r}\frac{\partial}{\partial r}\left[r\mu_{eff}\left(2\frac{\partial U_r}{\partial r}-\frac{2}{3}\left(\nabla.\vec{V}\right)\right)\right]-2\mu_{eff}\frac{U_r}{r^2}+\frac{2}{3}\frac{\mu_{eff}}{r}\left(\nabla.\vec{V}\right)+\rho\frac{U_\theta^2}{r}+F_r+S_{M_r}+\dot{S}U_r\end{aligned} \tag{A3}$$

θ-component (tangential velocity)

$$\begin{aligned}\frac{\partial}{\partial t}\left(\rho U_\theta\right)+\frac{1}{r}\frac{\partial}{\partial z}\left(r\rho U_z U_\theta\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho U_r U_\theta\right)=\frac{1}{r}\frac{\partial}{\partial z}\left[r\mu_{eff}\frac{\partial U_\theta}{\partial z}\right]+\\\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^3\mu_{eff}\frac{\partial}{\partial r}\left(\frac{U_\theta}{r}\right)\right]-\rho\frac{U_r U_\theta}{r}+F_\theta+S_{m_\theta}+\dot{S}U_\theta\end{aligned} \tag{A4}$$

Energy Conservation Equation

$$\frac{\partial(\rho h)}{\partial t}+\frac{\partial(\rho h U_r)}{\partial r}+\frac{\partial(\rho h U_z)}{\partial z}=\frac{\partial p}{\partial t}-\frac{\partial q_j}{\partial(z,r)}+Q_r-\dot{S}_E+\dot{S}\,h \qquad (A5)$$

$$q_j=\frac{c_p}{k}\left[-\frac{\partial h}{\partial(z,r)}+\left(1-\frac{1}{Le}\right)\sum_i^N h_i\frac{\partial Y_i}{\partial(z,r)}\right] \qquad (A6)$$

$$Le=\frac{\alpha}{D} \qquad (A7)$$

Species Conservation Equation

$$\frac{\partial(\rho Y_i)}{\partial t}+\frac{\partial(\rho Y_i U_r)}{\partial r}+\frac{\partial(\rho Y_i U_z)}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho D\frac{\partial Y_i}{\partial r}\right)+\frac{\partial}{\partial z}\left(\rho D\frac{\partial Y_i}{\partial z}\right)+R_i+\dot{S}_i+\dot{S}\,Y_i \qquad (A8)$$

Turbulent Kinetic Energy (k)

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho k U_r)}{\partial r}+\frac{\partial(\rho k U_z)}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial r}\right]+\frac{\partial}{\partial z}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial z}\right]+G_k+G_b-\rho\varepsilon-Y_M+\dot{S}\,k \qquad (A9)$$

Turbulent Kinetic Energy Dissipation Rate (ε)

$$\frac{\partial(\rho\varepsilon)}{\partial t}+\frac{\partial(\rho\varepsilon U_r)}{\partial r}+\frac{\partial(\rho\varepsilon U_z)}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial r}\right]+\frac{\partial}{\partial z}\left[\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial z}\right]+C_{1\varepsilon}\frac{\varepsilon}{k}\left(G_k+C_{3\varepsilon}G_b\right)-\rho C_{2\varepsilon}\frac{\varepsilon^2}{k}+\dot{S}\,\varepsilon \qquad (A10)$$

Where U_z, U_r and U_θ represent axial velocity, radial velocity and tangential velocity, respectively.