Chapter 5

MINIMISATION OF DISTRIBUTION COST USING CENTRE OF GRAVITY (CG) AND VEHICLE ROUTING PROBLEM (VRP) APPROACH

To reduce the distribution cost of milk, this chapter deals with formulation of an efficient vehicle routing problem (VRP). The work is done with existing routes and existing demand of co-operative dairy with the help of the centre of gravity and mathematical formulation for vehicle routing problem (VRP). The primary objective of this chapter is minimisation of the distribution cost with real world constraints.

5.1 Problem Definition

The co-operative dairy under study is experiencing gradual decline in sales. In year 2010 the sales were approximately 20,000 litres/day but 2015 the sales are approximately 10,000 per day only and the dairy has approximately 10% of the market value. 21% of the market is captured by the leading competitor of co-operative dairy and remaining is captured by some new entrants. The market survey, reported in Chapter 3, indicated some of the critical factors through which the co-operative dairy can sustain itself in the market. These are:

1. Market Planning
2. Minimisation of Distribution Cost & Time
3. Product Expansion

While, doing the research for the dairy, the distribution cost was found to be a major factor for low profits. Therefore, the focus is on minimisation of the distribution cost within the optimal time window, which should not exceed four hours.

In the current scenario, for milk distribution, the co-operative dairy hires and pays the fleet of vehicles according to number of litres of milk distributed by vehicle. The co-operative dairy pays to the vehicle driver, one helper and vehicle itself at the rate of INR 1.3 per litre.

The co-operative dairy currently has seven routes on which seven trucks of same capacity are running. Table 5.1 shows the data for existing routes and the requirement of milk on each route and cost incurred per day. The route no.1 has been discontinued due to lack of demand.

Table 5.1 Existing Routes, Milk Distributed and Cost for Each Route

Route	Requirement (Litre/Day)	Cost incurred (INR/Day)
No.-1	2629	3417.7
No.-2	931	1210.3
No.-3	1250	1625
No.-4	563	731.9
No.-5	1438	1869.4
No.-6	1896	2464.8
No.-7	1584	2059.2
Total	**10291**	**13378.3**

VRP is used to stablish a stockist in the route. These stockists are located on calculated centre of gravity (CG) points of existing routes of distribution of the co-operative dairy. The centre of gravity is calculated based on the demand and distance of the retailer located on existing route. The process of minimisation of distribution cost for milk is considered in to two stages. In first stage, the centre of gravity point for each route is identified, these centre of gravity points are made stockist to distribute the milk to other small retailers.

In second stage, after the stockist have been identified, the mathematical formulation was run with the data set to find the optimal route and number of vehicles to serve the seven stockists. After identifying the route for the seven stockists the mathematical formulation was used to reduce the number of vehicles for serving these points with maximum capacity of 2700 litres. For sub routes same mathematical modelling was rerun and for sub-routes the smaller capacity vehicles (500 litres) were used.

5.2 Mathematical Formulation

The centre of gravity / stockists' points was calculated by the following formula:

$$\mathrm{CG(x)} = \frac{\sum_{i=1}^{N} D(i)*x(i)}{\sum_{i=1}^{N} D(i)} \qquad \text{… Equation 5.1}$$

$$\mathrm{CG(y)} = \frac{\sum_{i=1}^{N} D(i)*y(i)}{\sum_{i=1}^{N} D(i)} \qquad \text{… Equation 5.2}$$

Where,

CG = centre of gravity

N = number of nodes in a route

D(i) = demand at node i

x(i) = x-coordinate of node i

y(i) = y-coordinate of node i

The mathematical model of VRP is explained in the following sub-section.

5.2.1 Indices

i, j, k   nodes for the vehicle route. Node 1 represents the depot whereas the remaining nodes corresponds to customers.

L   Number of delivery men that can be assigned to a single vehicle. Here, L = 1 is taken.

5.2.2 Parameters

N   Total number of nodes (the depot is represented by node i = 1 and customers by i = 2,3… N)

Q   Capacity of a vehicle.

C_L Cost of a deliveryman per day (Fixed cost)

C_d Cost of covering unit distance per day

D_j Delivery requirement of the customer at node j; j = 2,3…N

E_j Earliest allowed start time of service for the customer at node j

(j = 2,3...N)

L_j Latest allowed start time of service for the customer at node j;

(j = 2,3… N)

st_j Service time for the customer at node j; j = 2,3…N

d_{ij} Distance between node i and j (where node i, j = 1 denotes depot);

(i, j = 2,3…. N denotes customers) and $d_{ii}= d_{jj} = 0$.

t_{ij} Travel time from node i to j; i, j = 2,3…N and $T_{ii} = T_{jj} = 0$.

5.2.3 Decision Variables

Y_j Demand delivered up to node j; j = 1, 2…N.

t_j Service start time for the customer at node j; j = 2,3…N.

$$Xij = \begin{cases} 1, & if\ vehcile\ travels\ from\ node\ i\ to\ j \\ 0, & otherwise \end{cases}$$

The problem formulation can be expressed as follows.

Minimize Z = C_d*TD + L*C_L*TV Equation... 5.3

Where,

TV (Total number of Vehicle) = $\sum_{j=2}^{N} X_{1j}$ Equation... 5.4

TD (Total Distance) = $\sum_{i=1}^{N} \sum_{j=1}^{N} d_{ij} X_{ij}$ Equation... 5.5

After substituting TV and TD in the above equation 5.3, the final objective function is shown below in equation 5.6.

5.2.4 Mathematical Model

Minimize Z = $C_d * \sum_{i=1}^{N} \sum_{j=1}^{N} d_{ij} * X_{ij} + L * C_L * \sum_{j=2}^{N} X_{1j}$ Equation…5.6

Subject to:

$\sum_{\substack{i=1 \\ i \neq j}}^{N} X_{ij} = 1, (j = 2,3 \ldots N)$ Equation...5.7

$\sum_{\substack{i=1 \\ i \neq j}}^{N} X_{ij} = \sum_{\substack{k=1 \\ k \neq j}}^{N} X_{jk}, (j = 2,3 \ldots N)$ Equation…5.8

$t_j \geq (t_i+st_i+ t_{ij})X_{ij}$, $(i = 2,3...N, j = 1,2,3…N, j \neq i)$ Equation …5.9

$t_j \geq (t_{1j}) X_{1j}$, $(j = 2,3…N)$ Equation…5.10

$E_i \leq t_i \leq L_i$, $(i = 2,3…N)$ Equation…5.11

$\sum_{j=2}^{N} X_{1j} \geq [\sum_{i=2}^{N}(D_i/Q) + 0.99]$ Equation…5.12

$Y_j \geq Y_i + D_j – Q + Q (X_{ij} + X_{ji}) – \{(D_j+ D_i)X_j\}\dot{n}$ Equation…5.13

$X_{ii} = 0$ $(i = 1,2…N)$ Equation…5.14

The objective function equation 5.6 has two terms. The first term stands for vehicle movement cost in visiting various nodes in a route trip and the second term represents the cost of delivery men on a particular existing route. Constraint 5.7 ensures that every node is visited by exactly one vehicle. Constraint 5.8 is to make sure that a vehicle reaching a node is coming from only one other node and must go to some other node in the same mode. It basically takes care of flow conservation, meaning that a vehicle arriving at a node must leave it. Constraint 5.9 not only relates X_{ij} and st_i but also ensures that the vehicle arriving on time at a node j. Constraint 5.10 helps to determine the vehicle arriving time for nodes that are visited by the vehicle just after leaving the depot. Constraint 5.11 ensures that service should begin at the nodes in their specified time windows. Constraint 5.12 ensures that enough numbers of vehicles are leaving the depot (in a form of binary variable). If a vehicle is travelling from i to j then we can bound the

difference between the amount delivered up to node j (Y_j) and node i (Y_i) which can be done using constraint 5.13. Constraint 5.14 defines that no vehicle can travel inside a city. The mathematical modelling has been performed in the platform named LINGO **(Appendix E)**.

5.3 Methodology

The methodology to address the cost reduction of distribution of milk is given below:

***Step1*:** The calculation of CG/Bulk-breaking/stockiest points is obtained by using equation 5.1 and 5.2. The CG's (Centre of Gravity) are also called stockist points. Through equation 5.1 and 5.2 total seven stockiest were found, one stockiest for each route. The distance matrix along with the demand for stockists is shown below in Table 5.2.

Table 5.2: Distance-Demand Matrix for Stockist in Kilometres

From/to	Factory	stockist 2	stockist 3	stockist 4	stockist 5	stockist 6	stockist 7	stockist 8	Demand
Factory	0	18.6	14.1	13.2	14.8	15.1	9.9	15	0
stockist 2	18.6	0	1.7	3.1	4.6	7.5	6	3.8	2629
stockist 3	14.1	1.7	0	1.7	2.7	5.8	4.6	5.5	931
stockist 4	13.2	3.1	1.7	0	2	6.1	3.7	6	1250
stockist 5	14.8	4.6	2.7	2	0	5.6	1.8	7.4	563
stockist 6	15.1	7.5	5.8	6.1	5.6	0	5.1	11.1	1438
stockist 7	9.9	6	4.6	3.7	1.8	5.1	0	8.7	1896
stockist 8	15	3.8	5.5	6	7.4	11.1	8.7	0	1584

***Step2*:** By running the mathematical model described by equation 5.3 to 5.14, the seven stockiest were reduced to four clusters, with deployment of one vehicle for each cluster. In this case the cost of vehicle is taken as INR 25 per KM. The cost details are given in Table 5.3.

Table 5.3: The Cost Component for the Model

Type of Vehicle	Description	Labour Charge or fixed cost (per delivery man)	Transportation Charge (per KM)
Higher Capacity Vehicle (2700 litres)	Transport Milk from Depot to Stockist	300	25 INR
Small Capacity Vehicle (500 litres)	Transport Milk from Stockist to small Retailer	100	20 INR

***Step3*:** The model was rerun to obtain the optimal distribution sub-routes from the stockiest to small retailer, which is identified in step 1, with smaller capacity vehicles of 500 litres for which the cost details are given above in Table 5.3. The distance matrix for one such sub-route (from stockist 2 to its nearby retailers) is given below in Table 5.4.

Table 5.4: Distance Matrix for Sub Route (From Stockist 2 to Its Nearby Retailers)

From/to	stockist **(1)**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Demand**
stockist **(1)**	0	6.2	2.3	2.4	1.5	0.7	3	2.4	0
2	6.2	0	6.1	7.1	6.3	5.6	2.9	5.6	157
3	2.3	6.1	0	1	1	1.7	4.1	3.5	153
4	2.4	7.1	1	0	.85	2.2	4.5	4	116
5	1.5	6.3	1	.85	0	1.3	3.6	3.1	496
6	.7	5.6	1.7	2.2	1.3	0	2.4	1.8	486
7	3	2.9	4.1	4.5	3.6	2.4	0	1.8	148
8	2.4	5.6	3.5	4	3.1	1.8	1.8	0	120

Step4: Each stockist will be provided with storage capacity (deep freezer), incurring fixed cost or EMI, this cost is borne by the Co-operative dairy. By providing the storage capacity will be helpful in reducing the milk supply from two shifts to one shift.

5.4 Results

The results obtained from the calculation of CG/ stockists is shown in Figure 5.1.

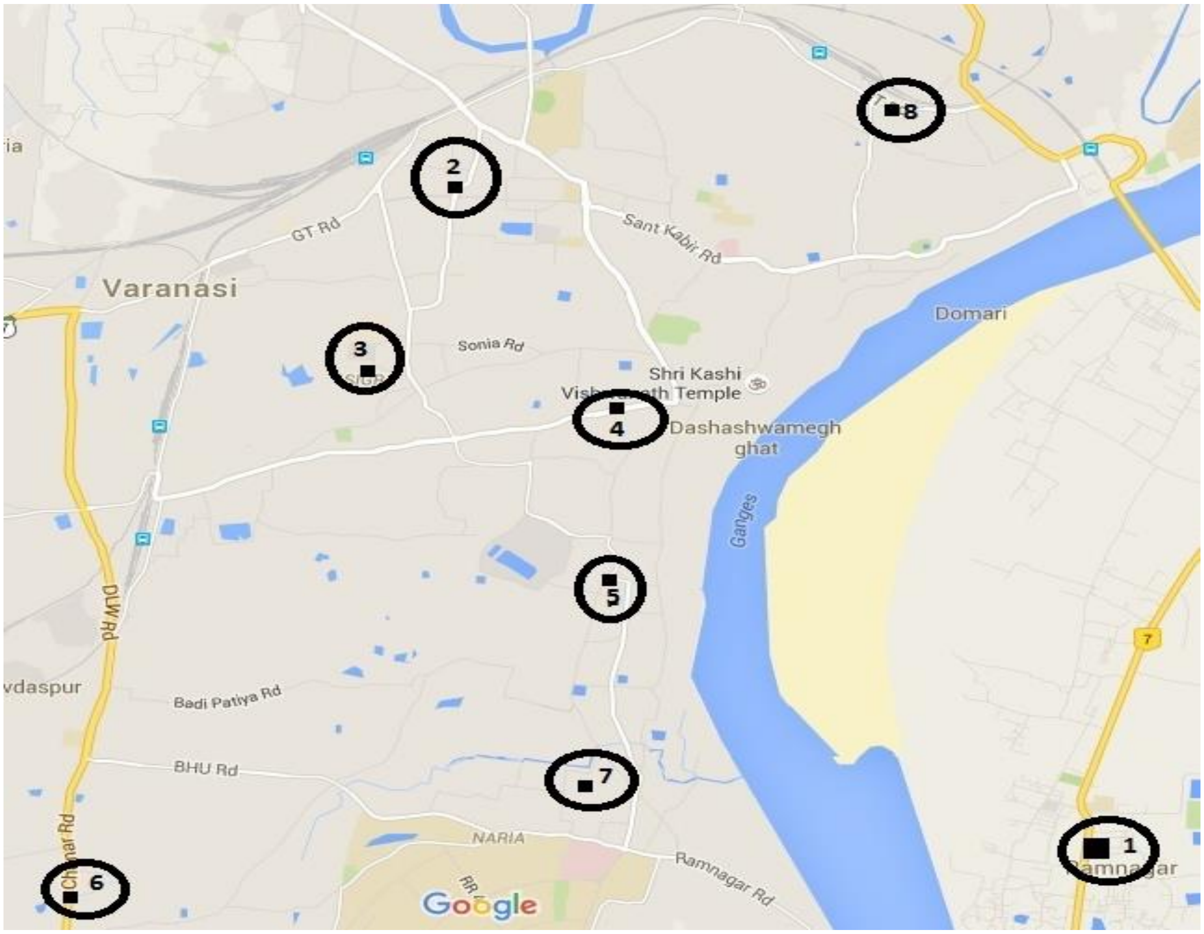

Figure 5.1 Graphical Representation of Cluster Formation of Centre of Gravity/ Stockists

Now, from step 2 (Section 5.3), running the mathematical model for the vehicles of higher capacity (2700 litres) gave the four clusters to be served by the four independent vehicles instead of seven, used by the co-operative dairy. In the next page, the result given by the model is shown in the Table 5.5.

Table 5.5 Cluster Formation of Centre of Gravity/ Stockists

No. of Trucks = 4	Total cost = INR 4517.7	Total Distance Travelled = 132.7 KM
Optimised routes:		
1 → 3 → 8 → 1 (Route 1)		
1 → 4 → 6 → 1 (Route 2)	1 Factory or main dairy	
1 → 2 → 1 (Route 3)		
1 → 5 → 7 → 1 (Route 4)	2 to 8 are stockists	

Figure 5.2 shows the graphical representation of the results given in Table 5.4.

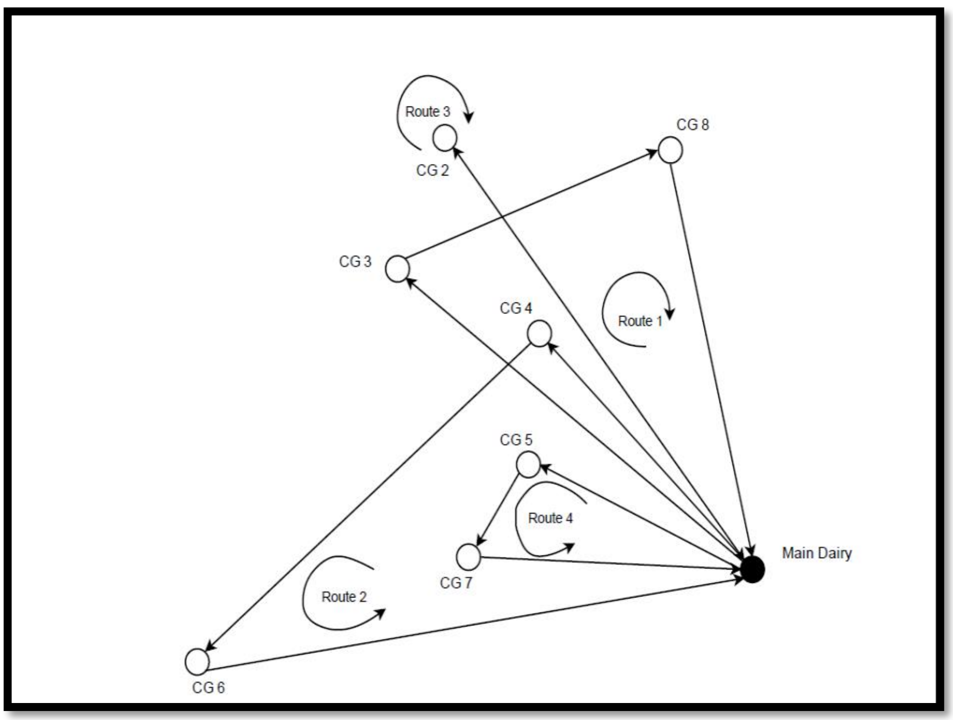

Figure 5.2 Vehicle Route Clusters for the Main Routes with Seven Centres of Gravity/Stockist

The cost of distribution calculated from the mathematical model is INR 4517.5 per day. After the stockist clusters and routing the sub-route clustering was obtained for each stockiest point.

The distance matrix for the sub-route (from Stockist Point (SP) 2 to nearby retailers) is given in Table 5.4.

The solution suggested by the model for the sub-route (from Stockist Point (SP) 2 to nearby retailers) is shown in Figure 5.5.

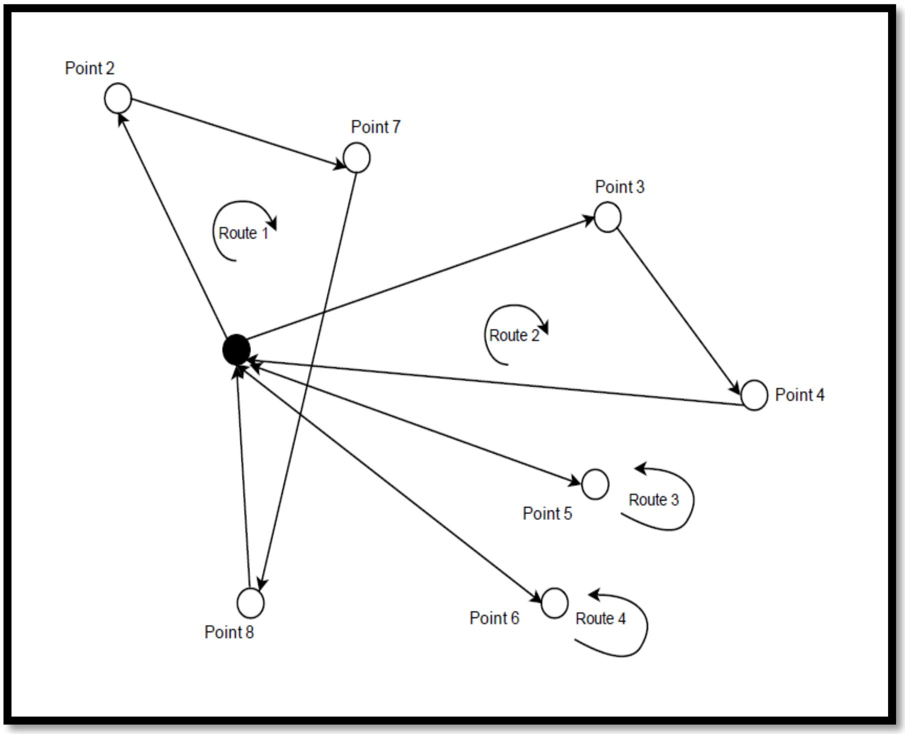

Figure 5.3: Solution Route for Sub-Route (From Stockist Point (SP)/CG 2 to Nearby Retailers)

The cost of this sub-route (from SP/CG 2 to nearby retailers) is INR 868 per day. The cost calculation for each stockist to their nearby retailers is given below in Table 5.6.

Table 5.6: Calculated Cost for Each Sub Routes from Respective Stockiest to their Nearby Retailers

Stockiest point	2	3	4	5	6	7	8	Total Cost
Cost (INR)	868.00	902.00	1006.00	828.00	952.00	814.00	1034.00	6064.00

The gross total for the transportation cost is calculated as INR 10921 per day. After addition the cost as well as the EMI (approximately INR 170 per day), the total distribution cost is INR 11091.5 per day. The original cost of the distribution was INR 13378.3 per day and this model gives a cost saving of INR 2286.8 per day. The saving is 17.09 % of the original distribution cost.

5.5 Conclusions

This chapter addressed a study of a co-operative dairy situated in, Varanasi. In Section 5.2 the VRP model was used with the aim of reducing the distribution cost of the co-operative dairy developed with the constraints like a time window, a vehicle capacity and demand matching followed a two-step procedure. First, stockist points were identified. Then a suitable clustering was obtained by the VRP model. In the original policy the company used seven vehicles for each route, which are now replaced by four vehicles to serve the four clusters of stockists, one route per cluster. After identifying the stockist, the optimal sub routes for serving the nearby retailers were also found. In addition to this, a one-time storage capacity was provided to these stockists by the co-operative dairy to cut down the number of trips from two to one. The original distribution cost of the co-operative dairy was calculated and compared with the cost obtained for optimised clusters, giving a substantial saving of 17.09% per day for the co-operative dairy.

The work in this chapter is done using existing routes of the co-operative dairy. Therefore, in the next chapter the new routes are identified to find out if this new approach can reduce the distribution costs further.

CHAPTER 6

MINIMISATION OF DISTRIBUTION COST USING K-MEANS AND CHEAPEST LINK ALGORITHM

The work of this chapter is done to explore if the new routes can reduce the distribution cost further. To identify the new routes the k-means clustering, and Cheapest Link Algorithm are used. Here, k-means clustering is clustering method and Cheapest Link Algorithm is applied for Vehicle Routing Problem method.

6.1 Methodology and Algorithm

In this part the Capacitated K-means Clustering was used to split delivery locations into similar size groups (i.e. clusters) based on proximity without exceeding a specified total cluster capacity. Each cluster was to be served by a local stockist. The CLA was then used to find delivery routes from dairy (i.e. depot) to stockist in each cluster and from stockist to all other delivery locations within the cluster.

The work is done in two phases. These phases are described as follows:

a) Clustering: To divide the problem into smaller parts, the service locations were grouped into clusters (Shieh and May, 2001). Delivery locations were grouped by their closeness to each other. Instead of solving the CVRP for all the delivery locations together, we solved the CVRP for each cluster separately, thus reducing complexity and increasing the optimality of our heuristic solution.