**CHAPTER 3**

# MAGNETIC FIELD TAPERING FOR THE GAIN AND EFFICIENCY OF THE FEL AMPLIFIERS



*Part of this work has been published as:

**Ram Gopal** and P. K. Jain, "Effect of magnetic field tapering on whistler-pumped FEL amplifiers in collective Raman regime operation", *International Journal of Engineering & Technology*, Vol. 7, No. 4, pp. 2044-2050, 2018.

# MAGNETIC FIELD TAPERING FOR THE GAIN AND EFFICIENCY OF THE FEL AMPLIFIERS

## 3.1. Introduction

The first relativistic beam experiments demonstrating stimulated scattering in the Raman regime were performed by Granatstein in 1976 [Granatstein *et al.* (1976), Granatstein *et al.* (1977)]. Utilizing intense relativistic electron beam generators, super radiant FEL oscillators were demonstrated by producing megawatt power levels in short interaction regions $\sim 30cm$ at wavelengths ranging from $2mm$ to $400\mu m$ and with efficiencies as high as $0.1\%$. More recently, McDermott reported the realization of a collective Raman FEL for the first time. The experiment was designed so as to permit several passes of feedback by employing a quasioptical cavity. Laser output of $1MW$ at $400\mu m$ and line narrowing to $\Delta\omega/\omega \approx 2\%$ were observed, compared to $\Delta\omega/\omega \approx 10\%$ for the earlier super radiant oscillator studies [McDermott (1978), Marshall and Schlesinger (1978), Walsh (1980), Roberson and Sprangle (1989), Tripathi and Liu (1989), Pellegrini (1990), Dattoli and Renieri (1993), Liu and Tripathi (1994), Oerle and Mathias (1997), Saldin *et al.* (1999), Shea and Freund (2001), Workie (2001)]. Danley proposed the scheme of gyrotron pumped free electron laser to produce FELs radiation at optical frequencies and followed extensive efforts to employ high power laser to produce much shorter wavelengths, including X-rays [Danly *et al.* (1987)]. FELs radiation is produced as stimulated Compton back scattering of wiggler electromagnetic wave (laser) by the relativistic electron beam [Scharlemann

(1986), Sprangle *et al.* (1979)]. The process can be viewed as a nonlinear coupling between the FELs radiation and negative energy beam mode. The wiggler provides phase synchronism for the process and stimulates it.

Freund and Ganguly have introduced a Nonlinear Simulation of a High-Power, using a uniform wiggler while tapered wiggler experiment achieved $35\%$ efficiency for same frequency using an $3.5MeV/850A$ electron beam in conjunction with a planar wiggler with tapered amplitude and explain the high-efficiency operation of FEL amplifiers [Ganguly and Freund (1988), Freund and Ganguly (1992)]. In 1993, Chung proposed the simulation technique of tapered FEL amplifiers in millimetre and infrared regions and developed the nonlinear theory and numerical simulation with the tapered wiggler and the axial guide magnetic field. They have also investigated the possibility of a tunable IR-FEL with a test linac of energy $20-60MeV$ for the feasible to amplify a $14kW$ signal of $10.6\mu m$ radiation to a $2GW$ level [Chung *et al.* (1993)]. Sharma and Tripathi is examined the feasibility of the device in the low-current Compton regime, practically, in the Compton regime, the whistler wave does not appear to be a suitable wiggler for FELs operation due to requirements of extremely high pump power density for reasonable growth rate [Sharma and Tripathi (1993)], hence including the effect of finite space charged mode, Raman Regime operation plays an important role in whistler-pumped FELs only [Pant and Tripathi (1994)]. Orzechowski examined the operation of FEL amplifiers at $35GHz$ with a peak output power of $180MW$ and powered by $3.6MeV/850A$ of electron beams and find out an extraction efficiency of $6\%$ with operating bandwidth of approximately $10\%$, while the amplifier saturates at a $1.4m$ wiggler lengths [Orzechowski *et al.* (1986)]. Gardelle and

Parker emphasized the effects of electron beam quality and space charge on FELs efficiency, a very good agreement has been found between experimental results and simulations. The collective FELs laser interaction has been studied at millimeter wavelengths and the Measurements in a super radiant amplifier configuration indicate the production of $35MW$ at $4mm$ with an efficiency of $2.5\%$ [Parker *et al*. (1982), Gardelle *et al*. (1994)]. Lefevre and their groups are also examined the measurements of microwave power and frequency in a pulsed-FEL amplifiers (P-FELA) [Lefevre *et al.* (1997)]. Baxevanis gives the general method for analyzing three-dimensional effects in free electron laser amplifiers [Baxevanis *et al.* (2013)]. The theory of longitudinal dynamics of high gain free electron laser amplifiers was proposed by Dattoli [Dattoli *et al.* (2013)]. Experimentally, Elias demonstrated the amplification of a $10.6\mu m$ laser beam and since then the name-FELs and the gain has been observed by $7\%$ per pass at an electron beam current of $70mA$, the experiments indicate the possibility of a new class of tunable high-power free electron lasers [Elias *et al*. (1976)]. FELs oscillator and amplifiers have been operated above threshold at a wavelength of $3.4\mu m$, experiments demonstrating wave-particle stimulated scattering with an output in the infrared have been performed at Stanford University using a low current, high energy electron beam from a linear accelerator [Deacon *et al.* (1977)], observed efficiencies were less than $0.01\%$ and attempts to improve the efficiency have focused on the use of storage rings to continuously recirculate the beam through the wave generation region.

Apart from the Ponderomotive potential, the self-consistent of free space charge potential $\Phi \approx e^{-i(\omega t - kz)}$ is also experienced on electrons to the high relativistic beam current as $I_b \geq 40kA$ [Pant and Tripathi (1994), Tripathi (2013)]. Since the beam current is very high, the susceptibility $(\chi_b)$ is greater than unity to the medium, i.e., $(\chi_b >> 1)$, hence, therefore the self-consistent of free space charge potential $(\Phi)$ is considered comparatively as Ponderomotive potential $(\Phi_{pb})$ ,i.e., $(\Phi >> \Phi_{pb})$ [Pant and Tripathi (1994), Tripathi (2013)].

This chapter of the thesis is organized as follows. In Section 3.2, Raman regime operation for linear tapering of the magnetic fields to the FEL amplifiers deals with an interaction of FEL amplifiers, dispersion curve and growth rate and their formalism analysis are discussed. Section 3.3 deals the FEL amplifiers gain and efficiency with phase and momentum, trapping of electrons and the gain function and efficiency in Raman regime operation. Further, the analytical results with the reported analytical values and their results are discussed in Section 3.4. The relevant conclusions are drawn in Section 3.5.

## 3.2. Raman Regime Operation

Raman regime is mode for the free space charge wave i.e., the self-consistent of free space charge potential is experienced a force, hence, feeding more and more negative energy to the FEL amplifiers. An interaction behavior, dispersive in nature and growth rate of the FEL amplifiers in Raman Regime mode is separately described in brief.

### 3.2.1. Interaction region of FEL amplifiers

In a free electron laser, consider the interaction region $0 < z < L$ and it comprises an electron plasma density $n_{op}^{o}$ immersed into a static magnetic field $B_s(\hat{z})$ at $B_s = B_{os}(1 - z/L)$, , therefore, it is written as [Pant and Tripathi (1994)],

$$\vec{B}_o = B_o(\hat{x} + i\hat{y})e^{ik_o dz}. \tag{3.1}$$

If the whistler wave propagates through the plasma along $-\hat{z}$ direction, then an electric field is as [Pant and Tripathi (1994), Tripathi (2013)],

$$\vec{E}_o = A_o(\hat{x}+i\hat{y})e^{-i(\omega_o t+\int k_o dz)}, \quad (3.2)$$

Where, $k_o = \frac{\omega_o}{c}\varepsilon^{1/2}$ $\varepsilon = [1-\frac{\omega_p{}^2}{\omega_o(\omega_o-\omega_c)}]$ and $\omega_c = \omega_{co}(1-\frac{z}{L})$, $\omega_{co} = \frac{eB_{os}}{mc}$, $\omega_p = (\frac{n_{op}^o e^2}{\varepsilon_o m})^{1/2}$. Here $\omega_c$ and $\omega_{co}$ is frequency of electron cyclotron and initial cyclotron frequency, $\omega_o$ is wiggler frequency and the wiggler wave number $k_o$, $\omega_p$ is plasma frequency and $n_{op}^o$ is density of the electron plasma, $-e$ and $m$ are the electronic charge and rest mass, $L$ is the length of interaction and $c$ is the speed of light in vacuum, $\varepsilon_o$ and $\varepsilon$ are free space and relative permittivity respectively.

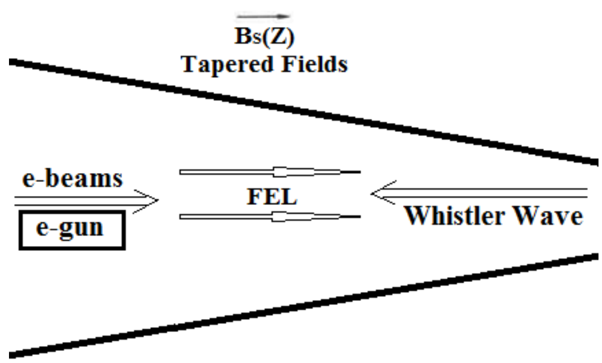


**Figure 3.1:** Beat-wave excitation as FEL amplifiers (FELA), $0 < z < L$.

A relativistic density of an electron beam $n_{ob}^{o}$ and relativistic velocity $v_{ob}^{o}\hat{z}$ propagates through the plasma with a given interaction region $0 < z < L$ as Fig. 3.1. The wiggler field appears as an electromagnetic wave frequency $\gamma_{ob}^{o} k_o v_{ob}^{o}$ and the guided axial magnetic field gives to the electrons cyclotron motion at cyclotron frequency $\omega_c$. It acquires an oscillatory velocity in transverse direction due to the whistler wave. Therefore, the relativistic equation of motion one obtains [Pant and Tripathi (1994), Tripathi (2013)],

$$m[\frac{\partial \gamma_{ob}^{o} \vec{v}_{ob}}{\partial t} + \vec{v}_{ob} \nabla.(\gamma_{ob}^{o} \vec{v}_{ob})] = -e\vec{E}_{ob} - e(\vec{v}_{ob} \times \vec{B}_o). \tag{3.3}$$

On solving the relativistic equation (3.3) and realizing by $\frac{\partial}{\partial t} = -i\omega$ and $\nabla = ik$, we have,

$$-i(\omega_o \gamma_{ob}^{o} \vec{v}_{ob}) + \gamma_{ob}^{o} (\vec{v}_{ob} \times \vec{k}_o \times \vec{v}_{ob} - \vec{v}_{ob}(k_o v_{ob}^{o})) = -\frac{e}{m}[\vec{E}_{ob} + e(\vec{v}_{ob} \times \frac{\vec{k}_o \times \vec{E}_{ob}}{\omega_o})],$$

$$\Rightarrow \vec{v}_{ob} = \frac{e\vec{E}_o (1 + k_o v_{ob}^{o} / \omega_o)}{imc\gamma_{ob}^{o} (\omega_o + k_o v_{ob}^{o} - \omega_c / \gamma_{ob}^{o})}. \tag{3.4}$$

Here, $\vec{B}_o = \frac{k_o(-\hat{z} \times \vec{E}_o)}{\omega_o}$, $\vec{k}_o = -k_o \hat{z}$, $\vec{v}_{ob} = \frac{\omega_1}{k_1}$ and $\gamma_{ob}^{o} = [1 - (v_{ob}^{o} / c)^2]^{-1/2}$. Where, $\gamma_{ob}^{o}$ is relativistic gamma factor. The plasma electrons also acquire a drift velocity, hence the equation of motion and Maxwell's third equation, one obtains [Tripathi (2013)],

$$m[\frac{\partial \vec{v}_{ob}}{\partial t} + \vec{v}_{ob} \nabla.(\vec{v}_{ob})] = -e\vec{E}_{ob}, and, \nabla \times \vec{E}_{ob} = -\frac{\partial \vec{B}_o}{\partial t}, \tag{3.5}$$

Now linearization equation (3.5), the electron cyclotron velocity in x-direction is,

$$v_x = \frac{eE_x}{im(\omega_o - \omega_c)}, \tag{3.6}$$

Similarly the velocity for the electron cyclotron in $y$-direction is as [Tripathi (2013)],

$$v_y = \frac{eE_y}{im(\omega_o - \omega_c)}, \tag{3.7}$$

Assuming here, the convective term is equal to be zero and $v_y = iv_x, and, E_y = iE_x$. Therefore, the total velocity for the electron cyclotron is [Pant and Tripathi (1994)],

$$\vec{v}_o = \frac{e\vec{E}_o}{im(\omega_o - \omega_c)}. \tag{3.8}$$

Now for launch a circularly polarized amplifiers radiation through the length of $z = 0$ end with electric field [Pant and Tripathi (1994), Tripathi (2013)],

$$\vec{E}_1 = A_1(\hat{x} + i\hat{y})e^{-i(\omega_1 t - k_1 z)}. \tag{3.9}$$

Where the radiation wave number of the amplifiers, $k_1 = \omega_1 / c$ and frequency of radiation, $\omega_1 >> \omega_p, \omega_c$. It imparts oscillatory velocities to beam and plasma. If radiated electrons move with a whistler wave, then the phase matching condition at, $(\omega, k)$, $\omega = \omega_1 - \omega_o$ and $k = k_o + k_1$ to be satisfied as [Pant and Tripathi (1994), Tripathi (2013)],

$$v_{ob} = \frac{\omega}{k} = \frac{\omega_1 - \omega_o}{k_o + k_1}, \tag{3.10}$$

Now on solving the above equation (3.10) at $v_{ob} = c$, we have [Tripathi (2013)],

$$\omega_1 \approx 2\gamma_{ob}^{o\,2} k_o c. \tag{3.11}$$

Where $\gamma_{ob}^{o} = [1-(v_{ob}/c)^2]^{-1/2}$, relativistic gamma factor $(\gamma_{ob}^{o})$ of electron beam. It is an important parameter for the characteristics. If the radiation wavelength of FELs, $\lambda_1$ and the wiggler wavelength, $\lambda_o$ then the relativistic gamma factor $(\gamma_{ob}^{o})$ related as $\lambda_1 \cong \lambda_o / 2\gamma_{ob}^{o\,2}$.

The whistler wave and the FELs radiation wave gives oscillatory velocities $v_{1b}, and, v_{1p}$ to beam and plasma electrons, then using equation of motion, one obtain,

$$m[\frac{\partial \gamma_{ob}^{o} \vec{v}_{1b}}{\partial t} + \vec{v}_{1b}(\nabla.(\gamma_{ob}^{o}\vec{v}_{1b})] = -e\vec{E}_{1b} - e(\vec{v}_{1b} \times \vec{B}_o), \tag{3.12}$$

Initially no force impart on perturb part to the wave, therefore, realizing by $\frac{\partial}{\partial t} = -i\omega$ and $\nabla = ik$ and ignoring second terms on both side, the beam velocity of electrons is as [Pant and Tripathi (1994), Tripathi (2013)],

$$\vec{v}_{1b} = \frac{e\vec{E}_1}{im\gamma_{ob}^{o}\omega_1}, \tag{3.13}$$

Therefore, the velocity for the electron plasma is [Tripathi (2013)],

$$\vec{v}_{1p} = \frac{e\vec{E}_1}{im\omega_1}. \tag{3.14}$$

If the whistler wave and FELs signal impart a Ponderomotive force on beam electrons at $(\omega_1, k_1)$, assuming here, $\omega = \omega_1 - \omega_o$ and $k = |k_o| + k_1$, and $\vec{\omega}_1 >> \vec{\omega}_c$, therefore the total Ponderomotive force is as [Pant and Tripathi (1994), Tripathi (2013)],

$$\vec{F}_{pb} = e\nabla\Phi_{pb} = -\frac{e}{2c}\vec{v}_{ob}^{*}\times\vec{B}_1 - \frac{e}{2c}\vec{v}_{1b}\times\vec{B}_o^{*}, \tag{3.15}$$

Now from Maxwell's Third equations $\nabla\times\vec{E}_1 = -\frac{\partial\vec{B}_1}{\partial t}$, and, $\nabla\times\vec{E}_o = -\frac{\partial\vec{B}_o}{\partial t}$ and after realizing by $\frac{\partial}{\partial t} = -i\omega$ and $\nabla = ik$ , one obtains [Tripathi (2013)],

$$\vec{B}_o = -\frac{k_o}{\omega_o}\hat{z}\times\vec{E}_o = \frac{\vec{k}_o\times\vec{E}_o}{\omega_o}, and, \vec{B}_1 = \frac{k_1}{\omega_1}\hat{z}\times\vec{E}_1 = \frac{\vec{k}_1\times\vec{E}_1}{\omega_1}, \tag{3.16}$$

Now from expressions (3.15) and (3.16), we have [Tripathi (2013)],

$$\vec{F}_{pb} = -\frac{e}{2c}[\{\frac{e(\omega_o + k_o v_{ob}^o)}{im\gamma_{ob}^o\omega_1\omega_o(\omega_o + k_o v_{ob}^o - \omega_c/\gamma_{ob}^o)}\}\{\vec{E}_o^{*}\times(\vec{k}_1\times\vec{E}_1)\} + \frac{e}{im\omega_1\omega_o\gamma_{ob}^o}\{\vec{E}_1\times(\vec{k}_o\times\vec{E}_o^{*})\}]. \tag{3.17}$$

Again using equations (3.2), (3.9) and (3.17) and on solving, we have,

$$\vec{F}_{pb} = -(\frac{e^2 A_o^{*} A_1}{imc\omega_1\omega_o\gamma_{ob}^o})[k_o + \frac{k_1(\omega_o + k_o v_{ob}^o)}{(\omega_o + k_o v_{ob}^o - \omega_c/\gamma_{ob}^o)}]\hat{z}e^{-i\psi}. \tag{3.18}$$

Where $\psi = \omega - \int(k_o + k_1)dz$ , $\omega = \omega_1 - \omega_o$ and $k = \int(k_o + k_1)dz$ .

Again we know that [Pant and Tripathi (1994), Tripathi (2013)],

$$\vec{F}_{pb} = e\nabla\Phi_{pb} = iek\Phi_{pb}\hat{z} = ie(k_o + k_1)\Phi_{pb}\hat{z}. \tag{3.19}$$

Now from equations (3.18) and (3.19), we have [Pant and Tripathi (1994), Tripathi (2013)],

$$\Phi = -\frac{eA_o^{*}\vec{E}_{1\perp}}{mc\omega_o\omega_1\gamma_{ob}^o(k_o + k_1)}[k_o + \frac{(\omega_o + k_o v_{ob}^o)k_1}{(\omega_o + k_o v_{ob}^o - \omega_c/\gamma_{ob}^o)}]. \tag{3.20}$$

Similarly, the Ponderomotive potential for electron plasma is as [Tripathi (2013)],

$$\Phi_{pp} = -\frac{eA_o^* \vec{E}_{1\perp}}{mc\omega_o\omega_1(k_o + k_1)}[k_o + \frac{\omega_o k_1}{(\omega_o - \omega_c)}]. \tag{3.21}$$

Now the Ponderomotive force $\vec{F}_p = e\vec{E}, \vec{E} = -\nabla\Phi, \Phi \approx e^{-i(\omega t - kz)}$ produces z-velocity, $v_2$ and density, $n_{2b}$ perturbation of the electron beam in z-direction, therefore, the equation of motion for fluid electron beam due to perturbation under the influence of the Ponderomotive force, the electrons acquire an axial velocity as [Tripathi (2013)],

$$m[\frac{\partial(\gamma_{ob}^o \vec{v}_{ob})}{\partial t} + \vec{v}_b.\nabla(\gamma_{ob}^o \vec{v}_{ob})] = -e\vec{E}_{pb} - e(\vec{v}_{ob}^* \times \vec{B}) - e(\vec{v}_{1b} \times \vec{B}_o^*), \tag{3.22}$$

First of all examine that the RHS term is only of z-component and the total velocity is in z-direction, i.e., $v_{ob} = v_{ob}^o + v_2$, need only z-component, this term is become zero, because beam velocity is in the z-direction, therefore, this force is perpendicular to z-axis and it is equal to zero. It is presuming that, any velocity of electrons perpendicular to z-axis, so this is the velocity of perturbation. Therefore, equation (3.22) can be written as,

$$m[\frac{\partial(\gamma_{ob}^o \vec{v}_{ob})}{\partial t} + \vec{v}_{ob}.\nabla(\gamma_{ob}^o \vec{v}_b)] = -e\vec{E}_{pb} + 0,$$

$$\Rightarrow m[\frac{\partial(\gamma_{ob}^o \vec{v}_{ob})}{\partial t} + \vec{v}_{ob}.\nabla(\gamma_{ob}^o \vec{v}_b)] = -e\vec{E}_{pb}, \tag{3.23}$$

Let me linearizing the equation of motion, treating electric field to be a perturbation of densities $n_{ob}^o$ and velocities $v_{ob}^o$ to be a perturbation and magnetic field of the wave is also to be a perturb. And here, relativistic gamma factor or Lorentz factor,

$\gamma_{ob}^{o}=[1-(v_{ob}^{o}/c)^{2}]^{-1/2}$ and velocity in z-direction have velocities $v_{ob}=v_{ob}^{o}+v_{2}$, due to perturbation and relativistic gamma factor $\gamma_{ob}^{o}$ is changed and equal to be $\gamma_{ob}^{o}=[1-(v_{ob}^{o}+v_{2})^{2}/c^{2}]^{-1/2}$. For the sake of simplicity, using binomial expansion and written as $\gamma=[1-(\frac{v_{ob}^{o}}{c})^{2}-(\frac{v_{2}}{c})^{2}-(\frac{v_{ob}^{o}.v_{2}}{c})]^{-1/2}$ or $\gamma=\gamma_{ob}^{o}+\gamma_{ob}^{o\,3}\frac{v_{ob}^{o}v_{2}}{c^{2}}$, where $\gamma_{ob}^{o}=[1-(v_{ob}^{o})^{2}/c^{2}]^{-1/2}$, for an initial electron velocity $v_{ob}$ in the z-direction. Therefore, equation (3.23), after rearrange and linearization by $\nabla=ik$ and $\frac{\partial}{\partial t}=-i\omega$, we have [Pant and Tripathi (1994)],

$$\frac{\partial\vec{v}_{2z}}{\partial t}+v_{ob}^{o}\frac{\partial\vec{v}_{2z}}{\partial z}=\frac{-e\vec{E}_{pb}}{m\gamma_{ob}^{o\,3}},$$

$$\Rightarrow v_{2z}=-\frac{ek\Phi_{pb}}{m\gamma_{ob}^{o\,3}(\omega_{1}-kv_{ob}^{o})}. \tag{3.24}$$

The resulting density perturbation, obtained by solving the equation of continuity and linearizing them, one obtains [Pant and Tripathi (1994), Tripathi (2013)],

$$\frac{\partial n}{\partial t}+\nabla.(nv_{b})=0,$$

$$\Rightarrow n_{2b}=\frac{kn_{ob}^{o}}{(\omega_{1}-kv_{ob}^{o})}.v_{2z}, \tag{3.25}$$

Now from expressions (3.24) and (3.25), we have [Pant and Tripathi (1994)],

$$n_{2b}=-\frac{ek^{2}n_{ob}^{o}\Phi_{pb}}{m\gamma_{ob}^{o\,3}(\omega_{1}-kv_{ob}^{o})^{2}}. \tag{3.26}$$

The oscillatory axial velocity $\vec{v}_{2z}$ interacts with the wiggler field $\vec{B}_o$, generating a nonlinear force $-\frac{e}{2}[v_{2z}(\hat{z}\times\vec{B}_o)]$ and the transverse velocity due to wiggler. Therefore, the electrons' response to this force is given as [Pant and Tripathi (1994), Tripathi (2013)],

$$mc[\frac{\partial\gamma_{ob}^{o}\vec{v}_{ob}}{\partial t}+\vec{v}_{ob}\nabla.(\gamma_{ob}^{o}\vec{v}_{ob})]=-e\vec{E}_1-\frac{e}{2}(\vec{v}_{ob}\times\vec{B}_o),$$

$$\Rightarrow\vec{v}_{2zo}=-\frac{ev_{2z}\vec{B}_o}{2\gamma_{ob}^{o}mc(\omega_1-\omega_c/\gamma_{ob}^{o})}=-\frac{e\vec{B}_o}{2\gamma_o mc(\omega_1-\omega_c/\gamma_{ob}^{o})}.\frac{n_{2b}(\omega_1-kv_{ob}^{o})}{kn_{ob}^{o}}. \qquad (3.27)$$

### 3.2.2. Dispersion curve

In the Raman Regime, a self-consistent of free space charge potential $\Phi\approx e^{-i(\omega t-kz)}$ is also experienced a force on electrons due to high beam current as $I_b\geq 40kA$ [Pant and Tripathi (1994), Tripathi (2013)]. Therefore, after replacing $\Phi_{pb}$ by $(\Phi+\Phi_{pb})$ from equations (3.24) and (3.26), one can get,

$$v_{2z}=-\frac{ek}{m\gamma_{ob}^{o\,3}(\omega_1-kv_{ob}^{o})}(\Phi+\Phi_{pb}), and, n_{2b}=-\frac{ek^2n_{ob}^{o}}{m\gamma_{ob}^{o\,3}(\omega_1-kv_{ob}^{o})^2}(\Phi+\Phi_{pb}). \qquad (3.28)$$

The nonlinear current density at $(\omega_1,k_1)$ can be written as [Pant and Tripathi (1994), Tripathi (2013)],

$$\vec{J}_{b\perp}^{1}=-en_{ob}^{o}\vec{v}_{1b}-\frac{1}{2}en_{2b}\vec{v}_{ob}-en_{ob}^{o}\vec{v}_{2zo},$$

$$\Rightarrow\vec{J}_{b\perp}^{1}=-\frac{e^2n_{ob}^{o}\vec{E}_{1\perp}}{im\gamma_{ob}^{o}\omega_1}+\frac{1}{2}\frac{e^2k^2n_{ob}^{o}\vec{v}_{ob}}{m\gamma_{ob}^{o\,3}(\omega_1-kv_{ob}^{o})^2}(\Phi+\Phi_{pb})+\frac{e^2n_{2b}(\omega_1-kv_{ob}^{o})\vec{B}_o}{2\gamma_{ob}^{o}kmc(\omega_1-\omega_c/\gamma_{ob}^{o})}. \qquad (3.29)$$

Here also the background plasma density can be obtained from equation (3.29) by taking by $\gamma_o = 1, \vec{v}_{ob} = 0$ and replacing $n_{ob}^o, and, n_{2b}$ by $n_{op}^o, and, n_{2p}$ respectively. Therefore, we have [Pant and Tripathi (1994), Tripathi (2013)],

$$\vec{J}_{p\perp}^1 = -\frac{e^2 n_{op}^o \vec{E}_{1\perp}}{im\omega_1} - \frac{e^3 k n_{op}^o \vec{B}_o}{2m^2\omega_1 c(\omega_1 - \omega_c)}(\Phi + \Phi_{pp}). \tag{3.30}$$

Here $n_{2p}$ and $\Phi_{pp}$ can be written from expressions (3.21) and (3.26), we have [Pant and Tripathi (1994), Tripathi (2013)],

$$n_{2p} = -\frac{ek^2 n_{op}^o}{m\omega_1^2}(\Phi + \Phi_{pp}),$$

and,

$$\Phi_{pp} = \frac{eA_o^* \vec{E}_{1\perp}}{mc\omega_o\omega_1(k_o + k_1)}[k_o + \frac{\omega_o k_1}{(\omega_o - \omega_c)}]. \tag{3.31}$$

Now the total current density of the beams in transverse direction $(\vec{J}_{T\perp}^1 = \vec{J}_{b\perp}^1 + \vec{J}_{p\perp}^1)$, using equation (3.29) and contribution of the background plasma equation (3.30) at $(\omega_1, k_1)$ is as [Pant and Tripathi (1994), Tripathi (2013)],

$$\vec{J}_{T\perp}^1 = -\frac{e^2 n_{ob}^o \vec{E}_{1\perp}}{im\gamma_{ob}^o\omega_1} + \frac{1}{2}\frac{e^3k^2 n_{ob}^o(\omega_o + k_o v_{ob}^o)(\Phi + \Phi_{pb})\vec{E}_o}{im^2\omega_o\gamma_{ob}^{o\,4}(\omega_1 - kv_{ob}^o)^2(\omega_o + k_o v_{ob}^o - \omega_c/\gamma_{ob}^o)}$$
$$-\frac{e^3 k n_{ob}^o \vec{B}_o}{2\gamma_{ob}^{o\,4} cm^2(\omega_1 - \omega_c/\gamma_{ob}^o)(\omega_1 - kv_{ob}^o)}(\Phi + \Phi_{pb}) - \frac{e^2 n_{op}^o \vec{E}_{1\perp}}{im\omega_1} - \frac{e^3 k n_{op}^o \vec{B}_o}{2m^2 c\omega_1(\omega_1 - \omega_c)}(\Phi + \Phi_{pp}). \tag{3.32}$$

In the case when beam current is large i.e., $(\chi_b >> 1)$, the free space charge potential can be considerable, hence, the Ponderomotive potential $(\Phi >> \Phi_{pb})$ is considered, therefore, rearranging expressions (3.31) and (3.32) can be written as [Pant and Tripathi (1994), Tripathi (2013)],

$$\vec{J}^{1}_{T\perp} = -\frac{1}{4\pi}[\frac{\omega_{pb}^2}{i\gamma_{ob}^{o}\omega_1} + \frac{\omega_p^2}{i\omega_1} + \frac{1}{2}\frac{e\omega_p^2\omega_c A_o^*}{m\omega_o\omega_1{}^2 c(\omega_1-\omega_c)}\{k_o + \frac{\omega_o k_1}{(\omega_o-\omega_c)}\}]\vec{E}_{1\perp}$$
$$+\frac{1}{2}\frac{1}{4\pi}\frac{ek^2\omega_{pb}^2(\omega_o + k_o v_{ob}^o)\vec{E}_o}{im\omega_o\gamma_{ob}^{o\,4}(\omega_1 - kv_{ob}^o)^2(\omega_o + k_o v_{ob}^o - \omega_c/\gamma_{ob}^o)}\Phi - \frac{1}{2}\frac{1}{4\pi}\frac{k\omega_{pb}^2\omega_c}{\gamma_{ob}^{o\,4}(\omega_1-\omega_c/\gamma_{ob}^o)(\omega_1 - kv_{ob}^o)}\Phi \quad (3.33)$$
$$-\frac{1}{2}\frac{1}{4\pi}\frac{k\omega_p^2\omega_c}{\omega_1(\omega_1-\omega_c)}\Phi.$$

Now using $\vec{J}^{1}_{T\perp}$ in the wave equation, one obtains [Pant and Tripathi (1994), Tripathi (2013)],

$$\nabla^2\vec{E}_{1\perp} + \frac{\omega_1{}^2}{c^2}\vec{E}_{1\perp} = -\frac{4\pi i\omega_1}{c^2}\vec{J}^{1}_{T\perp},$$

$$\Rightarrow (\omega_1{}^2 - k_1{}^2c^2)\vec{E}_{1\perp} = -4\pi i\omega_1\vec{J}^{1}_{T\perp}. \quad (3.34)$$

Comparing equations (3.33) and (3.34), we have [Pant and Tripathi (1994)],

$$[\omega_1{}^2 - k_1{}^2c^2 - \frac{\omega_{pb}^2}{\gamma_{ob}^{o}\omega_1} - \frac{\omega_p^2}{\omega_1} - \frac{1}{2}\frac{ei\omega_p^2\omega_c A_o^*}{m\omega_o\omega_1 c(\omega_1-\omega_c)}\{k_o + \frac{\omega_o k_1}{(\omega_o-\omega_c)}\}]\vec{E}_{1\perp} =$$
$$-\frac{k\omega_1\Phi}{2}[\frac{ek\omega_{pb}^2(\omega_o + k_o v_{ob}^o)\vec{E}_o}{m\omega_o\gamma_{ob}^{o\,4}(\omega_1 - kv_{ob}^o)^2(\omega_o + k_o v_{ob}^o - \omega_c/\gamma_{ob}^o)} - \frac{i\omega_{pb}^2\omega_c}{\gamma_{ob}^{o\,4}(\omega_1-\omega_c/\gamma_{ob}^o)(\omega_1 - kv_{ob}^o)} - \frac{i\omega_p^2\omega_c}{\omega_1(\omega_1-\omega_c)}],$$

$$\Rightarrow \mathrm{R}.\vec{E}_{1\perp} = \Phi.\mathrm{B}. \quad (3.35)$$

Where,

$$\mathrm{R} = \omega_1^{\,2} - k_1^{\,2}c^2 - \frac{\omega_{pb}^2}{\gamma_{ob}^o\omega_1} - \frac{\omega_p^2}{\omega_1} - \frac{1}{2}\frac{ie\omega_p^2\omega_c A_o^*}{m\omega_o\omega_1(\omega_1-\omega_c)}[k_o + \frac{\omega_o k_1}{(\omega_o-\omega_c)}], \tag{3.36}$$

and,

$$\mathrm{B} = -\frac{k\omega_1}{2}[\frac{ek\omega_{pb}^2(\omega_o + k_o v_{ob}^o)\vec{E}_o}{m\omega_o\gamma_{ob}^{o\;4}(\omega_1 - kv_{ob}^o)^2(\omega_o + k_o v_{ob}^o - \omega_c/\gamma_{ob}^o)} - \frac{i\omega_{pb}^2\omega_c}{\gamma_{ob}^{o\;4}(\omega_1-\omega_c/\gamma_{ob}^o)(\omega_1 - kv_{ob}^o)} - \frac{i\omega_p^2\omega_c}{\omega_1(\omega_1-\omega_c)}]. \tag{3.37}$$

Now for free space, from expressions (3.28) and (3.31) and using Poisson's equation $\nabla^2\Phi = 4\pi en$ and linearizing by $(ik)^2\Phi = en/\varepsilon, i.e., -\varepsilon k^2\Phi = n$ that yields [Pant and Tripathi (1994)],

$$\varepsilon\Phi = \frac{ek_b A_o^*\vec{E}_{\perp 1}}{mc\omega_o\omega_1\gamma_{ob}^o(k_o+k_1)}[k_o + \frac{(\omega_o + k_o v_{ob}^o)k_1}{(\omega_o + k_o v_{ob}^o - \omega_c/\gamma_{ob}^o)}] + \frac{ek_p A_o^*\vec{E}_{1\perp}}{mc\omega_o\omega_1(k_o+k_1)}[k_o + \frac{\omega_o k_1}{(\omega_o-\omega_c)}]. \tag{3.38}$$

Where $\varepsilon = 1 + k_p + k_b = 1 - \frac{\omega_p^{\;2}}{\omega_1^{\;2}} - \frac{\omega_{pb}^{\;2}}{\gamma_{ob}^{o\;3}(\omega_1 - kv_{ob}^o)^2}$, $\quad k_p = -\frac{\omega_p^{\;2}}{\omega_1^{\;2}}$, $\quad k_b = -\frac{\omega_{pb}^{\;2}}{\gamma_{ob}^{o\;3}(\omega_1 - kv_{ob}^o)^2}$

and $\varepsilon = 1 + \chi_b$. Here $\varepsilon$ is the permittivity in free space and $\chi_b$ is the medium susceptibility. Hence from equations (3.35) and (3.38) can be written as [Pant and Tripathi (1994)],

$$\mathrm{R}.\varepsilon = \mathrm{B}[\frac{ek_b A_o^*}{mc\omega_o\omega_1\gamma_{ob}^o(k_o+k_1)}\{k_o + \frac{(\omega_o + k_o v_{ob}^o)k_1}{(\omega_o + k_o v_{ob}^o - \omega_c/\gamma_{ob}^o)}\} + \frac{ek_p A_o^*}{mc\omega_o\omega_1(k_o+k_1)}.\{k_o + \frac{\omega_o k_1}{(\omega_o-\omega_c)}\}],$$

$$\Rightarrow \mathrm{R}.\varepsilon = Q. \tag{3.39}$$

Where,
$$Q=-\frac{k\omega_1}{2}\left[\frac{ek\omega_{pb}^2(\omega_o+k_ov_{ob}^o)\vec{E}_o}{m\omega_o\gamma_{ob}^{o\,4}(\omega_1-kv_{ob}^o)^2(\omega_o+k_ov_{ob}^o-\omega_c/\gamma_{ob}^o)}-\frac{i\omega_{pb}^2\omega_c}{\gamma_{ob}^{o\,4}(\omega_1-\omega_c/\gamma_{ob}^o)(\omega_1-kv_{ob}^o)}-\frac{i\omega_p^2\omega_c}{\omega_1(\omega_1-\omega_c)}\right].$$
$$.\left[\frac{ek_bA_o^*}{mc\omega_o\omega_1\gamma_{ob}^o(k_o+k_1)}\{k_o+\frac{(\omega_o+k_ov_{ob}^o)k_1}{(\omega_o+k_ov_{ob}^o-\omega_c/\gamma_{ob}^o)}\}+\frac{ek_pA_o^*}{mc\omega_o\omega_1(k_o+k_1)}\{k_o+\frac{\omega_ok_1}{(\omega_o-\omega_c)}\}\right].$$

Hence, the above equation (3.39), called the dispersion relation and plays an important role in Raman Regime operation for the whistler-pumped FEL amplifiers [Pant and Tripathi (1994)] i.e.,

$$\mathrm{R}=0,\varepsilon=0. \quad (3.40)$$

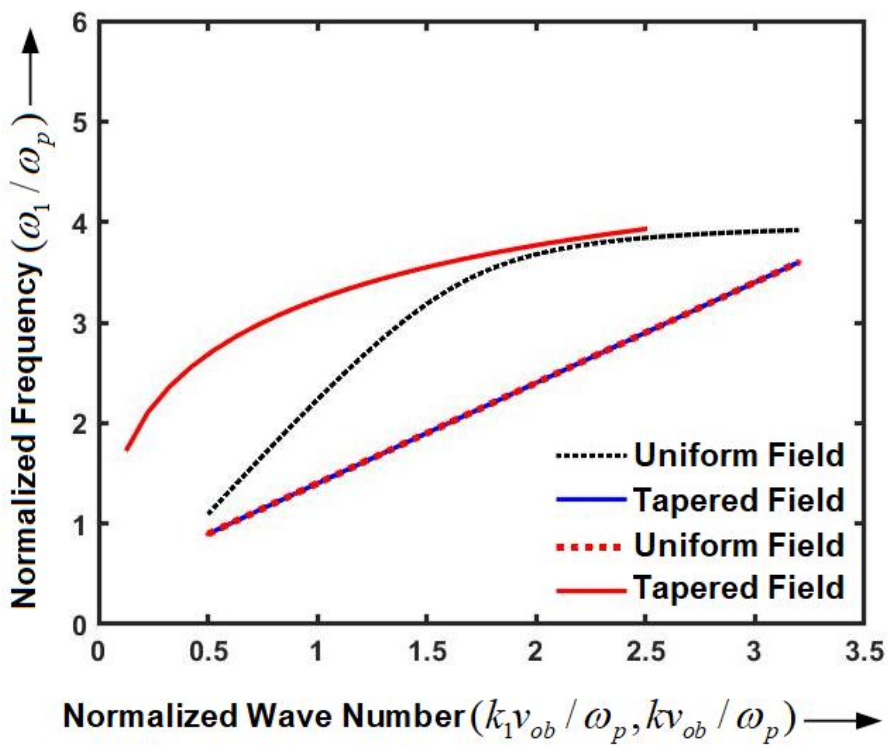


**Figure 3.2:** Dispersion Relation Curve to the Beam Modes $(\omega_1/\omega_p vs kv_{ob}/\omega_p)$ and Radiation Modes $(\omega_1/\omega_p vs k_1v_{ob}/\omega_p)$. Curve blue (Solid Line) and curve red (Dotted Line) represent the Beam Modes for tapered and uniform fields and Curve red (Solid Line) and curve black (Dotted Line) represent the Radiation Modes and the parameters are as: $\omega_{pb}=0.2\omega_p,\omega_c=4\omega_p,v_{ob}=0.4\mathrm{c},k_o=3cm^{-1},\omega_p=2\times10^{10}r/s.$

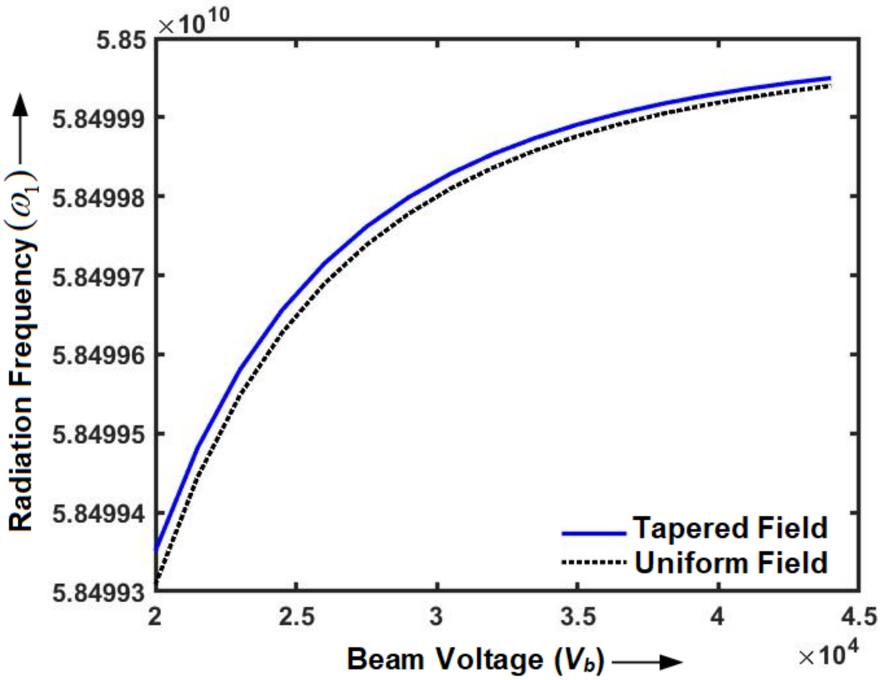


**Figure 3.3:** Radiation Frequency $(\omega_1)$ versus Beam Voltage $(V_b)$ for the given parameters: $\omega_{pb} = 0.2\omega_p, \omega_c = 4\omega_p, v_{ob} = 0.4\text{c}, k_o = 3cm^{-1}, \omega_p = 2\times10^{10} r/s, \omega_1 = 6.2\times10^{10} r/s, V_b \geq 40(kV)$.

**Table 3.1**
Analytical dispersive parameters of the FEL amplifiers
[Pant and Tripathi (1994)]

| **FEL amplifier parameters** | **Values** |
|---|---|
| Wiggler wave number $(k_o)$ | $3cm^{-1}$ |
| Plasma frequency $(\omega_p)$ | $2.0 \times 10^{10}$ r/s |
| Oscillatory beam velocity $(v_{ob})$ | $0.4c$ |
| Beams plasma frequency $(\omega_{pb})$ | $0.2\omega_p$ |
| Electron cyclotron frequency $(\omega_c)$ | $4\omega_p$ |
| Electron beams voltage $(V_b)$ | ≥40kV |
| Radiation frequency $(\omega_1)$ | $6.2 \times 10^{10}$ r/s |

The dispersion relation of the FEL amplifiers is sensitive to the tapered magnetic fields, electron cyclotron frequency and plasma frequency of electrons, which can plays an important role in the Raman Regime operations as shown in Fig. 3.2 and an emission of the FEL amplifiers frequency $(\omega_1)$ with the function of beam voltage $(V_b)$ has been shown in Fig. 3.3 for the typical parameters listed in Table 3.1.

### 3.2.3. Growth Rate

In this regime, the production of amplifier radiation $(\omega_1, k_1)$ is estimated through the coupling of whistler-pumped $(0, k_o)$ with negative energy mode $(\omega, k)$. Therefore, the free space charge mode is feeding more and more negative energy to the amplifier mode. Hence, the growth rate of the FEL amplifiers in this regime [Pant and Tripathi (1994)] is defined as,

$$\Gamma = [-Q\left|\frac{\partial \varepsilon}{\partial \omega_1}.\frac{\partial \mathrm{R}}{\partial \omega_1}\right|_{\omega_1=\omega_{1r}}^{-1}]^{1/2}. \tag{3.41}$$

Where $\omega_{1r}$ is the solution of equation (3.39) and from expressions (3.36) and (3.38), we have

$$\frac{\partial \mathrm{R}}{\partial \omega_1} = 2\omega_1 + \frac{\omega_{pb}^2}{\gamma_{ob}^o \omega_1^2} + \frac{\omega_p^2}{\omega_1^2} + \frac{1}{2}\frac{e\omega_p^2\omega_c A_o^*}{m\omega_o c}[k_o + \frac{\omega_o k_1}{(\omega_o - \omega_c)}][\frac{1}{\omega_1(\omega_1-\omega_c)^2} + \frac{1}{\omega_1^2(\omega_1-\omega_c)}] \text{ and } \frac{\partial \varepsilon}{\partial \omega_1} = \frac{2\omega_p^2}{\omega_1^3} + \frac{2\omega_{pb}^2}{\gamma_{ob}^{o\,3}(\omega_1 - kv_{ob})^3}.$$

The growth rate is larger as shown in Fig. 3.4, while in the Raman Regime operation, occurrences of radiation frequency are quite possible at higher frequencies increasing with beam voltage $(V_b)$ shown in Fig. 3.5, on other hand the growth rate decreases as increases radiation frequency while it is unaffected in Compton Regime for the typical parameters as Table 3.2 [Pant and Tripathi (1994)].

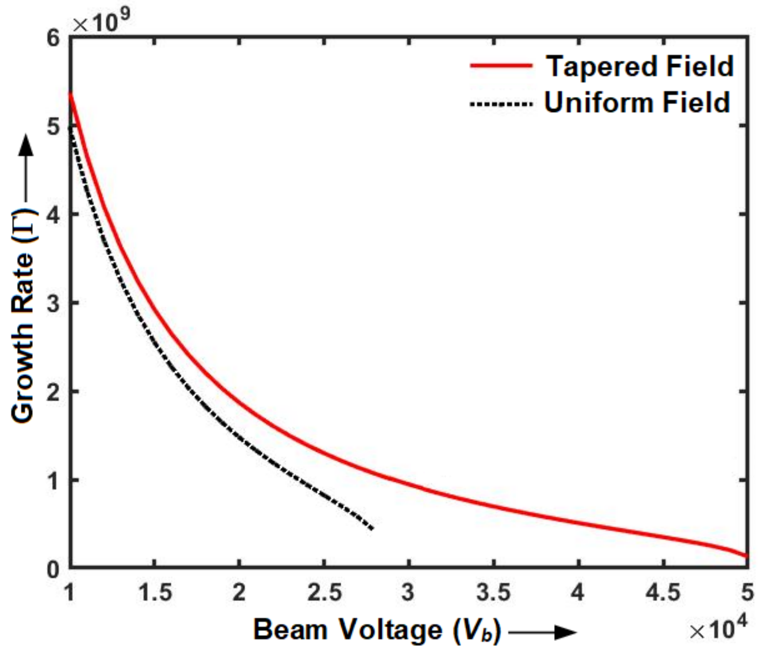


**Figure 3.4:** Growth Rate $(\Gamma)$ versus Beam Voltage $(V_b)$ for the Given Parameters: $\omega_{pb} = 0.2\omega_p, \omega_c = 4\omega_p, v_{ob} = 0.4c, k_o = 3cm^{-1}, \omega_p = 2\times10^{10} r/s, V_b \geq 40(kV).$

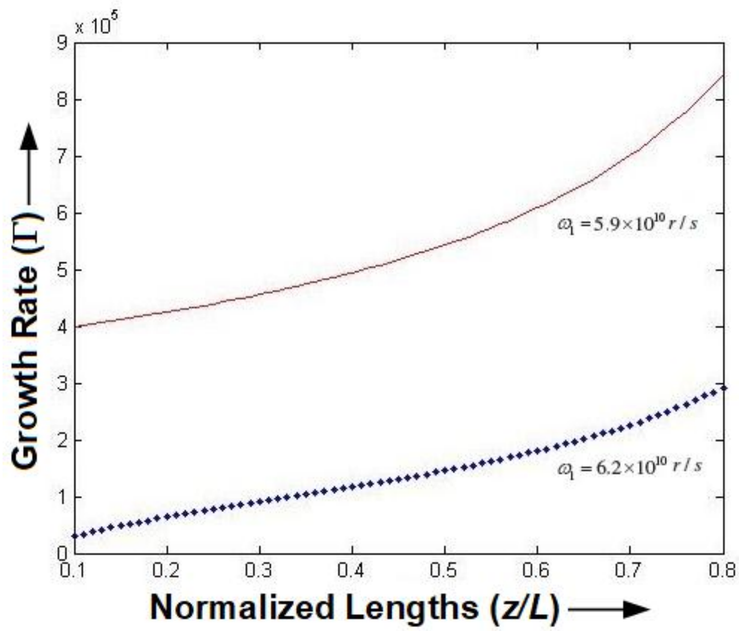


**Figure 3.5:** Growth Rate $(\Gamma)$ versus Normalized Lengths $(z/L)$ for the Given Parameters: $\omega_{pb} = 0.2\omega_p, \omega_c = 4\omega_p, v_{ob} = 0.4c, k_o = 3cm^{-1}, \omega_p =, 1.9\times10^{10}, 2\times10^{10} r/s, \omega_1 = 5.9\times10^{10}, 6.2\times10^{10} r/s, L = 40cm.$

**Table 3.2**
Analytical growth parameters of the FEL amplifiers
[Pant and Tripathi (1994)]

| **FEL amplifier parameters** | **Values** |
|---|---|
| Speed of light in vacuum *(c)* | 3.0 x $10^8$ m/s |
| Wiggler wave number $(k_o)$ | 3cm$^{-1}$ |
| Plasma frequency $(\omega_p)$ | 1.9 x $10^{10}$ r/s |
| | 2.0 x $10^{10}$ r/s |
| Oscillatory beam velocity $(v_{ob})$ | 0.4*c* |
| Beams plasma frequency $(\omega_{pb})$ | $0.2\omega_p$ |
| Electron cyclotron frequency $(\omega_c)$ | $4\omega_p$ |
| Wiggler frequency $(f_o)$ | 10*GHz* |
| Wiggler fields $(B_o)$ | 3*kG* |
| Axial fields $(B_s)$ | Up to 20*kG* |
| Electron beams voltage $(V_b)$ | ≥40kV |
| Lenghts of Interaction *(L)* | 40cm |
| Radiation frequency $(\omega_1)$ | 5.9 x $10^{10}$ r/s |
| | 6.2 x $10^{10}$ r/s |

## 3.3. FEL amplifiers gain and efficiency

In the Raman Regime (RR), amplitude of the beat wave is growing with trapped electrons while the instability may be saturated for the nonlinear state. Therefore under influence, an electric field $\vec{E}_{pb}$ can be written as [Pant and Tripathi (1994)],

$$\vec{E}_{pb} = (\frac{eA_o^* A_1}{im\omega_1\omega_o\gamma_{ob}^o})[k_o + \frac{k_1(\omega_o + k_o v_{ob}^o)}{(\omega_o + k_o v_{ob}^o - \omega_c / \gamma_{ob}^o)}]\hat{z}e^{-i\psi}. \tag{3.42}$$

Where $\psi = \omega - \int (k_o + k_1)dz$, $\omega = \omega_1 - \omega_o$ and $k = \int (k_o + k_1)dz$. Hence the phase momentum, trapping of electrons, the gain function $'G'$ variation with electron momentum $(P_{in})$ and their efficiency are describes as below.

### 3.3.1. Phase and momentum equation

Considering here that the z-component of momentum equation under the beat Ponderomotive force can be written as, $\frac{dP_{zb}}{dt} = v_{zb}\frac{dP_{zb}}{dz} = mc^2\frac{d\gamma_e}{dz} = F_{pbz}$ or $\frac{d\gamma_e}{dz} = \frac{1}{mc^2}F_{pbz}$.

Here $\gamma_e$ denotes the electrons relativistic energy at a given point $z$. Therefore, taking real part of the above equation (3.18), one obtained [Pant and Tripathi (1994)],

$$\frac{d\gamma_e}{dz} = -\frac{eA_{pb}}{mc^2}\cos\psi. \tag{3.43}$$

Where $\vec{A}_{pb} = -i\vec{E}_{pb}$, $E_{pb} = (\frac{a_1 A_o^* c}{\omega_o\gamma_{ob}^o})[k_o + \frac{k_1(\omega_o + k_o v_{ob}^o)}{(\omega_o + k_o v_{ob}^o - \omega_c / \gamma_{ob}^o)}]$ and $a_1 = \frac{eA_1}{m\omega_1 c}$.

Now defining variables $\Delta\gamma_e = \gamma_e - \gamma_r$, $\gamma_r$ is the constant resonant energy. Considering here, $\gamma_e = (1+\frac{P_\perp^2 + P_z^2}{m_o^2 c^2})^{1/2} = (1-\frac{v_{o\perp}^2}{c^2}-\frac{v_z^2}{c^2})^{-1/2}, E_p = k|\phi_p|$, P is the electron momentum and has written as $\frac{d\gamma_e}{dz} = \frac{1}{mc^2} F_{pbz}$ or $\frac{dP_{zb}}{dt} = v_{zb}\frac{dP_{zb}}{dz} = mc^2\frac{d\gamma_e}{dz} = F_{pbz}$. If an electron is moving with the phase velocity of the wave, then the resonant gamma factor $(\gamma_r)$ i.e. $\gamma_r = (1-\frac{v_{o\perp}^2}{c^2}-\frac{\omega^2}{k^2c^2})^{-1/2}$. Again when $\gamma_e$ falls to $\gamma_r$ the beam can no longer transfer energy to the wave. The deviation of energy, $\Delta\gamma_e = \gamma_e - \gamma_r$ are also effective in resonant case [Pant and Tripathi (1994)]. Hence,

$$\frac{d\Delta\gamma_e}{dz} = \frac{d\gamma_e}{dz} - \frac{d\gamma_r}{dz}, \tag{3.44}$$

Let $\Delta\gamma_e = \gamma_e - \gamma_r, \psi = kz - \omega t$ to write [Pant and Tripathi (1994), Tripathi (2013)],

$$\frac{d\Delta\gamma_e}{dz} = -\frac{eA_{pb}}{mc^2}\cos\psi. \tag{3.45}$$

Now the equation $\psi = \omega t - \int(k_o + k_1)dz = \omega_1 t - \int(k_o + k_1)dz$ is governing $\psi$, then differentiate w. r. to $z$, we have [Pant and Tripathi (1994), Tripathi (2013)],

$$\frac{d\psi}{dz} = \frac{\omega_1}{v_{zb}} - (k_o + k_1), \tag{3.46}$$

Using assumption, $k = k_o + k_1 = \frac{\omega_1}{c}(\frac{1}{1-1/\gamma_r^2})^{1/2} = \frac{\omega_1}{c(1-1/\gamma_r^2)^{3/2}}$ [Pant and Tripathi (1994), Sharma and Tripathi (1996), Tripathi (2013)], then,

$$\frac{d\psi}{dz} = \frac{\omega_1 \Delta\gamma_e}{2c(\gamma_r^{\,2} - 1)^{3/2}}, \tag{3.47}$$

The expressions (3.43) and (3.44), represent the evaluation of energy and momentum equation. Now from equation (3.47), after dimensionalizing $z$ by $\xi = z/L, i.e., dz = Ld\xi$, we have [Pant and Tripathi (1994), Tripathi (2013)],

$$\frac{d\psi}{d\xi} = \frac{\omega_1 L \Delta\gamma_e}{2c(\gamma_r^{\,2} - 1)^{3/2}}. \tag{3.48}$$

The above equation (3.48) is called Ponderomotive wave momentum i.e., $P = \frac{d\psi}{d\xi}$ [Pant and Tripathi (1994), Tripathi (2013)], therefore,

$$\frac{d\psi}{d\xi} = P = \frac{\omega_1 L \Delta\gamma_e}{2c(\gamma_r^{\,2} - 1)^{3/2}}. \tag{3.49}$$

Again equation (3.48), differentiate w. r. to $z$ and from equation (3.45), putting the value of $\frac{d\Delta\gamma_e}{dz} = -\frac{eA_{pb}}{mc^2}\cos\psi$, we have [Pant and Tripathi (1994), Tripathi (2013)],

$$\frac{dP}{dz} = \frac{d}{dz}\left(\frac{d\psi}{d\xi}\right) = \frac{\omega_1 L}{2c(\gamma_r^{\,2} - 1)^{3/2}} \frac{d\Delta\gamma_e}{dz}, \tag{3.50}$$

or,

$$\frac{dP}{dz} = -\frac{eA_{pb} L \omega_1}{2mc^3(\gamma_r^{\,2} - 1)^{3/2}} \cos\psi. \tag{3.51}$$

After dimensionalizing $z$ by $\xi = z/L, i.e., dz = Ld\xi$, the above equation (3.51) can be written as [Pant and Tripathi (1994), Tripathi (2013)],

$$\frac{dP}{d\xi} = -\frac{eA_{pb}L^2\omega_1}{2mc^3(\gamma_r{}^2 - 1)^{3/2}}\cos\psi. \tag{3.52}$$

Expressions (3.49) and (3.52) defined the energy and phase momentum equations respectively and can be rewritten as [Pant and Tripathi (1994), Tripathi (2013)],

$$\frac{dP}{d\xi} = -A\cos\psi,$$

and,
$$\frac{d\psi}{d\xi} = P. \tag{3.53}$$

Where $A$ is constant i.e., $A = \frac{eA_{pb}L^2\omega_1}{2mc^3(\gamma_r{}^2 - 1)^{3/2}}$ and $P = \frac{\omega_1 L\Delta\gamma_e}{2c(\gamma_r{}^2 - 1)^{3/2}}$.

### 3.3.2. Trapping of electrons

Since an electron can lose the energy and transfer it to the wave between $-\pi/2$ to $\pi/2$. Therefore integrating equation (3.53) w.r.to $\psi$ and it can be written as [Pant and Tripathi (1994), Tripathi (2013)],

$$P^2 = -2A\sin\psi + P_{in}^2 + 2A\sin\psi_{in}, \tag{3.54}$$

Where, the values of electron momentum ( $P$ ) and phase ( $\psi$ ) at the entry point $z = 0$, are $P_{in} = P_{\xi=0}$ and $\psi_{in} = \psi_{\xi=0}$ respectively.

Equation (3.54) shows the trajectories of the trapped electrons with the phase space $(P,\psi)$ at different values of ( $P_{in},\psi_{in}$ ). If $P_{in}^2 + 2A\sin\psi_{in} \le 2A$ and $P^2$ >0, all values of $\psi$

are not accessible i.e., trajectories of electrons are representing localized and trapped. Therefore, the separatrix is given by [Pant and Tripathi (1994), Tripathi (2013)],

$$P^2 = 2A(1-\sin\psi). \tag{3.55}$$

Initially, at $z = 0$ electron lies uniformly with $P = P_{in}$ i.e., horizontal line for all times in the $(P,\psi)$ plane. Therefore, the separatrix of electrons are trapped as Fig 3.6. If the electrons are inside the separatrix, called trapped electrons and some ones are outside the separatrix, called untrapped electrons. If $z > 0$, i.e., electrons are some gain energy or lose energy.

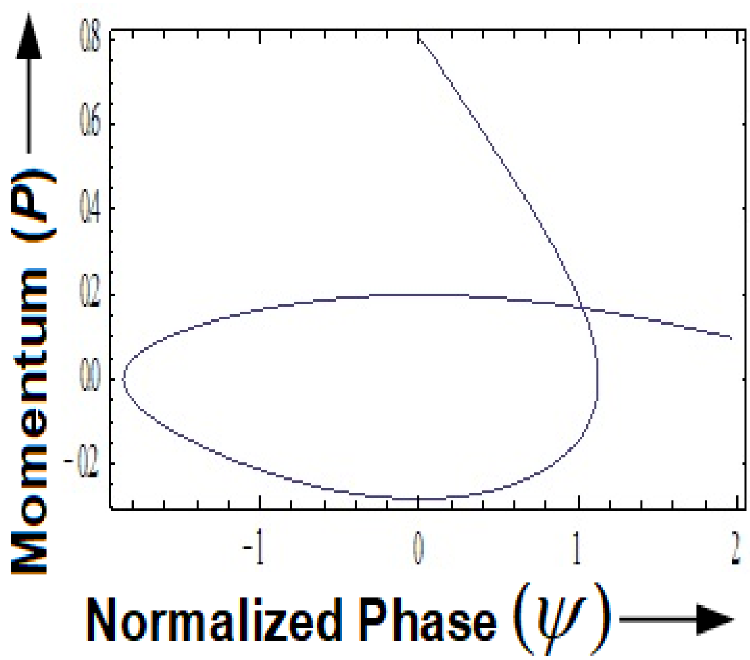


**Figure 3.6:** Normalized momentum $(P)$ versus Normalized phase $(\psi)$ of beam electrons for the parameters. $\gamma_o = 1, a_1 = 0.2, a_o = 0.028, \omega_0 = 1.5, \omega_{co} = 1.6.$

### 3.3.3. Gain function and efficiency

The electron bounces back and forth inside a potential energy well. Such electrons are trapped electrons. Since the ponderomotive wave slows down with $z$, the trapped electrons also slow down losing energy to FELs radiation. The energy losing by an electron is $\Delta P \equiv P_{in} - P(\xi = 1)$ or $\Delta P \equiv -(P_1 + P_2)_{\xi=1}$. Now the average values of $< \Delta P >$ over the initial phases yields,

$$< \Delta P > = -\frac{A^2}{8}\frac{d}{dx}(\frac{\sin^2 x}{x^2}). \tag{3.56}$$

Where, $x = \frac{P_{in}}{2}$, hence the Fig. 3.6, shows the gain function as, $G \equiv -\frac{d}{dx}(\frac{\sin^2 x}{x^2})$ with initial values of $P_{in}$ or $\gamma_o - \gamma_r$. If $\gamma_o > \gamma_r$, the net electrons energy is transfer to the waves [Pant and Tripathi (1994)].

Figure 3.7, shows the gain as a function $'G'$, with the variation of electron momentum $(P_{in})$ and the net electrons energy is transfer to the ponderomotive wave which is quite considerable at $V_b \geq 40KV$. An efficiency of the wave is enhanced adiabatically with slowing down the Ponderomotive waves also. Hence, the magnetic field tapering is improved the performance of the FEL amplifiers.

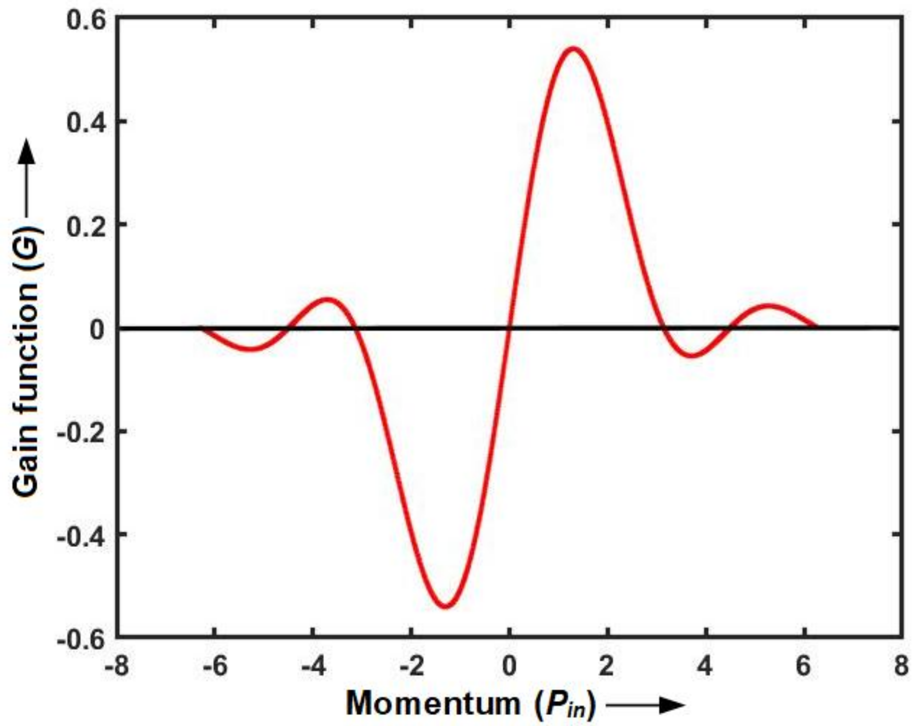


**Figure 3.7:** Gain as a function $(G)$ Versus Momentum $(P_{in})$ of trapped electrons [Pant and Tripathi (1994), Tripathi (2013)].

Now the net radiated energy by the trapped electrons [Liu and Tripathi (1994)] for the value of $\gamma_r$ and $P$ at $\xi = 0$ and $\xi = 1$ is as,

$$\Delta\varepsilon = mc^2[\gamma_r(0) - \gamma_r(1) + \frac{L(\omega_1 - \omega_0) < P(0) - P(1) >}{2c(\gamma_r^{\,2}(0) - 1)^{3/2}}]. \tag{3.57}$$

Where < > denotes the average value of the trapped electrons and $< P(0) - P(1) >$ can be find out by equation (3.53). Hence the efficiency by trapped electrons is as,

$$\eta_{tr} = \frac{\Delta\varepsilon}{mc^2(\gamma_r(0) - 1)}, \tag{3.58}$$

Now the total efficiency of the wave for a phase ($\psi$) between $-\pi/2$ to $\pi/2$ in the FEL amplifiers is, [Liu and Tripathi (1994)],

$$\eta = \frac{\gamma_r(\xi=0)-\gamma_r(\xi=1)}{\gamma_r(\xi=0)-1}. \tag{3.59}$$

For the following set of parameters: $\gamma_o = 1, a_1 = 0.2, a_o = 0.028, \omega_0 = 1.5, \omega_{co} = 1.6$, an efficiency of a trapped electron is estimated as $20\%$ (Fig. 3.8), while the reduction along the interaction region of about 10 % [Pant and Tripathi (1994)] with the variation of magnetic field tapering. Hence, the intensity of FEL amplifiers are influenced little with dynamics of beams by tapering but it might not be detrimental to the instability in FEL amplifiers.

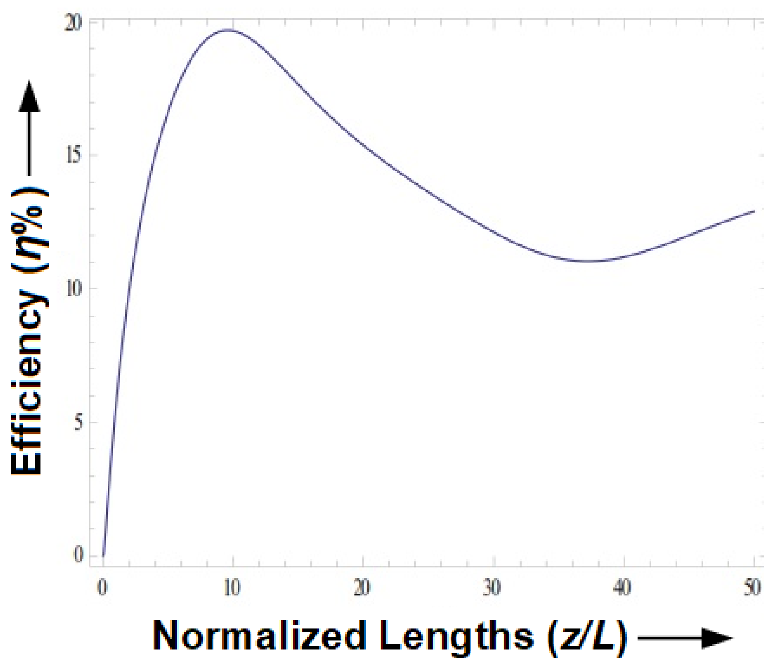


**Figure 3.8:** Efficiency $(\eta)$ versus normalized distance $(\xi \rightarrow z/L)$ for the parameters, $\gamma_o = 1, a_1 = 0.2, a_o = 0.028, \omega_0 = 1.5, \omega_{co} = 1.6, L = 40cm.$

Tapering of the DC magnetic field is a crucial role to enhance the efficiency of the net transfer energy of the wave, which is estimated as 20% achievable efficiency than that of the 5% for uniform FEL amplifies [Pant and Tripathi (1994)]. It is also higher than experimentally demonstrated the high power FEL amplifiers (FELA) using relativistic electron beams (REBs) at *35GHz* for *1.2dB/cm* growth rates and estimated an experimental efficiency >*3%* with 50dB gain for uniform amplifiers while examined an effect of tapering on axial magnetic field to enhance the efficiency and power of the device that indicate the production of *>75MW* at *75GHz* with experimental efficiency of *6%* [Gold *et al.* (1984)].

## 3.4. Result and discussion

The Normalized dispersion curve for the beam modes $(\omega_1/\omega_p vs k v_{ob}/\omega_p)$ and radiation modes $(\omega_1/\omega_p vs k_1 v_{ob}/\omega_p)$ are shown in Fig. 3.2 for the typical parameter as Table 3.1. The dispersion relation of the FEL amplifiers is sensitive to the tapered magnetic fields, electron cyclotron frequency and plasma frequency of electrons, which plays an important role in this configuration. An emission of the FEL amplifiers frequency $(\omega_1)$ with the function of beam voltage $(V_b)$ has been showed in Fig. 3.3 for the typical parameter as Table 3.1. In this device, we study the operation of Raman Regime to the generation of $\omega_1 = 6.2\times10^{10} rad/\sec$ frequency for radiation mode using with IREBs as Fig. 3.5 for the typical parameter as Table 3.2. The growth rate decreases as increases with frequency of operation of the amplifier while it is unaffected in Compton Regime [Pant and Tripathi (1994)]. It is also clear that the growth rate is inversely proportional to the radiation frequency of the device, however, in the Raman Regime operation, the growth rate is larger while occurrences of radiation is quite possible at higher frequency increasing with beam voltage $(V_b)$ of the amplifiers as Fig 3.4 for the typical parameter as Table 3.2. The mechanism of background plasma density can serve for tenability of the higher frequency of the device. Although the frequency of radiation can be tuned by very small wiggler period and/or higher electron beams energy, however, in practically, it is not accessible more easily for high beams energy as well as shortening the wiggler periods. The separatrix of the trapped electrons are shown as Fig 3.6, means passing electrons are outside the separatrix i.e., electrons are some gain energy or lose energy to the waves.

Fig. 3.7 shows the gain function $'G'$ with the variation of electron momentum $(P_{in})$ and the net transfer energy is quite considerable at $V_b \geq 40KV$, however, tapering of the magnetic field is a crucial role for enhancing the efficiency of the net transfer energy of the wave (Fig. 3.8) and estimated as 20% which is higher than that of the 5% [Pant and Tripathi (1994)]. An intensity of FEL amplifiers can be influenced little with dynamics of beams by tapering but it might not be detrimental to the instability in FEL amplifiers.

## 3.5. Conclusion

The analytical formalism of the magnetic field tapering on Whistler-Pumped FEL amplifiers in Collective Raman Regime operation is developed. The tapering raises the efficiency of the device to 20% for typical parameters. The gain function $'G'$ with the variation of electron momentum $(P_{in})$ and the net transfer energy is quite considerable at $V_b \geq 40KV$, however, tapering of the magnetic field is a crucial role for enhancing the efficiency of the net transfer energy of the wave and estimated as 20% which is higher than that of the 5%. The dispersion relation of the FEL amplifiers is sensitive to the linear tapered magnetic fields, electron cyclotron frequency and plasma frequency of electrons. For the synchronism of the pumped frequency, it is closed to electron cyclotron frequency which is resonantly enhanced the wiggler wave number that produces the amplifier radiation for higher frequency from sub-millimeter wave to optical ranges. Gyrotron may be envisaged to launch a whistler in a magnetized plasma channel. As the beam slows down by imparting energy to FEL amplifiers radiation, the wiggler wave number of the whistler wave is enhanced by reducing the cyclotron frequency to maintain phase synchronism of radiation with the beam space charge mode while the cyclotron frequency is taken to have a negative tapering of the amplifiers. The frequency and power of the FEL amplifiers can be controlled by tuning the magnetic field and/or plasma density also by increasing the energy of electron beams. Presence of plasma ensures the space charge and current neutralization and larger power handling capacity of the devices.