CHAPTER 2

ANALYSIS OF THE FREE ELECTRON LASER AMPLIFIERS

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2.1. Introduction

The FEL amplifier (FELA) can be operated with multiple wavelengths and allows tuning of the wavelengths continuously with some ranges. However, it does not require a metallic structure for the interaction which is the major advantage of this device. Consequently, it has the potential to either radiate of very high power with metallic walls or generation at millimetre to sub millimetre wavelengths, infrared, terahertz radiation, visible, UV, XUV or X-ray, where no other sources achieved [Jia (2011), Chen and Joshi (1980), Tripathi and Liu (1989), Pellegrini (1990), Oerle and Mathias (1997), Wenlong (2000), Pelka *et al.* (2010), Baxevanis *et al.* (2013)]. The conventional sources of radiation offer very little at terahertz range. The microwave sources, for instance, operate below 60GHz, while lasers operate above 30THz and gyrotrons are limited to $30GHz$-$200GHz$ range. Free electron lasers can offer an alternative. However, conventional magnetic wiggler with wiggler period $\geq 1cm$ requires electron beams of energy $\geq 3MeV$, which escalates the size and cost considerably. Using accelerator facilities with intense bunches of electron beams via transition radiation or synchrotron, higher pulse energies is generated [Sharma and Tripathi (1996)]. Recently, the range of energies as $10-100\mu J$ per pulse, intense THz frequency has been generated via transition radiation using accelerated electron beam passing through plasma to vacuum. Hence there have been efforts to produce THz radiation by alternate methods [Tripathi and Liu (1989), Pellegrini (1990)]. Since FELs are an extremely adaptable light sources and fascinating devices that produce tunable coherent radiation over a wide frequency range from sub millimeter wavelengths to visible region with high efficiency and huge power levels using energetic electron beams. It comprises a high voltage $(>1MV)$ power supply (accelerator) and an electron gun, an interaction region with a strong wiggler magnetic field, beam pump, radiation coupler (mirror) and diagnostics. The device is tunable by tuning the beam voltage. The FELs have a magnetic field perpendicular to the beam velocity i.e., the main components, hence, therefore, the electrons have an oscillatory motion in transverse direction, which is suitable for interaction with either TE mode or TEM mode (TWT is always interacts with the TM mode whereas in gyrotron interaction is always with the TE mode only). As fast-wave device, gyrotron is interacts with an electromagnetic wave (uniform or periodic magnetic field) by phase velocity equal or slightly larger than the light velocity, c, whereas, in FELs, the electron oscillation is in transverse direction while its bunching process is in longitudinal direction similar to TWT. Hence the free electron lasers can offer an alternative to generate the ranges of sub millimetre wavelengths to x-rays.

The experiments were performed that led to the present day FELs that evolved along two separate paths as the type of accelerator and the regime of operations. Experimentally in occurrences of FELs, the two types of scattering processes are used as one is Compton (wave-particle) scattering and others Raman (wave-wave) scattering [Pourkey and Toepfer (1974)]. If the Debye wavelength is much greater than pumped wavelengths, the wave-particle process dominates and is called Compton scattering (i.e., off single “shielded” particles), whereas, if the Debye wavelength is much smaller than pumped wavelengths, then the wave-wave process dominates and Raman scattering occurs [Darke *et al.* (1974), Manheimer and Ott (1974)]. Experimentally, Elias demonstrated the

FELs and observed their amplification gain by 7% per pass of a $10.6\mu m$ laser beam with $70mA$ beam current which is taked the new possibility of high power tunable FELs [Elias *et al*. (1976)]. After experiments at Stanford University, free electron laser oscillators (FELO) have demonstrated for wave-particle simulated scattering above the $3.4\mu m$ threshold wavelengths using high electron beams energy with low current linear accelerator [Deacon *et al.* (1977)]. Observed efficiencies were less than 0.01% and attempts to improve the efficiency have focused on the use of storage rings to continuously recirculate the beam through the wave generation region.

The first stimulated scattering experiments in the Raman regime using relativistic electrons beam was performed by Granatstein in 1976 [Granatstein *et al.* (1976), Granatstein *et al. (*1977)]. Through the use of intense REBs generators, super radiant FEL oscillators were developed by producing megawatt power levels in short interaction regions $\sim 30cm$ at wavelengths ranging from $2mm$ to $400\mu m$ and with efficiencies as high as 0.1%. More recently, McDermott reported the realization of a collective Raman FEL for the first time. The experiment was designed so as to permit several passes of feedback by employing a quasioptical cavity. A laser output of $1MW$ were observed at $400\mu m$ and narrowing line with $\Delta\omega/\omega \approx 2\%$ and compared to $\Delta\omega/\omega \approx 10\%$ for the earlier super radiant oscillator studies [McDermott, Marshall and Schlesinger (1978)]. An alternative is the tapered wiggler and axial fields, which drastically changed the scenario and lead to improve the efficiency of the device. It is also observed that, an efficiency of the transfer energy enhanced reduction with interaction region along axis [Pant and Tripathi (1994)].

This chapter of the thesis is organized as follows. In Section 2.2, working principal of FEL amplifiers, frequency of operations, mechanism of radiation, phase coherence and bunching, Madey's theorem for gain, stimulated emission by Madey's theorem, principles of energy conservation by Madey's theorem have been discussed. The Raman regime operation in FEL amplifiers, nonlinear states of Raman regime and gain estimate of Raman regime in FEL amplifiers have been presented and their behavior of interaction are discussed in Section 2.3. The conclusions are drawn in Section 2.4.

2.2. Working principle of FEL amplifiers

In this section, we discussed the working principle of FEL amplifiers as frequency of operation, mechanism of radiation emission, phase coherence and bunching/ pre-bunching, ponderomotive force and growth rate and finally Madey's theorem and their applications.

2.2.1. Operating Frequency

In a wiggler, if the electron beam is travelling through the structure in the presence of an electromagnetic wave, then the interaction of the electron beam is almost zero with an electromagnetic wave but in the presence of the wiggler magnetic field, it is possible. Hence an arrangement of magnets plays important role to produce wiggler magnetic field as shown Fig. 2.1.

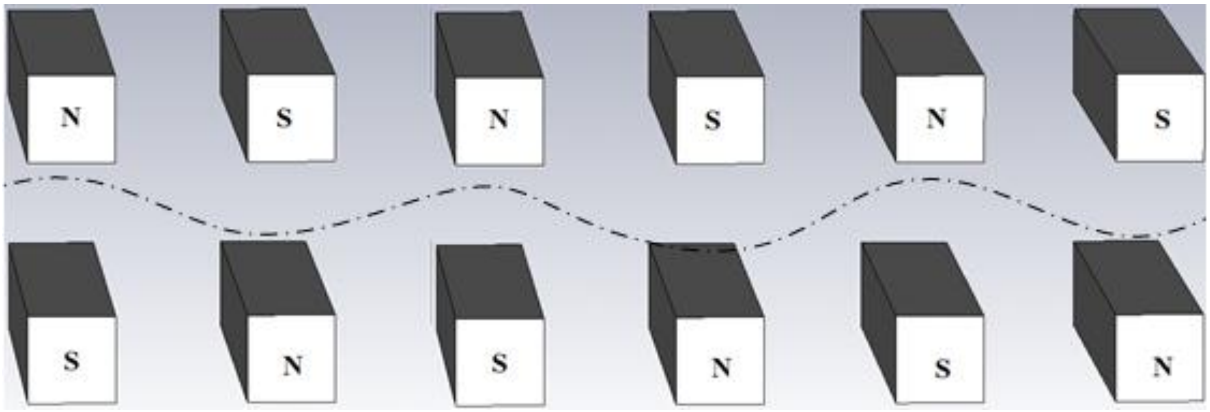

Figure 2.1: Permanent magnets arrangement for wiggler.

Since the electron beam emerging out of the electron gun (with cathode-anode potential difference V_o) possesses kinetic energy E_b as [Liu and Tripathi (1994), Schachter (2011)],

$$E_b = m_e c^2(\gamma_o - 1) = eV_o,$$

and,

$$\gamma_o = 1 + \frac{eV_o}{m_e c^2}. \quad (2.1)$$

Where $\gamma_o = (1-(v_b/c)^2)^{-1/2}$, is called beam Lorentz factor or the relativistic gamma factor, $-e$ is the charge of electrons, m_e is rest mass of electrons, v_b is drift velocity and c, light speed in vacuum. The momentum, P and energy, E_b of the electron can be express as,

$$P = m_e c(\gamma_o^2 - 1)^{1/2}. \quad (2.2)$$

Now squaring expression (2.2), we have [Liu and Tripathi (1994), Schachter (2011)],

$$P^2 = m_e^2 c^2 (\gamma_o^2 - 1). \tag{2.3}$$

Therefore, from expression (2.2) and (2.3), we get that,

$$E_b = m_e c^2 (1 + P^2 / m_e^2 c^2)^{1/2}. \tag{2.4}$$

After squaring above equation (2.4), one obtains [Liu and Tripathi (1994), Schachter (2011)],

$$E_b^2 = m_e^2 c^4 + P^2 c^2. \tag{2.5}$$

This is called the dispersion relation of an electron to the infinite length of interaction from initial to final stage changes. Hence, the resultant of emission has exactly the same difference of energy and momentum i.e., $E_i = E_f + E_{ph}$ and $P_i = P_f + P_{ph}$. Where E_i, E_f are the initial and final beam energy and P_i, P_f initial and final momentum and photon energy and momentum is E_{ph}, P_{ph} respectively. Therefore the dispersion relation is written as $E_b = (m_e^2 c^4 + P^2 c^2)^{1/2}$ for an infinite length of interaction and $E_b = c.P$ as shown in Fig. 2.2 (a). The dispersion relation of the wave is also the asymptote of the dispersion relation of the electron. Consequently, it cannot change its state along a line parallel to the asymptote. In other words, energy and momentum cannot be conserved simultaneously in vacuum. While for the finite length of interaction, the interaction is possible since although the energy conservation remains unchanged i.e., $E_i = E_f + \hbar\omega_L$ and the constraint on momentum conservation is released somewhat and it is written as, $\left| P_i - P_f - \hbar \frac{\omega_L}{c} \right| < \frac{\hbar}{cT}$, which is clearly less strength than infinite length of interaction of electron. Where, $T = 2\pi / \omega_L$ is the scale of the radiation period and $\hbar = 1.05457\times10^{-34} J.\text{sec}$ is the Planck constant. Hence the dispersion relation for the finite length of interaction of a free electron $E_b = (m_e^2c^4 + P^2c^2)^{1/2}$ and an electromagnetic plane wave in vacuum, $E_b = c.P$ are given below Fig. 2.2 (b). The constraint on the momentum conservation is less stringent because the interaction occurs in a finite length [Schachter (2011)].

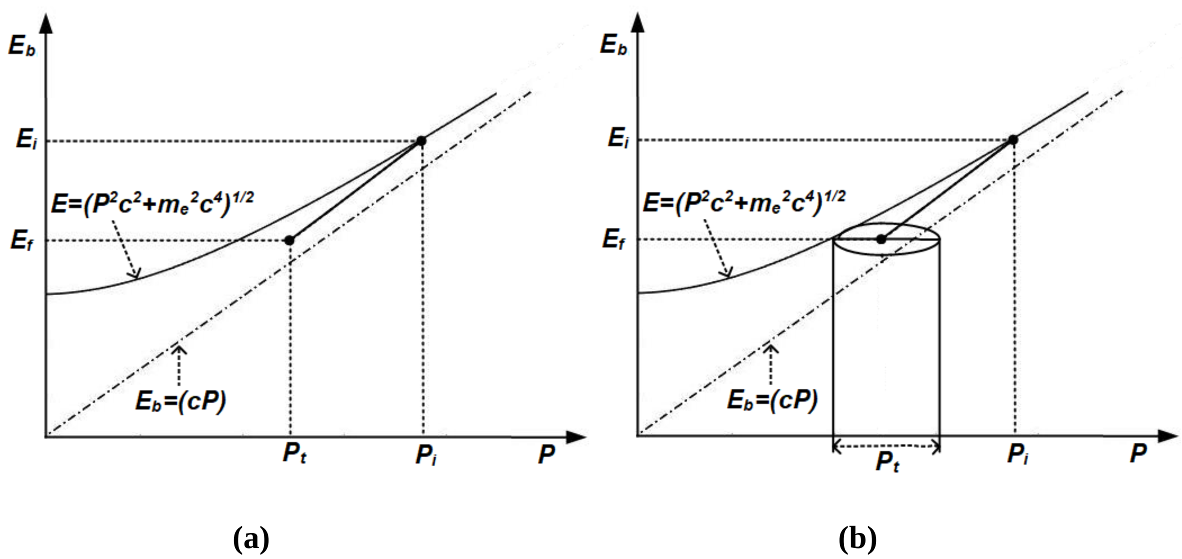

Figure 2.2: (a) The dispersion relation for the infinite length of interaction and (b) The dispersion relation for the finite length of interaction [Schachter (2011)].

Again when an electron emits a photon of energy $\hbar\omega_L$ and momentum $\hbar\vec{k}_L$ and transferred part of momentum $\hbar\vec{k}_w$ to the wiggler (where ω_L, radiation frequency, $\vec{k}_L$ is the wave number, $\vec{k}_w$ is wiggler wave number and $\hbar = 1.05457\times10^{-34} J.\text{sec}$ is the Planck constant). If energy and momentum are represented as $E_f = E_b - \hbar\omega_L$ and $[\vec{P} - \hbar(\vec{k}_L + \vec{k}_w)]$ respectively, then the emission is satisfied a relation similar as equation (2.5).

Therefore, phase synchronism of the electron with beam energy to leads to electron bunching and the growth of the radiation wave i.e.,

$$\omega_L = (k_L + k_w)v_b, \tag{2.6}$$

Since $k_L = \omega_L / c$, this equation gives [Liu and Tripathi (1994), Schachter (2011)],

$$\omega_L \approx 2\gamma_o^2 k_w c = 2\gamma_o^2 \omega_w,$$

or,

$$\lambda_L = \lambda_w / 2\gamma_o^2. \tag{2.7}$$

If the wiggler period is shorter, then the radiation frequency is shorter and reduces the wavelengths of radiation by increasing the energy of the electron beams. In a wiggler, the wiggler wavelength is related as $\lambda_L = \dfrac{\lambda_w}{\beta_b(1+\beta_b)\gamma_o^2} = (\dfrac{\lambda_w}{2\gamma_o^2})(1+\dfrac{a_w^2}{2})$ at $\beta_{\mathrm{b}} = v_{\mathrm{b}} / \mathrm{c}$. Where $a_w = eB_w\lambda_w / 2\pi mc^2 = eB_w / k_w mc^2$ is called wiggler constant or wiggler parameter [Roberson (1989), Pellegrini (1990)]. Assuming for operating state $v_b = c$, the operating wavelength or radiation wavelength of FELs λ_L scales with wiggler period (λ_w). In Fig 2.3, the line $\omega_L = k_L c$ and the beam line $\omega_L = k_L v_b$ have plotted. Fig 2.3 shows the operating point of FELs that an electron loses momentum by $\hbar\vec{k}_w$ (where, $\hbar = h/2\pi$, h is Plank constant and k_w is wiggler wave number), it's also loses energy by $\hbar\omega_L = \hbar k_L v_b$ and satisfy the equation $\varepsilon_b = m_o\,\mathrm{c}^2(1+P_b^2 / m_o^2 c^2)^{1/2}$ [Marshall (1985), Liu and Tripathi (1994)].

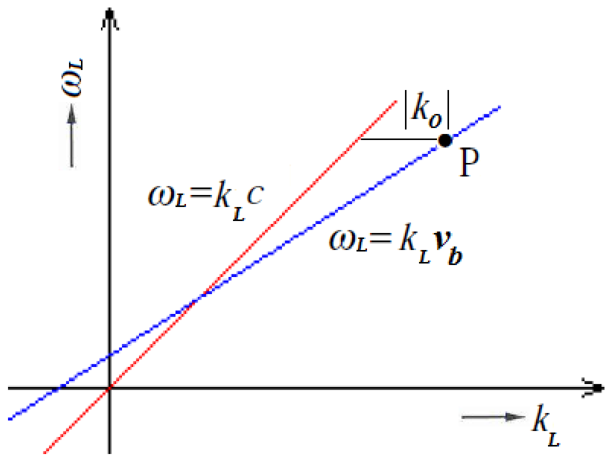

Figure 2.3: Operating point of FELs [Liu and Tripathi (1994)].

For $\omega_L = k_L c$, an electron loses more momentum in free space without a wiggler, means there is no emission. In a FEL, the difference in momentum between that given by electrons and absorbed by photons is taken up by the wiggler. Some experiments have been proved that the generation of ultraviolet or violet or visible radiation by using gyrotron as a pump which is produces radiation in the millimetre wavelength regimes also.

It has a tremendous advantage over the conventional lasers. Because the frequency of this device is tunable, wave length can be changed by changing the energy of the electron beam. This tunability is a very important consideration and more over the efficiency of the device which is much higher than the conventional laser efficiency. So, it is a very important device that which produces radiation over a very wide frequency ranges [Tripathi, NPTL (2013)]. Some frequency ranges from millimetre to sub millimetre wavelengths (between electronics and photonics regime) are given below Fig. 2.4 [Maria (2009), Chen *et al.* (2016)].

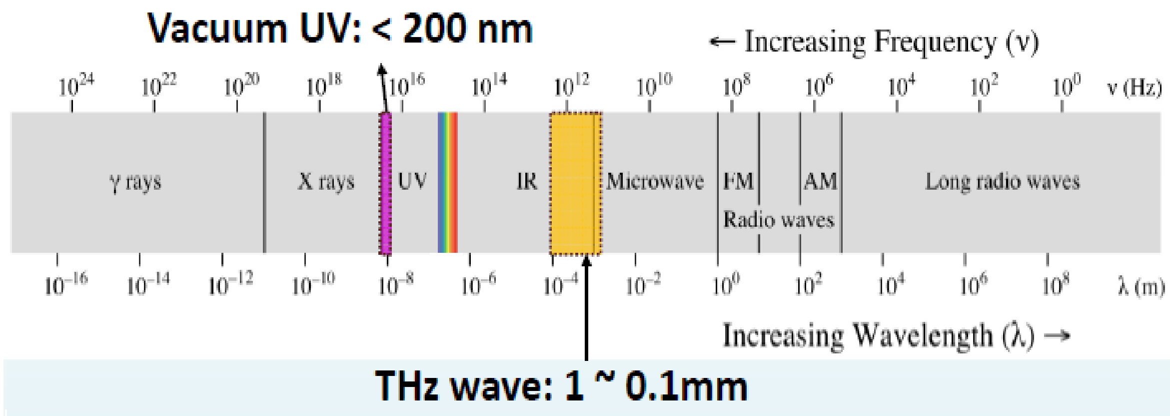

Figure 2.4: Frequency ranges from millimeter to sub millimeter wavelengths (between electronics and photonics regime) [Maria (2009), Chen *et al.* (2016)]

2.2.2. Mechanism of Radiation Emission, Phase Coherence and Bunching

In a conventional LASER, the amplification comes from the stimulated emission of electrons bound to atoms, whereas, in FELs, the "free" (unbound) electrons are the medium of amplification. The free electrons have been stripped from atoms in an electron gun and accelerated with relativistic velocities. The electromagnetic fields emitted by the bunched electrons are superimposed in phase and the total field amplitude increases. Thus the bunching mechanism of an electron energy is stronger shown in Fig. 2.5 [Marshall (1985), Liu and Tripathi (1994), Prosnitz and Swingle (1982)]. The electrons are decelerated by the field $(\vec{J}.\vec{E}_z < 0)$ which is emitted coherent radiation with experienced a ponderomotive force $\vec{F}_P = \hat{z}F_o\cos(\omega_L t - k_p z)$, where $k_p = k_L + k_w$. The phase velocity of these forces is $(v_p = \omega_L / k_p) < c$. When the velocity of the beam $v_L \approx \omega_L / k_p$ (at the frequency $\omega_L \approx 2\gamma_o^2 k_w c$), the wave appears almost a static field and capable to accelerating or decelerating electrons efficiently. If the beam velocity equal to the phase velocity (operating frequency of the device, $v_L = \omega_L / k_p$) i.e., purely static, accelerated and retarded half of the electrons. The resultant is no loss or gain. The situation is different when beam velocity, v_b is slightly higher than phase velocity, there are two kinds of regions (i) the accelerated zones where $-e\vec{E}_z > 0$ and (ii) the decelerated zones where $-e\vec{E}_z < 0$.

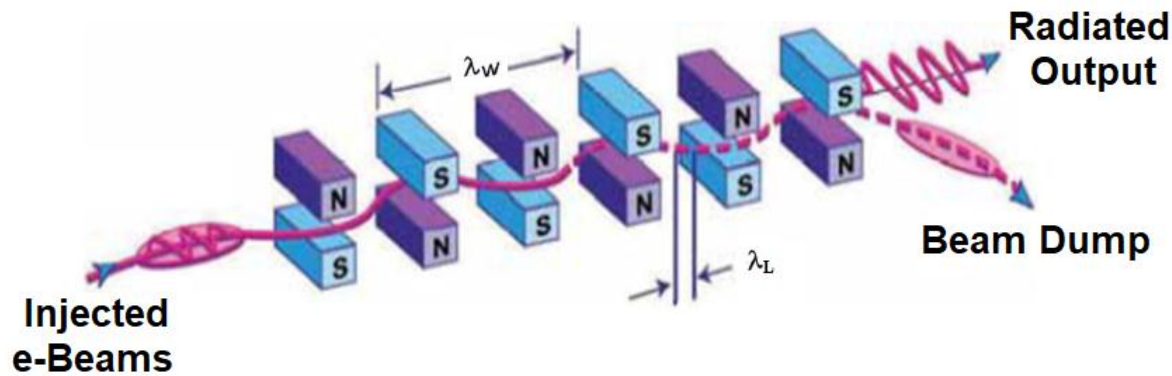

Figure 2.5: Schematic of physical mechanism interaction between electron beams and planner wiggler [Marshall (1985), Liu and Tripathi (1994), Sirigiri (2001)].

Initially electrons are uniformly distributed at all $\hat{z}$. However, the ones zones (i) are accelerated and quickly move over to zone (ii), whereas those in zones (ii) are retarded, spending more time here. Thus there is a net build-up of electrons in the retarding zone, resulting is the net transfer of energy from electrons to the wave which is causes of growth or amplification [Walsh (1980), Marshall (1985), Liu and Tripathi (1994)].

2.2.3. Ponderomotive Force and Growth of Rate

The transportation of the beam from the source to the interaction region is also a big issue. The wiggler magnetic fields play an important role to the transportation of the beams into the interaction chamber of the device. There are two kinds of wigglers, one is a permanent magnetic wiggler in which have essentially an arrangement of magnets is called a linear wiggler or planner wiggler and second is a circularly polarized wiggler or helical wiggler. Therefore, the permanent magnets can place on top in x-axis and underneath on the y-axis, then produce a wiggler magnetic field as [Marshall (1985), Tripathi (2013)],

$$\vec{B}_w = \vec{B}_{wx} + \vec{B}_{wy} = A_w(\hat{x} + i\hat{y})\,\mathrm{e}^{\mathrm{i}k_w z}. \tag{2.8}$$

Here the complex notation, which certainly this expression implies that the real part of the right hand side (RHS) is to be taken. It means that the wiggler magnetic field have an x-component because of these magnets and y-component because the magnets placed along the y-axis, with alternate polarities and the net magnetic field in this system is, take the real part, hence the wiggler magnetic field in x-direction is $B_{wx} = A_w \cos k_w z$ and $B_{wy} = A_w \sin k_w z$ in y-direction. Where k_w is the wiggler number and A_w, B_w is the wiggler amplitudes and these two components are out of phase by $\pi/2$, this is called circularly polarized wiggler. The more interesting things are the coherent radiation that is generated by the electron beams. The kinetics process of the radiation by examining the response of electrons to wiggler and the radiation signal, called FELs signal.

There are three issues that the response of electrons to a wiggler magnetic field, the response of electrons to a radiation field and both either wiggler or electrons i.e., evaluation of the non-linear force (called as the ponderomotive forces) that arise due to the coupling between the wiggler and the radiation signal, deals with millions of electron volts energy of the beam. The consideration of the ponderomotive forces by the relativistic equation of motion for electron beams and ignoring the effect of space charge, DC space charge of the beam, is define as the rate of change of momentum [Marshall (1985), Tripathi (2013)], i.e.,

$$\frac{\partial \vec{P}_b}{\partial t} + \vec{v}.\nabla \vec{P}_b = -e\vec{E}_b - e(\vec{v}_b \times \vec{B}_w). \tag{2.9}$$

Where momentum $\vec{P}_b = m\gamma \vec{v}_b$, $\gamma = [1-(v_b / c)^2]^{-1/2}$ is relativistic gamma factor or Lorentz factor, m is rest mass of electron, $\vec{v}_b$ is the drift velocity of electrons beam and c is speed of the light. Now the equation of motion for fluid electron beam as,

$$m[\frac{\partial (\gamma \vec{v}_b)}{\partial t} + \vec{v}_b.\nabla(\gamma \vec{v}_b)] = -e\vec{E}_b - e(\vec{v}_b \times \vec{B}_w). \tag{2.10}$$

This equation has two terms on the right hand side (RHS), the first is called the force due to the electric and other is called the magnetic force due to wiggler.

If the magnetic field is time independent then the force on the electron is $(\vec{v}_b \times \vec{B}_w)$, which is always perpendicular to velocity $(\vec{v}_b)$ and this does not give rise to any energy exchange. Here γ is becomes a great simplification, means velocity is changes but magnitude of velocity does not changes and relativistic gamma is written as $\gamma = \gamma_o = [1-(v_o / c)^2]^{-1/2}$. Since there is no electric field, the electric force is zero while the wiggler magnetic force is presented. Therefore, the equation of motion after linearization is written as [Marshall (1985), Tripathi (2013)],

$$m\gamma_o[\frac{\partial \vec{v}_w}{\partial t}+(v_o\hat{z}+\vec{v}_w).\nabla\vec{v}_w]=-e[(v_o\hat{z}+\vec{v}_w)\times\vec{B}_w],$$

$$\Rightarrow m\gamma_o[\frac{\partial \vec{v}_w}{\partial t}+(v_o.\nabla\vec{v}_w)\hat{z}+\vec{v}_w.\nabla\vec{v}_w]=-e[v_o(\hat{z}\times\vec{B}_w)+(\vec{v}_w\times\vec{B}_w)]. \quad (2.11)$$

Here $(\vec{v}_w.\nabla\vec{v}_w)$ and $(\vec{v}_w\times\vec{B}_w)$ is a product to perturbed quantities, which is very small and ignore it. Therefore, from equation (2.11), we have [Marshall (1985), Tripathi (2013)],

$$m\gamma_o[\frac{\partial \vec{v}_w}{\partial t}+v_o\frac{\partial \vec{v}_w}{\partial z}]=-e(v_o\hat{z}\times\vec{B}_w). \quad (2.12)$$

The response $\vec{v}_w$ is due to the effect of wiggler magnetic field, $\vec{B}_w$ i.e., a force creates with velocity $\vec{v}_w$, which is in the quasi study state and z-dependence as the source and very small which is written as $\vec{v}_w = a_w\,\mathrm{e}^{\mathrm{i}k_w z}$, where a_w is an amplitude of velocity due to wiggler. Since the RHS of equation (2.12) has time independent (no dependent of time), therefore from equation (2.8) and (2.12) and after linearization by $\frac{\partial}{\partial z}=ik$, we have,

$$0+m\gamma_o ik_w v_o\vec{v}_w=-ev_oA_w(\hat{z}\times(\hat{x}+i\hat{y})\,\mathrm{e}^{\mathrm{i}k_w z},$$

$$\Rightarrow \vec{v}_w=\frac{e\vec{B}_w}{m\gamma_o k_w}=\frac{\vec{\omega}_c}{\gamma_o k_w}. \quad (2.13)$$

If assume that the free electron laser amplifier as an electromagnetic wave with electric field $\vec{E}_L$, then we have [Marshall (1985), Tripathi (2013)],

$$\vec{E}_L=A_L(\hat{x}-i\hat{y})\,\mathrm{e}^{-\mathrm{i}\omega_L t+ik_L z}=A_L(\hat{x}-i\hat{y})\,\mathrm{e}^{-\mathrm{i}(\omega_L t-k_L z)}. \quad (2.14)$$

Where k_L, ω_L, are wave number and frequency of the FEL, A_L is amplitude and $\vec{B}_L$ is the magnetic field produced by transverse electromagnetic wave. These electromagnetic waves want to amplify at the expanse of beam energy in a transverse electromagnetic wave. Therefore, using Maxwell Third Equation, we have [Marshall (1985), Tripathi (2013)],

$$\nabla \times \vec{E}_L = -\frac{\partial \vec{B}_L}{\partial t}. \tag{2.15}$$

This is the faraday's law of electromagnetic induction and varying with time. After linearization by $\nabla = ik$ and $\frac{\partial}{\partial t} = -i\omega$, putting the value in expressions (2.14) and (2.15), one obtains,

$$\vec{B}_L = \frac{ik_L}{\omega_L}\vec{E}_L. \tag{2.16}$$

Since the beam velocity due to electric field is became $\vec{v}_b = v_o\hat{z} + \vec{v}_L$, therefore, the relativistic gamma factor is written as $\gamma = [1-(v_o + v_L / c)^2]^{-1/2}$. If $\vec{v}_L$ is very small as compare to $v_o\hat{z}$, the relativistic gamma factor is written as $\gamma = [1-(v_o / c)^2]^{-1/2} = \gamma_o$. When $\vec{v}_L$ is perpendicular to z-axis then gamma turns out to be unmodified within the limit of perturbation analysis. Therefore, the equation of motion after linearization to be [Marshall (1985), Tripathi (2013)],

$$m\gamma_o[\frac{\partial \vec{v}_L}{\partial t} + (v_o\hat{z} + \vec{v}_L).\nabla\vec{v}_L] = -e[\vec{E}_L + (v_o\hat{z} + \vec{v}_L)\times\vec{B}_L],$$

$$\Rightarrow m\gamma_o[\frac{\partial \vec{v}_L}{\partial t} + (v_o.\nabla\vec{v}_L)\hat{z} + \vec{v}_L.\nabla\vec{v}_L] = -e[\vec{E}_L + v_o(\hat{z}\times\vec{B}_L) + (\vec{v}_L\times\vec{B}_L)]. \tag{2.17}$$

Here $(\vec{v}_L.\nabla\vec{v}_L)$ and $(\vec{v}_L \times \vec{B}_L)$ is a product to perturbed quantities, which is very small and ignore it. Therefore, from equation (2.17), we have [Marshall (1985), Tripathi (2013)],

$$m\gamma_o[\frac{\partial\vec{v}_L}{\partial t}+(v_o.\frac{\partial\vec{v}_L}{\partial z})]=-e[\vec{E}_L+v_o(\hat{z}\times\vec{B}_L)]. \qquad (2.18)$$

For a FEL signal, assume here, the beam velocity is $\vec{v}_L = a_L\, e^{-i(\omega t - k_L z)}$, where a_L, is the constant amplitude of velocity. Therefore, from expressions (2.14) and (2.18) and after linearization by $\nabla = ik$ and $\frac{\partial}{\partial t}=-i\omega$, we have,

$$\vec{v}_L=\frac{e\vec{E}_L}{im\gamma_o\omega_L}. \qquad (2.19)$$

Here expressions (2.13) and (2.19), have obtained a linear responses of electrons to wiggler magnetic field as well as to FEL laser signal independently.

Since the Ponderomotive force exerted by the regular magnetic field and radiated signal, which evolve in time or space. It imparts an oscillatory velocity to electrons with $\vec{v}_L$, is the electron velocity due to the laser signal. The electrons experienced force due to electric and field magnetic field both, are called Lorentz force or Ponderomotive force and this is caused by the interaction of (i) wiggler velocity to laser magnetic field and (ii) laser velocity to wiggler magnetic field (both are transverse forces or transverse quantities) and it is written as [Marshall (1985), Tripathi (2013)],

$$\vec{F}_p=-e(\vec{v}_w+\vec{v}_L)\times(\vec{B}_w+\vec{B}_L)=-e[(\vec{v}_w\times\vec{B}_L)-e(\vec{v}_L\times\vec{B}_w)]. \qquad (2.20)$$

Here $(\vec{v}_w \times \vec{B}_w)$ and $(\vec{v}_L \times \vec{B}_L)$ is a product to perturbed quantities which resultants became zero. The term $(\vec{v}_w \times \vec{B}_L)$ is act as force which is retarded the electrons and gain energy. It is the real part of the complex quantity. Hence we have,

$$\vec{F}_p = -\mathrm{e}[\mathrm{Re}\,\vec{v}_w \times \mathrm{Re}\,\vec{B}_L)] + [\mathrm{Re}\,\vec{v}_L \times \mathrm{Re}\,\vec{B}_w]. \tag{2.21}$$

The product of two parentheses in above equation implies the product of their real parts and using identity $\mathrm{Re}\,\vec{a} \times \mathrm{Re}\,\vec{b} \cong \frac{1}{2}\mathrm{Re}(\vec{a} \times \vec{b} + \vec{a} \times \vec{b}^*)$ and we get that [Tripathi (2013)],

$$\vec{F}_p = -\frac{e}{2}\{\frac{iek_L}{m\gamma_o k_w \omega_L}\mathrm{Re}[(\vec{B}_w \times \vec{E}_L) + (\vec{B}_w \times \vec{E}_L^*)] + \frac{e}{im\gamma_o \omega_L}\mathrm{Re}[(\vec{E}_L \times \vec{B}_w) + (\vec{E}_L \times \vec{B}_W^*)]\}, \tag{2.22}$$

Now solving the above equation (2.22) part by part and one obtains [Tripathi (2013)],

$$\vec{F}_p = -\frac{e^2 A_w A_L k_L}{m\gamma_o \omega_L k_w}\hat{z}\cos(\omega_L\,\mathrm{t} - \mathrm{k_L z} - \mathrm{k_w z}) = -\hat{z}A_p\cos(\omega_L\,\mathrm{t} - k_p\,\mathrm{z}). \tag{2.23}$$

Where, $A_p = \frac{e^2 A_w A_L k_L}{m\gamma_o \omega_L k_w}$ at $1 << \frac{k_L}{k_w}$ and $k_p = (k_L + k_w)$, here phase velocity of ponderomotive wave is $v_p = \frac{\omega_L}{k_p} = \frac{\omega_L}{(k_L + k_w)} < c$. If $\frac{\omega_L}{k_p} < c$, it is possible to respond that this force act resonantly. This is the beauty of free electron laser i.e., Ponderomotive force is resonantly interact with the beams and the amplitude of this quantity is proportional to the amplitude of the regular field. It is also proportional to the amplitude of the laser field and the examination of the energy and gain is improved.

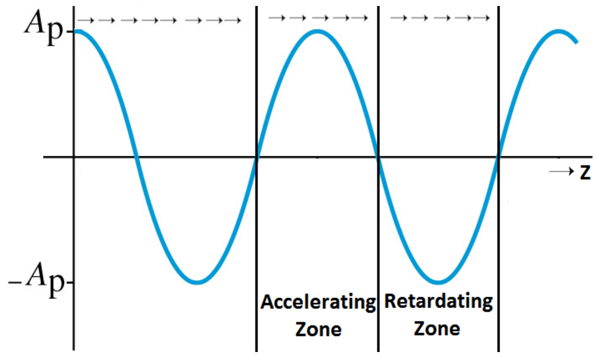

Figure 2.6: Accelerating zones and retarding zone by Ponderomotive force with A_p [Tripathi (2013)].

In a frame consideration, if the Ponderomotive force move with velocity $\vec{v}_p$ in the z-direction, it appears with new frame as $\vec{F}_{pz} = -A_p \cos k'_p \mathrm{z}'$, where, Doppler shifted frequency of this force is zero. This is called Lorentz transformation of the force. The regions of values of x prime for which $\vec{F}_{pz}$ is positive, are called the accelerating zones and the regions of values of x prime for which $\vec{F}_{pz}$ is negative, are called retarding zone as Fig. 2.6. As a resultant, the more electrons crossed from the accelerating zones to the retarding zones and less leave the retarding zone. So, there is a net bunching of electrons, in the retarding zones. Hence there is a net retardation of an electrons and net transferred of energy from the electron to wave. The equation of motion as energy balance of an electron beam is defined as [Tripathi (2013)],

$$\varepsilon_b = m\mathrm{c}^2 \gamma. \tag{2.24}$$

Where m is the rest mass, c is the velocity of light in free space and γ is the relativistic gamma factor or Lorentz factor. This is the rate of gain of energy by the electron per second, which is equal to the net electric force by wiggler velocity and written as,

$$m\mathrm{c}^2 \frac{\partial \gamma}{\partial t} = -e(\vec{E}_L.\vec{v}_w). \tag{2.25}$$

Now, there is only one electric field in the system and the velocity is sum of original velocity i.e., $(v_o\hat{z} + \vec{v}_L + \vec{v}_w)$. The dot products of all quantities with $\vec{E}_L$ are turned out to be zero. Only the quantity $\vec{E}_L.\vec{v}_w$ is survived. So, the electrons gain energy because of this term and it is useful for the evaluation of the energy of the electron as the beam travels with distance. Therefore, using equation (2.23) and rearranging equation (2.25), one obtains [Tripathi (2013)],

$$\frac{\partial \gamma}{\partial \mathrm{z}} = -\frac{e^2 A_w A_L}{\mathrm{m}^2\mathrm{c}^2\, \gamma_o v_o k_w} \cos(\omega_L\, \mathrm{t} - k_p\, \mathrm{z}) = -A\cos\psi. \tag{2.26}$$

Where $A = \frac{e^2 A_w A_L}{\mathrm{m}^2\mathrm{c}^2\, \gamma_o v_o k_w}$, $\frac{\partial t}{\partial \mathrm{z}} = \frac{1}{v_o}$ and $\psi = k_p\, \mathrm{z} - \omega_L\, \mathrm{t}$. Initially the beam velocity $v_o \geq v_p = \omega_L / k_L$, which is governed the evaluation of the energy for Ponderomotive force of an electron. So, the energy of an electron is loses, when $\cos\psi$ is positive. Then γ is decrease with z i.e., regular amplitude which are also depends on the regular wave number k_w besides the energy of the electron beam. Therefore, the phase of the wave as seen by the moving electron evolves according to the equation as [Tripathi (2013)],

$$\psi = k_p\, \mathrm{z} - \omega_L\, \mathrm{t},$$

$$\Rightarrow \frac{\partial \psi}{\partial z} = k_p - \omega_L \frac{\partial t}{\partial z} = k_p - \frac{\omega_L}{v_z}. \tag{2.27}$$

If the velocity of the electron is v_z then the relativistic gamma factor or Lorentz factor is as, $\gamma = 1/\sqrt{1-(v_z/c)^2}$ and the equation (2.26) is governed by $\gamma = 1/\sqrt{1-(v_z/c)^2}$. The energy transfer from the electron to the wave is weak; the resonant gamma factor (γ_r) is equal to $\gamma_r = 1/\sqrt{1-\omega_L^2/c^2 k_p^2}$. Therefore the deviation, $\Delta\gamma$ is introduces as $(\gamma-\gamma_r)$ by small modification between γ and γ_r. Hence using Taylor to the binomial expansion and one obtain [Tripathi (2013)],

$$\frac{\partial \psi}{\partial z} = \frac{k_p \Delta\gamma / \gamma_r}{(\gamma_r^2 - 1)} = \frac{k_p \Delta\gamma}{\gamma_r(\gamma_r^2 - 1)}. \tag{2.28}$$

This equation (2.28) is governed the evaluation of phase momentum to the Ponderomotive force. If the length of the interaction chamber is L and varies between from $z = 0$ to $z = L$ then the dimensionalizing equation (2.28) by $\xi = z/L, i.e., \partial z = L\partial\xi$ and written as,

$$P = \frac{\partial \psi}{\partial \xi} = \frac{L k_p \Delta\gamma}{\gamma_r(\gamma_r^2 - 1)}. \tag{2.29}$$

This is called momentum of the wave i.e., $P = \frac{\partial \psi}{\partial \xi}$, and again equation (2.29), differentiate w. r. to z, we have,

$$\frac{\partial P}{\partial z} = \frac{L k_p}{\gamma_r(\gamma_r^2 - 1)} \frac{\partial \Delta\gamma}{\partial z}. \tag{2.30}$$

Now dimensionalizing equation (2.30) by $\xi = z/L, i.e., \partial z = L\partial\xi$, we have,

$$\frac{\partial P}{\partial \xi} = \frac{L^2 k_p}{\gamma_r(\gamma_r^2 - 1)} \frac{\partial \Delta\gamma}{\partial z}. \tag{2.31}$$

Therefore, expressions (2.31) by (2.26) and (2.29), constitute the energy and phase evolution equations and can be rewritten as [Liu and Tripathi (1994), Tripathi (2013)],

$$\frac{\partial P}{\partial \xi} = -A\cos\psi + \alpha,$$

and,

$$\frac{\partial \psi}{\partial \xi} = P. \tag{2.32}$$

Where $A = \frac{L^2 \alpha k_p}{\gamma_r(\gamma_r^2 - 1)}$, $\alpha = \frac{e^2 A_w A_L}{m^2 c^2 \gamma_o v_o k_w}$ and $P = \frac{L k_p \Delta\gamma}{\gamma_r(\gamma_r^2 - 1)}$. Now from equation (2.32) and divided by each other, one can get,

$$\frac{\partial P}{\partial \psi}.P = -A\cos\psi, \tag{2.33}$$

After rearrange $\frac{\partial P}{\partial \xi} = (\frac{\partial P}{\partial \psi})(\frac{\partial \psi}{\partial \xi}) = (\frac{\partial P}{\partial \psi}).P = \frac{\partial (P^2/2)}{\partial \psi}$ and integrating equation (2.33) w. r. to ψ, we have,

$$\frac{P^2}{2} = -A\sin\psi + C_1. \tag{2.34}$$

Now considering here all the electrons are moves at instant time with an initial value of normalized energy $P = P_{in} = P_o$ at $\psi = \psi_{in}$, therefore C_1 is turn out and written as,

$$C_1 = \frac{P_o^2}{2} + A\sin\psi_{in}. \tag{2.35}$$

Since P^2 cannot be less than zero. It is become negative or positive due to $\sin\psi$ vary between $+1$ to -1. Hence C_1 is depends on the phase of the wave and the values are either $C_1 > A, C_1 < A$ or $C_1 = A$. If $C_1 > A$, all values of the phase of the wave are accessible i.e., all values of ψ are permissible and such electrons are called passing electrons or untrapped electrons. In the second case if $C_1 < A$, in that case all value of ψ are not permissible because of both side of the above equation (2.34) is become positive, means, the electrons cannot move in a way, such electrons are called trapped electrons. And in third case, if $C_1 = A$, the boundary between trapped and untrapped electrons and decides the separatrix. Therefore, from above equation (2.34), we have [Liu and Tripathi (1994), Tripathi (2013)],

$$P^2 = 2A(1 - \sin\psi). \tag{2.36}$$

Initially when electron beam launched with some finite energy and the electrons are evenly distributed and they are entering the wave at regular intervals with the proper phase. If they are inside the separatrix i.e., trapped electrons and those electrons are outside the separatrix, are called untrapped electrons shown as Fig. 2.7.

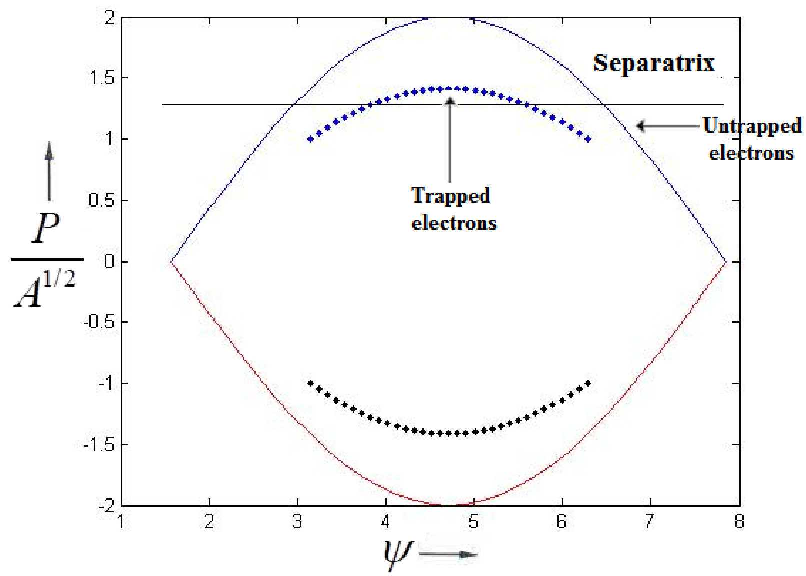

Figure 2.7: Phase (P,ψ) space trajectories of separatrix of the trapped-untrapped electrons [Liu and Tripathi (1994), Tripathi (2013)].

Since the ponderomotive waves are trapped and move inside the separatrix with a small value of momentum P, then they are lose energy and amplify signal to the radiation. Therefore, the estimation of that radiation energy of the electrons can be trapped within the potential well of the Ponderomotive force and using expressions (2.34) and (2.35), employed the expression for momentum of a trapped electron is,

$$\frac{P^2}{2} = -A\sin\psi + C_1,$$

$$\Rightarrow P = [P_o^2 + 2A(\sin\psi_{in} - \sin\psi)]^{1/2}. \quad (2.37)$$

Now from expressions (2.29) and (2.37), it is also written as,

$$\frac{\partial \psi}{\partial \xi} = [P_o^2 + 2A(\sin\psi_{in} - \sin\psi)]^{1/2}. \tag{2.38}$$

Therefore, solving this equation (2.38) iteratively to different powers for $P_o^2 >> 2A$, then binomial expands as [Liu and Tripathi (1994), Tripathi (2013)],

$$\frac{\partial \psi}{\partial \xi} = P = P_o - \frac{A}{P_o}(\sin\psi - \sin\psi_{in}) - \frac{1}{2}\frac{A^2}{P_o^3}(\sin\psi - \sin\psi_{in})^2. \tag{2.39}$$

This is the sort of expansion of P to different powers in amplitude of the Ponderomotive force.

Now the Ponderomotive force produces oscillatory electron z-velocity, $v_p\hat{z}$ and density, n_p having perturbation of the electron beam in z-direction with phase variation $\psi = \omega t - (k_L + |k_o|)z$, here $\omega = \omega_L, and, k_p = k_L + |k_o|$, therefore, the equation of motion for fluid electron beam due to perturbation as [Tripathi (2013)],

$$m[\frac{\partial(\gamma\vec{v}_b)}{\partial t} + \vec{v}_b.\nabla(\gamma\vec{v}_b)] = -e\vec{E}_L - e(\vec{v}_b \times \vec{B}_w) - e(\vec{v}_b \times \vec{B}_L), \tag{2.40}$$

First of all examine that the term is only of z-component and the total velocity is in z-direction, i.e., $v_b = v_o + v_p$, therefore, the force $-e(\vec{v}_b \times \vec{B}_w) - e(\vec{v}_b \times \vec{B}_L)$ is became zero due to perpendicular to z-axis. It is written as [Liu and Tripathi (1994), Tripathi (2013)],

$$m[\frac{\partial(\gamma\vec{v}_b)}{\partial t} + \vec{v}_b.\nabla(\gamma\vec{v}_b)] = -e\vec{E}_L, \tag{2.41}$$

Here relativistic gamma factor or Lorentz factor, $\gamma = [1-(v_b / c)^2]^{-1/2}$ and due to perturbation, the velocity in z-direction have $v_b = v_o + v_p$. Hence, the relativistic gamma factor γ is written as $\gamma = [1-(v_o + v_p)^2 / c^2]^{-1/2}$. For the sake of simplicity, using a binomial expansion and written as $\gamma = \gamma_o + \gamma_o^3 \dfrac{v_o v_p}{c^2}$. Therefore, equation (2.41) and after linearization by $\nabla = ik$ and $\dfrac{\partial}{\partial t} = -i\omega$, we have,

$$\vec{v}_p = \frac{e\vec{E}_L}{im_o\gamma_o^3(\omega - k_p v_o)}. \tag{2.42}$$

And the continuity equation of motion and linearization by $\nabla = ik$ and $\dfrac{\partial}{\partial t} = -i\omega$, we have,

$$\frac{\partial n}{\partial t} + \nabla.(nv_b) = 0,$$

$$\Rightarrow n_p = \frac{kn_o v_p}{(\omega - k_p v_o)} \tag{2.43}$$

Now from expressions (2.42) and (2.43), one obtains [Liu and Tripathi (1994), Tripathi (2013)],

$$n_p = \frac{en_o k_p E_L}{im_o\gamma_o^3(\omega - k_p v_o)^2}. \tag{2.44}$$

Therefore the nonlinear current density at (ω_L, k_L) can be written as [Liu and Tripathi (1994), Tripathi (2013)],

$$\vec{J} = -en_o\vec{v}_L - \frac{e}{2}n_p\vec{v}^*_w = -\frac{n_o e^2}{im_o\gamma_o\omega}(1+\frac{\omega_c^2 k_p^2 / k_o^2}{4\gamma_o^4(\omega-k_p v_o)^2})\vec{E}_L. \tag{2.45}$$

Now using wave equation for growth rate [Liu and Tripathi (1994), Tripathi (2013)],

$$\nabla^2 \vec{E} - \nabla(\nabla.\vec{E}) = -\mu_o(\frac{\partial \vec{J}}{\partial t} + \varepsilon_o \frac{\partial^2 \vec{E}}{\partial t^2}). \tag{2.46}$$

After linearization equation (2.46), by $\nabla = \frac{\partial}{\partial z} = ik, \frac{\partial}{\partial t} = -i\omega$ and putting the value of $\vec{J}$ from equation (2.45), one obtain [Liu and Tripathi (1994), Tripathi (2013)],

$$k_L^2 = \frac{\omega^2}{c^2}(1 - \frac{\omega_p^2}{\omega^2\gamma_o}[1+\frac{\omega_c^2 k_p^2 / k_w^2}{4\gamma_o^4(\omega-k_p v_o)^2}]), \tag{2.47}$$

Rearranging above terms and we get [Liu and Tripathi (1994), Tripathi (2013)],

$$(\omega^2 - k_L^2 c^2 - \frac{\omega_p^2}{\gamma_o})(\omega - k_p v_o)^2 = \frac{\omega_p^2 \omega_c^2 k_p^2}{4\gamma_o^5 k_w^2},$$

$$\Rightarrow (\omega^2 - k_L^2 c^2 - \frac{\omega_p^2}{\gamma_o})(\omega - k_p v_o)^2 = R. \tag{2.48}$$

Where $R = \frac{\omega_p^2 \omega_c^2 k_p^2}{4\gamma_o^5 k_w^2}$, the equation (2.48) is called the dispersion relation of FEL amplifiers and $\omega_p = (n_0 e^2 / \varepsilon_o m_o)^{1/2}$, plasma frequency of an electron cyclotron, m_o is rest mass of the electrons, n_0 is the density of the medium, ε_o is the free space permittivity and c is the speed of light in vacuum.

Now the solution of equation (2.48) is around zero simultaneously and there are two factors on LHS, i.e. [Liu and Tripathi (1994), Tripathi (2013)],

$$(k_L^2 c^2 + \frac{\omega_p^2}{\gamma_o})^{1/2} = k_p v_o, \tag{2.49}$$

Now expressing $\omega - k_p v_o = \delta$ or $\omega = k_p v_o + \delta$, around the simultaneous zeros, we get [Liu and Tripathi (1994), Tripathi (2013)],

$$\delta^3 = \frac{R}{2k_p v_o} e^{i2l\pi},$$

$$\Rightarrow \delta = (\frac{R}{2k_p v_o})^{1/3}(-\frac{1}{2} + \frac{\sqrt{3}}{2}). \tag{2.50}$$

Here $l = 0,1,2$ and for $l = 1, \delta$, we get the unstable root with positive imaginary part that gives the growth rate (Γ) of the wave [Liu and Tripathi (1994), Tripathi (2013)],

$$\Gamma = \text{Im}\,\delta = \frac{\sqrt{3}}{2}(\frac{R}{2k_p v_o})^{1/3},$$

or,

$$\Gamma = \frac{\sqrt{3}}{2}(\frac{\omega_p^2 \omega_c^2 k_p}{8\gamma_o^5 k_w^2 v_o})^{1/3}. \tag{2.51}$$

The growth rate scales as $B_w^{2/3}, k_p^{1/3}, \gamma_o^{-5/3}$ and one third power of beam current. In equation (2.50), when $\frac{\omega_p^2}{\gamma_o} << k^2c^2$, reduces equation (2.51) which gives the operating frequency of FEL amplifier and written as [Liu and Tripathi (1994), Tripathi (2013)],

$$\Gamma = \frac{\sqrt{3}}{2\gamma_o}(\frac{\omega_p^2\omega_c^2}{4k_w c})^{1/3}. \tag{2.52}$$

For fixed k_w and plans to increase the operating frequency of the FEL, its growth rate falls down as $\omega^{1/2}$. Then the power conservation efficiency of the device is,

$$\eta \approx \Gamma / 2\omega = \frac{\sqrt{3}}{4\omega\gamma_o}(\frac{\omega_p^2\omega_c^2}{4k_w c})^{1/3}. \tag{2.53}$$

So far we have ignored here the space charge effect at low beam current, however, at high beam current, the space charge effect becomes important for the collective approaches and shifting the FEL operation from the Compton to Raman regime and Cerenkov FEL [Walsh (1980), Marshall (1985), Liu and Tripathi (1994)].

2.2.4. Madey's Theorem for Gain

The FEL gain from the stimulated emission can be computed in a very simple way by using Madey's theorem [Madey (1979)]. There was the two parts of Madey's theorem, stated by Luchini, Motz and Sirigiri [Luchini and Motz (1990), Sirigiri (2001)]. The authors Grover and Pantell also treat each period of the wiggler in an FEL as a radiator and deduce the gain equation by using an array of radiators [Grover and Pantell (1985)]. The energy lost by an electron in each section of the wiggler is found by computing the decelerating force on the electron through the wiggler. Then using the conservation of energy argument one may compute the emitted radiation. The analysis is carried out in the single electron limit where the Coulomb interaction between electrons can be neglected. The Madey's theorem [Madey (1979)] comprises of two parts, the first part relates the stimulated emission to the spontaneous emission and the second part provides an expression for spontaneous emission in terms of the energy of the beam.

2.2.4.1. Stimulated emission by Madey's theorem

The rate of stimulated emission is [Luchini and Motz (1990), Sirigiri (2001)],

$$\alpha_{stim.} = N_2 W_{emiss.} - N_1 W_{absorp.}. \tag{2.54}$$

Where $\alpha_{stim.}$ is the net rate of stimulated emission, $W_{absorp.}$ and $W_{emiss.}$ are the upward and downward transition probabilities per atom per unit time and N_2 and N_1 is the electrons number in the upper and lower state respectively. If the rate of emission and absorption of the number of photons are approximately n times and the rate of spontaneous emission $W_{spont.}$, then the equation (2.54) can be written as [Luchini and Motz (1990), Sirigiri (2001)],

$$\alpha_{stim.} = (N_2 n W_{spont.}(E + h\nu) - N_1 W_{spont.}(E)). \tag{2.55}$$

Where E, is the electron energy and photons energy is $h\nu$ with photon frequency ν. In FEL, the gain is observed between spontaneous emissions through the wiggler for the finite difference. Therefore, from equation (2.55), carrying out the expansion by using Taylor's theorem and the first part of Madey's theorem is as [Luchini and Motz (1990), Sirigiri (2001)],

$$\alpha_{stim.} = Nnh\nu(\frac{d}{dE}W_{spont.}). \tag{2.56}$$

Since spontaneous emission is not a function of the energy of the particle, therefore, for the conventional laser theorem, the above equation (2.56) is neglected; however, in a FEL the above term is significant due to spontaneous emission. Thus one may tune the radiation frequency of an FEL by varying the electron energy that makes it a very attractive source for a variety of applications.

2.2.4.2. Principles of energy conservation by Madey's theorem

The spontaneous emission for the perturbation of first order electron energy is γ_1, then the second part of Madey's theorem for the spontaneous emission are as [Luchini and Motz (1990), Sirigiri (2001)],

$$\frac{d^2 I_o}{d\omega d\Omega} = \frac{m^2 c\omega^2}{8\pi^2 \varepsilon_o E_o^2} < \gamma_1^2 > . \tag{2.57}$$

Where $\frac{d^2 I_o}{d\omega d\Omega}$, called radiation intensity per unit frequency and per unit solid angle, E_o is the strength of electric field, ω is electromagnetic radiation frequency and the permittivity of free space is ε_o. Hence, the second part of Madey's theorem is also written as [Luchini and Motz (1990), Sirigiri (2001)],

$$< \gamma_2 > = \frac{1}{2}\frac{d}{d\gamma} < \gamma_1^2 > . \tag{2.58}$$

Where γ_2, the stimulated emission by the energy conservation and $<\gamma_1^2>$ is average spontaneous emission for the Madey's theorem [Shih and Yariv (1981)].

Let us consider the Lorentz force to an electron traversing through a uniform planar wiggler is [Luchini and Motz (1990), Sirigiri (2001)],

$$m\mathrm{c}^2 \frac{d\gamma}{dt} = -e\vec{E}.\vec{\mathrm{v}}. \tag{2.59}$$

If the TEM electric field seen by the electron is $\vec{\mathrm{E}} = \hat{x}E_o \cos(\omega\mathrm{t} - \mathrm{kz} + \phi)$. Therefore, linearizing equation (2.59), one obtains,

$$m\mathrm{c}^2 \frac{d\gamma_1}{dt} = -\hat{x}e\,\mathrm{v}_o\,E_o \cos(\omega\mathrm{t} - \mathrm{kz} + \phi). \tag{2.60}$$

Assuming here γ_1 is the first order perturbation to the relativistic Lorentz factor and v_o is the unperturbed dc beam velocity. If the transverse velocity of the electron is $\mathrm{v}_{xo}\cos k_w z$ (due to the wiggler magnetic field), where $k_w = 2\pi/\lambda_w$ and λ_w is the wiggler period. Hence equation (2.60) can be written as,

$$m\mathrm{c}^2 \frac{d\gamma_1}{dt} = -e\,\mathrm{v}_{xo}\,E_o \cos(\omega\mathrm{t} - \mathrm{kz} + \phi)\cos(k_w z),$$

$$\Rightarrow \frac{d\gamma_1}{dt} = -\frac{e\,\mathrm{v}_{xo}\,E_o}{2m\mathrm{c}^2}[\cos(\omega\mathrm{t} - \mathrm{kz} - k_w z + \phi) + \cos(\omega\mathrm{t} - \mathrm{kz} + k_w z + \phi)]. \tag{2.61}$$

At this point it must be mentioned that for the Doppler up-shifted scattered wave to be synchronous with the electron beam the following relation is hold as,

$$\frac{\omega}{\mathrm{v}_{zo}} - \mathrm{k} - k_w \approx 0, \tag{2.62}$$

Again integrating equation (2.61) w. r. to time and noting that $\mathrm{v}_{zo} = dz/dt$ is the longitudinal velocity of the electron beam, assuming here $\Delta k = \frac{\omega}{\mathrm{v}_{zo}} - \mathrm{k} - k_w$ and l is the length of the interaction region. Therefore, we have [Luchini and Motz (1990), Sirigiri (2001)],

$$\gamma_1 = -\frac{e\,\mathrm{v}_{xo}\,E_o}{2m\,\mathrm{c}^2\mathrm{v}_{zo}}\left(\frac{\sin(\Delta kl + \phi) - \sin(\phi)}{\Delta k}\right). \tag{2.63}$$

In equation (2.61), the terms arising from the integration of the second term $\cos(\omega \mathrm{t} - \mathrm{kz} + k_w z + \phi)$ is dropped because at synchronism given by equation (2.62) while the term $\cos(\omega \mathrm{t} - \mathrm{kz} - k_w z + \phi)$ is dominant term and have also assumed that v_{zo} to be constant and neglected the phase excursions that produced due to the variation in v_z. Now equation (2.63), squaring both side and averaging γ_1 over ϕ which gives [Luchini and Motz (1990), Sirigiri (2001)],

$$<\gamma_1^2> = \frac{1}{2}\left(\frac{eE_o\,\mathrm{v}_{xo}}{2m\,\mathrm{c}^2\mathrm{v}_{zo}}\right)^2\left(\frac{\sin(\frac{\Delta kl}{2})}{(\frac{\Delta kl}{2})}\right)^2. \tag{2.64}$$

Since the conservation of canonical momentum, which follows that $(P - eA)$ is a constant, here P is the momentum and A is the vector potential of the wiggler field. Therefore the rms value of the magnetic wiggler initially and by using $B_w = \nabla \times A_w$, can be written as,

$$A_w = \frac{\lambda_w B_w}{2\pi}, \qquad (2.65)$$

Where A_w and B_w are the rms value of the vector potential and magnetic field of the wiggler respectively. The transverse component of the canonical momentum is zero to ensure along the z-axis, then $P_{xo} = eA_o$ and we have [Luchini and Motz (1990), Sirigiri (2001)],

$$\beta_{xo}\big|_{rms} = \frac{eA_w}{\gamma mc} = \frac{\alpha_w}{\gamma}. \qquad (2.66)$$

Where, $\alpha_w = \frac{eA_w}{mc} = \frac{eB_w}{k_w mc}$. Now from expressions (2.64), (2.65) and (2.66), we have [Luchini and Motz (1990), Sirigiri (2001)],

$$<\gamma_1^2>= \frac{1}{2}\left(\frac{eE_o l}{2m\mathrm{c}^2}\right)^2\left(\frac{\alpha_w}{\gamma\beta_{zo}}\right)^2\left(\frac{\sin(\frac{\Delta kl}{2})}{(\frac{\Delta kl}{2})}\right)^2. \qquad (2.67)$$

Finally seeing that the spontaneous emission is depends on $(\sin(\frac{\Delta kl}{2})/(\frac{\Delta kl}{2}))^2 = (\sin(\phi)/(\phi))^2$.

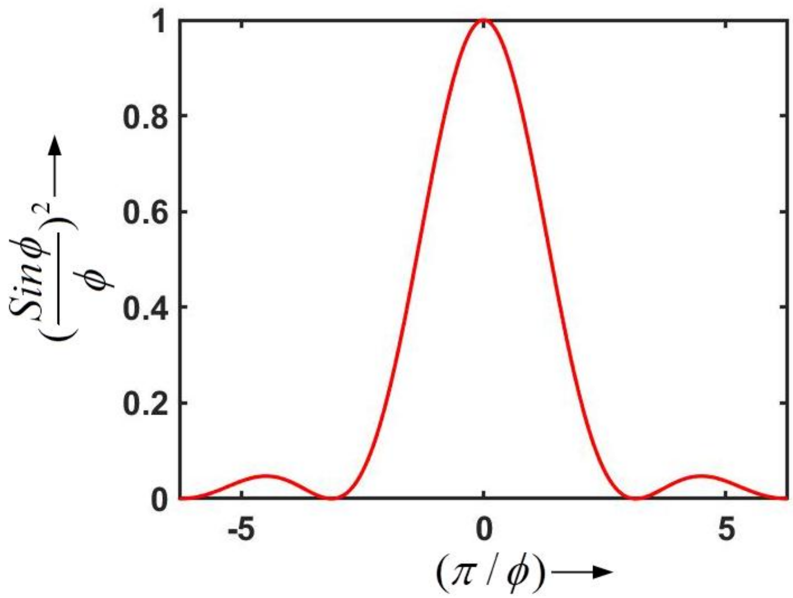

Figure 2.8: Spectral function for the spontaneous emission of a cold beam helical wiggler [Gover *et al.* (1984)].

The spontaneous emission spectrum peaks for a zero frequency mismatch, i.e., $(\phi = 0)$ is given in Fig. 2.8 have assumed a planar wiggler configuration in this analysis and ignored the effect of the planar wiggler by assuming that the longitudinal velocity is conserved. This effectively leads to an FEL in a helical wiggler and hence the plot in Fig. 2.8 depicts the emission spectrum for an FEL in a helical wiggler also [Sirigiri (2001), Gover *et al.* (1984)]. Now substituting equation (2.67) into Madey's theorem equation (2.57), we have obtained the expression for the spontaneous emission as [Luchini and Motz (1990), Sirigiri (2001)],

$$\frac{d^2 I_o}{d\omega d\Omega} = (\frac{\mu_o}{\varepsilon_o})^{1/2} (\frac{eIk^2 l^2}{32\pi^3 \beta_{zo}^2})(\frac{\alpha_w}{\gamma})^2 (\frac{\sin(\frac{\Delta kl}{2})}{(\frac{\Delta kl}{2})})^2. \tag{2.68}$$

Where I , is the beam current, $\beta_{zo} = \mathrm{v}_{zo} / \mathrm{c}$ is the normalized longitudinal velocity of the electron and μ_o is the permeability of free space.

The first order perturbation of the electrons by the electromagnetic field causes bunching of the electrons without a net energy transfer. Hence the second part of Madey's theorem as equation (2.58) and the principle of conservation of energy the gain of the FEL is equal to be [Luchini and Motz (1990), Sirigiri (2001)],

$$Gain = -\frac{mc^2 \frac{I}{e}}{\frac{1}{2}(\varepsilon o E_o^2 c A_{area})} < \gamma_2 > . \tag{2.69}$$

Where A_{area} is the cross-sectional area of an optical wave field and assuming that it is much greater than that of electrons in the FEL amplifiers. Now using expressions (2.58), (2.86) and (2.69) and the expression is as [Luchini and Motz (1990), Sirigiri (2001)],

$$< \gamma_2 > = \frac{1}{2}\left(\frac{eE_o \alpha_w l}{2mc^2 \gamma \beta_{zo}}\right)^2 \frac{d}{d\phi}\left(\frac{\sin\phi}{\phi}\right)^2 \Bigg|_{\phi=(\frac{\Delta kl}{2})} \frac{d}{d\gamma}\left(\frac{\Delta kl}{2}\right). \tag{2.70}$$

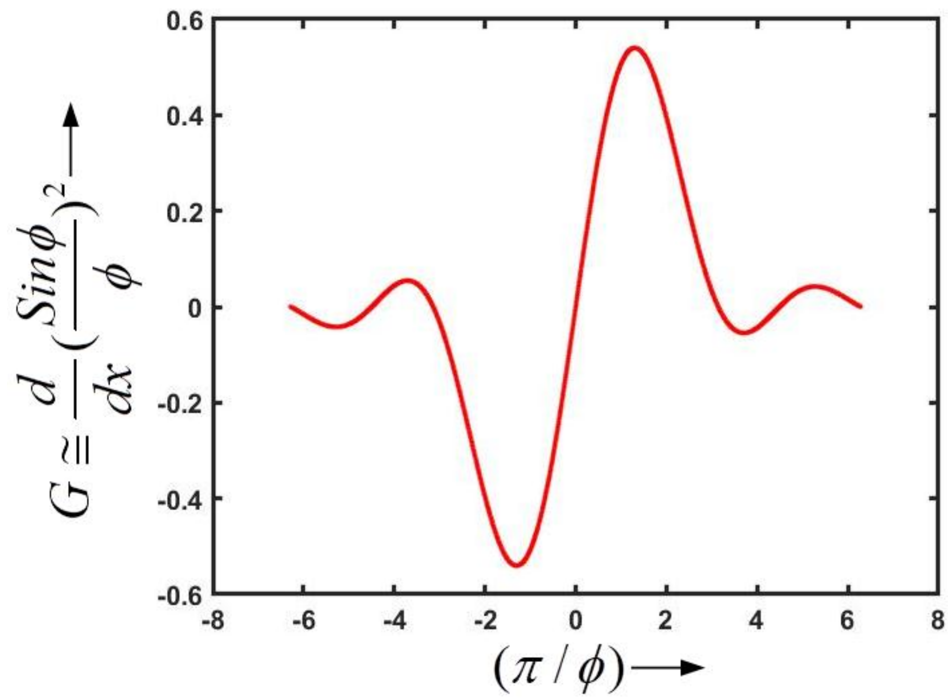

Figure 2.9: Gain function for the spontaneous emission of a cold beam helical wiggler [Luchini and Motz (1990), Sirigiri (2001)].

The gain function for the spontaneous emission of a cold beam helical wiggler is shown in Fig. 2.9. Here ignored the variation of (v_{xo}/v_{zo}) with γ as compared to the variation of Δkl with γ and the derivative of Δkl with respect to γ can be evaluated. Hence the expression for the total gain of an FEL is as [Luchini and Motz (1990), Sirigiri (2001)],

$$Gain = \frac{1}{8}(\frac{\mu_o}{\varepsilon_o})^{1/2}(\frac{eI}{mc^2})(\frac{\alpha_w}{\gamma})^2 \frac{kl^3(1+\alpha_w^2)}{2\beta_{zo}^5\gamma^3}\frac{d}{d\phi}(\frac{\sin\phi}{\phi})^2\Bigg|_{\phi=(\frac{\Delta kl}{2})} . \qquad (2.71)$$

Also the Gain function for the spontaneous emission of a cold beam helical wiggler express as Fig.2.9, that for a positive gain i.e., $-\pi < \phi < 0$ and in particular the maximum gain occurs at $\phi = -1.3$. This value of ϕ corresponds to the electrons travelling slightly faster than the electromagnetic wave and transferred energy to the wave when they tend to get pulled back into phase. The spontaneous and stimulated emission is obtained by using Madey's theorem as done [Luchini and Motz (1990), Sirigiri (2001)].

2.3. Raman regime operation in FEL amplifiers

Raman regime is mode for the free space charge wave. Consider a uniform beam of cold electrons of density n_o and the velocity $v_b\hat{z}$ is subjected to an electrostatic perturbation $\vec{E} = -\nabla\Phi, \Phi \approx e^{-i(\omega t - kz)}$. Here is the consideration only of $(\omega, k + k_o)$ and real part of the forces. The product of two parentheses in above equation implies the product of their real parts and using identity $\mathrm{Re}\,\vec{a} \times \mathrm{Re}\,\vec{b} \cong \frac{1}{2}\mathrm{Re}(\vec{a} \times \vec{b} + \vec{a} \times \vec{b}^*)$ and we get that [Walsh (1980), Marshall (1985), Liu and Tripathi (1994)],

$$\vec{F}_p = -\frac{e}{2}[(\vec{v}_b \times \vec{B}_o^*) + (\vec{v}_o^* \times \vec{B}_b)]. \tag{2.72}$$

Let the wiggler magnetic field and the laser fields be [Marshall (1985), Liu and Tripathi (1994)],

$$\vec{E}_o = \hat{x} A e^{-i(\omega_o t - k_o z)},$$

and,

$$\vec{B}_o = \frac{\vec{k}_o \times \vec{E}_o}{\omega_o}. \tag{2.73}$$

Therefore, from equation of motion and Maxwell's third equation for relativistic electron beams, we have [Marshall (1985), Liu and Tripathi (1994)],

$$m[\frac{\partial \gamma_o v_b}{\partial t} + v_b \nabla .(\gamma_o v_b)] = -e\vec{E}_b - e(\vec{v} \times \vec{B}_o),$$

and,
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}. \tag{2.74}$$

Assuming that the convective term is equal to be zero and $\omega_c = \frac{eB_o}{m}$. Now the linearization of equation (2.74), by $\nabla = ik$ and $\frac{\partial}{\partial t} = -i\omega$, one obtain [Marshall (1985), Liu and Tripathi (1994)],

$$-i\omega_o \gamma_o v_b = -\frac{e\vec{E}}{m} - (v_b \times \omega_c), \tag{2.75}$$

and,
$$ik\vec{E} = -(-i\omega_o)\vec{B},$$

$$\Rightarrow \vec{B} = \frac{k \times \vec{E}}{\omega_o}. \tag{2.76}$$

Since wave are circularly polarized then separate an impart oscillatory wave by equations (2.74) and (2.75) in x direction and y direction as $\vec{v}_{oy} = i\vec{v}_{ox}, and, \vec{v}_{oz} = \vec{v}_b$, we have,

$$\vec{v}_o = \hat{x}\frac{eB_o}{im\gamma_o k_o},$$

and,
$$v_b = \hat{z}\frac{eE}{im\gamma_o k_o}. \tag{2.77}$$

Now from expressions (2.72), (2.73), (2.76) and (2.77), we get that [Marshall (1985), Liu and Tripathi (1994)],

$$\vec{F}_p = -\frac{e}{2}[(\vec{v}_b \times \vec{B}_o^{\,*}) + (\vec{v}_o^{\,*} \times \vec{B}_b)] = \hat{z}\mathrm{i}\frac{e}{2}(\frac{\omega_c k}{\omega\gamma_o k_o} A)e^{-(\omega t - kz)}. \tag{2.78}$$

Then the ponderomotive force $\vec{F}_p = e\vec{E}, \vec{E} = -\nabla\Phi, \Phi \approx e^{-i(\omega t - kz)}$, produces z-velocity, v_2 and density, n_2 perturbation of the electron beam, hence from equation (2.78), one obtain [Marshall (1985), Liu and Tripathi (1994)],

$$\vec{v}_2 = -\frac{\vec{F}_p}{im\gamma_o^3(\omega - kv_o)},$$

and,

$$n_2 = -\frac{n_o e k^2 \Phi}{m\gamma_o^3(\omega - kv_b)^2}. \tag{2.79}$$

Which on using the Poisson's equation $\nabla^2\Phi = 4\pi en, (ik)^2\Phi = en/\varepsilon, \varepsilon\Phi = 0$ that yields for free space [Marshall (1985), Liu and Tripathi (1994)],

$$\varepsilon = 1 + \chi_b = 1 - \frac{\omega_p^2}{\gamma_o^3(\omega - kv_b)^2}. \tag{2.80}$$

Where ε is the permittivity and χ_b is the susceptibility of the medium in wave and here $\varepsilon = 0$, gives the space charge mode [Marshall (1985), Liu and Tripathi (1994)], i.e.,

$$\omega = kv_b \pm \frac{\omega_p}{\gamma_o^{3/2}}. \tag{2.81}$$

The one with the lower sign has $\omega\partial\varepsilon/\partial\omega < 0$, i.e., mode of energy is negative and the coupling of negative energy mode (ω, k) with the wiggler $(0, k_o)$ that produces the amplifier radiation $(\omega_{1,} k_1)$ in the operation of Raman Regime (RR). The effect of space charge feeds energy to amplifier which is become more and more negative that leading to the simultaneous growth of the beam space charge and radiation charge mode [Marshall (1985), Liu and Tripathi (1994)].

The density perturbation can be written from equations (2.79) and (2.80) with space charge effect potential (Φ) and ponderomotive potential (Φ_P) and (Φ) is replaced by $(\Phi+\Phi_P)$ i.e., $n_2 = -[n_o e k^2(\Phi+\Phi_P)/m\gamma_o^3(\omega-kv_b)^2]$. The Poisson's equation yields as,

$$\varepsilon\Phi + \chi_b\Phi_P = 0,$$

$$\Rightarrow \varepsilon\Phi = -\chi_b\Phi_P. \tag{2.82}$$

Then the current density at $(\omega_{1,} k_1)$, using continuity equation, we have,

$$\vec{J}_1 = -en_o\vec{v}_1 - \frac{1}{2}en_2 v_o = -\frac{n_o e^2\vec{E}}{im\gamma_o\omega} + \frac{k^2}{8\pi}\Phi\, v_o. \tag{2.83}$$

Now using $\vec{J}_1$ in the wave equation, one obtains [Marshall (1985), Liu and Tripathi (1994)],

$$k_1 E_{1\perp} - \frac{\omega_1^2}{c^2}E_{1\perp} = \frac{4\pi i\omega_1}{c^2}\vec{J}_{1\perp},$$

$$\Rightarrow (\omega^2 - \frac{\omega_p^2}{\gamma_o} - k_1^2c^2)E_{1\perp} = -\frac{i\omega_1}{2}k^2\Phi\, v_o. \tag{2.84}$$

Comparing expressions (2.82) and (2.84), we have [Marshall (1985), Liu and Tripathi (1994)],

$$(\omega^2 - \frac{\omega_p^2}{\gamma_o} - k_1^2 c^2)\varepsilon = -\frac{k^2 \left|\mathrm{v}_{o\perp}\right|^2}{2} \chi_b. \tag{2.85}$$

The simultaneous zero of the two factors on LHS gives the frequency of operation of the FEL and the equation (2.85), called the dispersion relation of FEL amplifiers [Marshall (1985), Liu and Tripathi (1994)].

$$\omega = k v_b - \frac{\omega_p}{\gamma_o^{3/2}}. \tag{2.86}$$

For $\omega \approx \frac{\omega_p}{\gamma_o^{1/2}}, \omega \approx 2\gamma_o^2 (k_o v_b - \frac{\omega_p}{\gamma_o^{3/2}})$, around this frequency we expand $\omega = \omega_r + i\delta$ and from equation (2.85) to obtain growth rate (Γ) as [Marshall (1985), Liu and Tripathi (1994)],

$$\Gamma = \mathrm{Im}\,\delta = \frac{k\left|\mathrm{v}_{o\perp}\right|}{2} (\frac{\omega_p}{\gamma_o^{3/2}\omega})^{1/2}. \tag{2.87}$$

The neglect of collective effects implies $\chi_b \approx 1$, so that $\frac{\omega_p}{\gamma_o^{3/2}} < \Gamma$. This defines the boundary condition between Raman and Compton regimes.

2.3.1 Nonlinear state of Raman regime in FEL amplifiers

To understand the dynamic of trapped electrons in the ponderomotive wave (neglecting space charge effects), the single particle equation of motion,

$$\frac{\partial \gamma_e}{\partial z} = -\frac{eE_p}{mc^2}\cos(\omega t - kz). \tag{2.88}$$

Where, $\gamma_e = (1+\frac{P_\perp^2 + P_z^2}{m_o^2 c^2})^{1/2} = (1-\frac{v_{o\perp}^2}{c^2}-\frac{v_z^2}{c^2})^{-1/2}, E_p = k\left|\phi_p\right|$, P is the momentum of electrons and has written as $\frac{\partial P_z}{\partial t} = v_z \frac{\partial P_z}{\partial z} = \frac{mc^2 \partial \gamma_e}{\partial z}$. It is very useful for defining the resonant gamma factor, γ_r of an electron moving with the phase velocity of the wave $\gamma_r = (1-\frac{v_{o\perp}^2}{c^2}-\frac{\omega^2}{k^2c^2})^{-1/2}$.

Let $\Delta\gamma_e = \gamma_e - \gamma_r, \psi = kz - \omega t$ to write [Marshall (1985), Liu and Tripathi (1994)],

$$\frac{\partial \Delta\gamma_e}{\partial z} = -\frac{eE_p}{mc^2}\cos\psi. \tag{2.89}$$

And differentiate $\psi = kz - \omega t$ w.r.to z and taking $k = \frac{\omega}{c}(\frac{1}{1-1/\gamma_r^2})^{1/2}$ or $\frac{\omega}{c}(1-1/\gamma_r^2)^{-1/2}$, then,

$$\frac{\partial \psi}{\partial z} = k - \frac{\omega \partial t}{\partial z} = k - \frac{\omega}{v_z} = \frac{\omega}{c}[(1-1/\gamma_r^2)^{-1/2} - (1-1/\gamma_e^2)^{-1/2}],$$

$$\Rightarrow \frac{\partial \psi}{\partial z} = \frac{\omega \Delta\gamma_e}{2c(\gamma_r^2 - 1)^{3/2}}. \tag{2.90}$$

Now dimensionalizing 'z' by the length of the interaction region L, i.e., $\partial z = L\partial\xi$ and putting in equation (2.90), we have [Marshall (1985), Liu and Tripathi (1994)],

$$\frac{\partial \psi}{\partial \xi} = \frac{\omega L \Delta\gamma_e}{2c(\gamma_r^2 - 1)^{3/2}}. \tag{2.91}$$

This is called momentum of the wave i.e., $P = \frac{\partial \psi}{\partial \xi}$,

$$P = \frac{\partial \psi}{\partial \xi} = \frac{\omega L \Delta \gamma_e}{2c(\gamma_r^2 - 1)^{3/2}}. \tag{2.92}$$

Now again differentiate w.r.t. to ξ, and putting the value of $\frac{\partial \Delta \gamma_e}{\partial z} = -\frac{eE_p}{mc^2}\cos\psi$, we get [Marshall (1985), Liu and Tripathi (1994)],

$$\frac{\partial P}{\partial \xi} = \frac{\partial}{\partial \xi}\left(\frac{\partial \psi}{\partial \xi}\right) = -\frac{eE_p L^2 \omega}{2mc^3(\gamma_r{}^2 - 1)^{3/2}}\cos\psi. \tag{2.93}$$

This is called equation of energy evaluation equation of the wave. Hence [Marshall (1985), Liu and Tripathi (1994)],

$$\frac{\partial P}{\partial \xi} = -A\cos\psi,$$

and,

$$P = \frac{\partial \psi}{\partial \xi}. \tag{2.94}$$

Where, $A = \frac{eE_p L^2 \omega}{2mc^3(\gamma_r{}^2 - 1)^{3/2}}$. This is constitute the set of energy and momentum equations with the consideration of constant A, however, an electron lose energy to the wave between $-\pi/2$ to $\pi/2$ with change in ψ at the exit point, $(\xi = 1)$ is π, i.e., $P_{exit} - P_{entry} \approx \pi$ [Marshall (1985), Liu and Tripathi (1994)]. Hence the FEL energy conversion efficiency is as,

$$\eta = \frac{\gamma_e(\xi = 0) - \gamma_e(\xi = 1)}{\gamma_e(\xi = 0) - 1} \approx \frac{\pi}{k_o L}. \tag{2.95}$$

2.3.2. Gain estimate of Raman regime in FEL amplifiers

Since the single pass amplification of radiation is small. It is worthwhile on solving equation (2.94), it can be obtain an interesting results for the constant value of A and written as [Marshall (1985), Liu and Tripathi (1994)],

$$P^2 = -2A\sin\psi + P_{in}^2 + 2A\sin\psi_{in}. \tag{2.96}$$

Where $P_{in} = P_{\xi=0}$ and $\psi_{in} = \psi_{\xi=0}$ are the values of P and ψ at the entry point $z = 0$. Equation (2.96) gives Phase (P,ψ) space trajectories of the trapped particles for different values of P_{in}, ψ_{in} as given in Fig. 2.10 [Marshall (1985), Liu and Tripathi (1994)]. For $P_{in}^2 + 2A\sin\psi_{in} \le 2A$, all value of ψ are not accessible (since P^2 has to be >0), i.e., the trajectories of particles are localized, representing trapped particles. Therefore, the separatrix is given by,

$$P^2 = 2A(1-\sin\psi). \tag{2.97}$$

Fig. 2.10 follows equation (2.97). If the initial conditions of an electrons is $C_1 < A$, i.e., inside the circle are called trapped electrons while outside the circle i.e., $C_1 > A$, the electrons trajectory are called untrapped. If $C_1 = A$, the boundary between trapped and untrapped electrons are called the separatrix. Here, $C_1 = \frac{P_o^2}{2} + A\sin\psi_{in}$. Initially when electron beam launched with some finite energy and the electrons are evenly distributed and they are entering the wave at regular intervals with the proper phase.

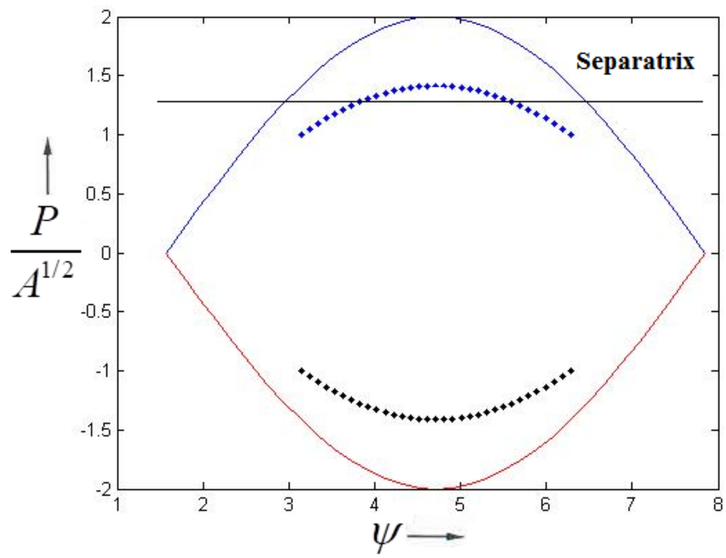

Figure 2.10: Phase (P,ψ) space trajectories of the trapped electrons [Marshall (1985), Liu and Tripathi (1994)].

If they are inside the separatrix i.e., trapped electrons and those electrons are outside the separatrix, are called untrapped electrons. Since the ponderomotive waves are trapped and move inside the separatrix with a small value of momentum P, then they are lose energy and amplify signal to the radiation.

Then the energy lost by an electron $\Delta P \equiv P_{in} - P(\xi = 1)$ in passing through the interaction region is $\Delta P \equiv < -(P_1 + P_2) >_{\xi=1}$. Therefore the average ΔP over the initial phases yields [Marshall (1985), Liu and Tripathi (1994)],

$$< \Delta P >= \frac{A^2}{P_{in}^3}[1 - \cos P_o - \frac{P_o}{2}\sin P_o] = -\frac{A^2}{8}\frac{d}{dx}(\frac{\sin^2 x}{x^2}) = \frac{A^2 G}{8}. \tag{2.98}$$

Where, $x = \frac{P_{in}}{2}$ or $x = \frac{P_o}{2}$. Hence the Fig.2.11, shows the variation of gain as function, $G \equiv -\frac{d}{dx}(\frac{\sin^2 x}{x^2})$, as a function of $P_{in} = P_o$ or $\gamma_o - \gamma_r$. If $\gamma_o > \gamma_r$, the net transfer energy from the electrons to the wave [Marshall (1985), Liu and Tripathi (1994)], however, an estimate of efficiency, in the Compton regime of FEL operation, electron gives energy to the wave as long as $v_b > \omega / k$. Hence the gain function $'G'$, with the variation of electron momentum (P_o) and the net electrons energy is transfer to the ponderomotive wave which is quite considerable at $V_b \geq 40KV$ [Pant and Tripathi (1994)]. An efficiency of the wave is enhanced adiabatically with slowing down the Ponderomotive waves also. The positive gain function for the spontaneous emission of a cold beam helical wiggler expressed between $\pi < x < 0$ and the maximum gain occurs at $x = -1.3$. This value of x corresponds to the electrons travelling slightly faster than the electromagnetic wave and transferred energy to the wave. During the emission process, electron velocity thus falls from $v_b = (\omega + \delta_r)/k$ to $v_b = \omega / k$, gives an efficiency [Liu and Tripathi (1994)],

$$\eta = \frac{\gamma_e(\frac{\omega+\delta_r}{k}) - \gamma_e(\frac{\omega}{k})}{\gamma_e(\frac{\omega+\delta_r}{k}) - 1} = (\frac{\delta_r}{\omega})\frac{\gamma_o^3 \pi}{\gamma_o - 1}. \tag{2.99}$$

In the Raman case the frequency $\omega = kv_b - \frac{\omega_p}{\gamma_o^{3/2}}$, of the space charge mode is detuned as the beam loses energy.

$$\eta \approx (\frac{\gamma}{\omega_o})\frac{\gamma_o^3}{\gamma_o - 1}. \tag{2.100}$$

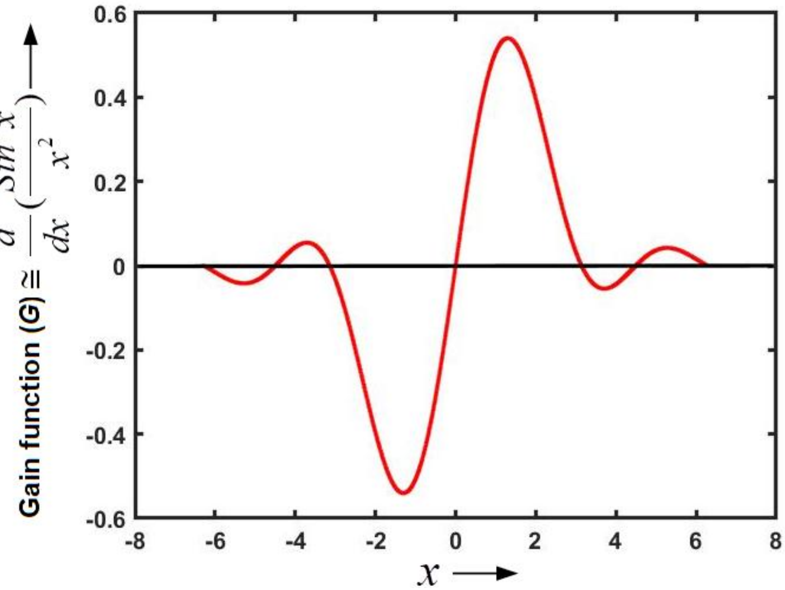

Figure 2.11: Gain (G) as a function of initial electron energy [Marshall (1985), Liu and Tripathi (1994)].

2.4. Conclusion

The fundamental analysis of the linear and nonlinear formalism has been presented to explore the analysis of FEL amplifiers and their behavior into the interaction region. In the present chapter, the working principal of FEL amplifier, frequency operations, mechanism of radiation, phase coherence and bunching, Madey's theorem for gain, stimulated emission by Madey's theorem, principles of energy conservation by Madey's theorem are discussed here. The Raman regime operation, nonlinear state of Raman regime and gain estimate of Raman regime in FEL amplifier has been presented to investigate the beam-wave interaction behavior in an interaction chamber of the FEL amplifier. The studies of the FEL amplifiers are used in the subsequent chapters for their design analysis and performance improvements.