Chapter 5

Conclusions and Future Scope

5.1 Conclusions

The discussion presented in this entire work considers mainly the non-differentiability and uncertainty aspects of the problems arising in control theory. In this context, the relevance of considering these issues with respect to the control design methodology has been shown by considering three problems which are related to the general nonlinear systems. These are tracking problem, differentiator design problem and stabilization under uncertainty. Appropriate examples have been given to relate the problems with physical systems. The problems occurring due to non-differentiability and uncertainty have been solved by using fractional-order operators. The beauty of using fractional calculus in the design of controllers is demonstrated with the help of such problems. It can be concluded that using generalized expressions of the operators in differential equations, much broader details of the related phenomena can be captured and the same information can be used in the design of control laws to deal with the same issues for which the classical integer-order calculus fails. It results in a deeper understanding of the system and more accurate results are obtained which can be related to the actual variables of the physical systems in a more meaningful way. The ideas presented in this text can be utilized further and beyond the limits of the classical techniques. Certainly, generalization of a technique at the fundamental level creates an immense possibility to explore the answers of many puzzling questions.

5.2 Contribution

The main contribution of the work is summarized as follows:

- A technique based on fractional-order operators is proposed to relax the constraint on the integer-order derivative of the reference function in the control design related to tracking problems for nonlinear systems. In this context, sliding mode control has been considered and the relaxed condition to be satisfied by the reference function is derived for the case of a switch-controlled RL circuit for which the simulation results have also been shown. The proposed approach has been generalized for general class of nonlinear systems.
- A super-twisting algorithm based differentiator is designed using fractional-order operators. Conventional integer-order approach results in a constraint on the second-order derivative of the signal to be differentiated. This issue is highlighted and the constraint is relaxed. Further, discretization of the proposed fractional-order differentiator is done. In order to suppress chattering, implicit Euler method is used for discretization. Comparison of the proposed technique with that of the classical one is made through simulation.
- An approach to robust control design is proposed for uncertain fractional-order systems. The classical sliding mode based design guarantees robustness only when the states are on the sliding surface. During the reaching phase, the system still suffers from uncertainty and disturbances. Though integral sliding mode control ensures robustness throughout the state-space, it is based on full order of the system. In order to deal with such issues, a sliding mode control scheme is proposed which is free from any reaching phase and preserves the reduced-order design methodology.

5.3 Future Scope

The approach presented in this text has several aspects which make it applicable to a number of problems in future. The operation of taking derivatives and integrals is very common in the study of the dynamic behaviour of a system. So, there always lies a possibility of an entirely new model of the phenomenon by taking the most suitable real value of the order of the operators involved. This will need model validation with the sets of actual data. Several existing fractional-order models also suggest immense potential in the future. These include fractional-order modeling of batteries, supercapacitors, fuels cells and photovoltaic cells [232].

Considering the work presented in this text, the tracking problems can be studied in a specific application field like robotics [25] [68] [203] [204] [230] [175] [169]. Robotic manipulator arms are required to follow a desired trajectory smoothly to achieve some pre-specified objective. The proposed technique can be applied to such systems to track non-differentiable reference trajectories. Attitude tracking of spacecrafts is another such problem [216]. The problem of achieving consensus in multi-agent systems can be solved with possibly new outcomes by using fractional-order operators [210] [211] [212] [213]. Image edge detection can be further done by using fractional-order operators for possibly improved results [231]. As discussed previously, trajectory tracking by the piezoelectric actuators of the atomic force microscopes can be explored by using this approach [224]. The differentiator design problem has been one of the interesting problems in the research community. Similarly, new problems can be identified and solved by using fractional-order operators to relax the general mathematical constraints imposed on the system variables. Further, stability results can be obtained using such operators and more efficient numerical techniques can be proposed [42] [165]. Fractional-order derivatives and integrals can be explored in the context of dealing with uncertainties. The existing assumptions on the disturbances and uncertainties can be relaxed. More efficient control techniques can be proposed to effectively reduce the undesired effect of the uncertainties.

Control laws can be improved in a similar manner increasing their effectiveness. The notions of finite-time stability in fractional-order systems can be extended to arbitrary-time convergence of the fractional-order dynamics. In the context of general nonlinear fractional-order systems, the notion of sector design can be studied. For linear fractional-order systems, the corresponding linear matrix inequality conditions can be explored. As the corresponding optimization problem becomes inherently non-convex for fractional-order systems, there lies a huge possibility of generalizing the classical approach of determining stability for linear systems [108] [109]. A recent theory for showing stability is the contraction theory which uses the information of the virtual dynamics of the system trajectories to comment on their convergence. Using fractional-order operators in this stability theory may result in some significant results. Obtaining consensus for multiple agents in fractional-order systems is also one of the areas which can be explored. The fractional-order operators can result into more generalized approaches. These are some of the possible directions which may have interesting results in the context of fractional-order systems.