## CHAPTER 8 **CONCLUSIONS**

In this thesis various reduction methods are developed for continuous and discrete time interval systems. All the proposed techniques are developed for SISO systems and few of them are applicable to MIMO systems and are illustrated through the aid of examples. The work included here in deals with frequency domain techniques only. The concluding chapter is devoted to summarize the main contributions of the work.

## **8.1 SUMMARY OF RESULTS**

The first introductory chapter lists the various possible reasons for going in for reduced order models. A brief review of present day available techniques for model order reduction is given in this chapter. The reason for using reduced order models and a broad classification of model order reduction techniques for both fixed coefficients and interval systems are also given in this chapter. A brief review of stability analysis of interval systems and controller design of uncertain systems are also discussed in the same chapter.

A review of existing stability analysis of interval systems is given in chapter 2. Stability analyses for both continuous and discrete time interval systems are explained by theorem and illustrated through the aid of examples. Kharitinov theorem and its alternative method is also discussed in this chapter.

A review of existing model reduction techniques for interval systems in frequency domain is given in chapter 3. Different Routh based techniques for reduction of continuous time interval systems are also given and while for others only appropriate references are mentioned. The model order reduction technique based on Routh- Pade [156],  $\gamma - \delta$  Routh approximation [157],  $\gamma$  Routh approximation [161], Dolgin-Zeheb method [162], Routh- Factor division method [169] and Evolutionary techniques based on Routh approximation [172] obtain reduced order models by these techniques show large inaccuracies in the transient period, or may be unstable and of nonminimum phase.

In chapter 4, various mixed methods are proposed based on the common theme of selecting the denominator of the reduced order transfer function a- priority to ensure the stability while the numerator by using Factor division method and Cauer second form. In this chapter, Mihailov criterion, direct truncation method and differentiation method attempt to overcome the drawback of the methods based on Routh approximation [156, 157, 161, 162, 169, 172]. All these three methods are computationally simple, possess merit of flexibility and give stable reduced order models. In the methods for Direct Routh approximation method and Direct truncation methods are easily extended to MIMO continuous time interval systems. The concept of applying truncation method to numerator and denominator terms successively until the low order model with satisfactory performance is achieved. This method does not require any computation. Successive differentiations for the denominator and its reciprocal for finding the reduced denominators. The numerator coefficients of the model are found by using factor division method and Cauer second form. All these methods are conceptually simple and give stable reduced order models.  $\alpha$  – Factor division and  $\alpha$  – Cauer second form methods are also proposed to SISO continuous time interval systems. However,  $\alpha$  - truncation method for reduction of higher order denominator interval polynomial has been proposed involving three steps basically: (1) reciprocal transformation of the higher order denominator interval polynomial, (2) truncation, and (3) second reciprocal of the reduced order denominator interval polynomial. But, Direct Routh approximation method (DRAM) requires single step of truncation without using reciprocal transformation. The DRAM is extended to MIMO systems and illustrated example indicated that the proposed method yield better approximation than many other methods.

In Chapter 5, Differentiation method has been proposed based on modified Routh array. The comparison of the reduced order interval model step responses provides an excellent accurate approximation to the original interval model. The proposed work proves that the zeroth-order interval time moments of the original model and reduced order model are equal, whereas same is not the case with other existing methods. ISE and IAE values are measured and compared with the proposed method and with existing techniques to show the high quality and efficiency of the proposed method. It is determined that the proposed reduced interval model demonstrates good performance and majority of the necessary characteristics of the original interval system are preserved. To prove the powerfulness of the proposed method, three numerical examples of continuous interval systems are considered. The proposed method facilitates easy computation of the reduced interval models. It is confirmed from the illustrative examples that the proposed method of extension of differentiation method to interval systems will guarantee the stability of the reduced model as compared to other available methods. In this work, modified Schwarz approximation (MSA) technique has been applied to model order reduction of interval systems. With examples it has been proved that many existing methods [156, 157,161, 162, 169,172] fails to preserve the stability of reduced order systems whereas the proposed method guarantee the stability of the reduced order interval systems. The modified Schwarz approximation is computationally simple to apply for continuous time interval systems.

In Chapter 6,  $\alpha - \beta$  truncation,  $\alpha$  - factor division method and modified differentiation method applied for discrete time interval systems using linear transformation. It is shown through the example that these two methods give good approximation.

All these methods are computationally simple than the time domain methods.

In Chapter 7, stability boundary locus is used to design the PI controller for interval system.

## **8.2 HIGHLIGHTS OF CONTRIBUTIONS**

The main characteristics of these methods may be summed up as:

- (1) In Chapter 4, existing techniques are extended to interval system for the order reduction of high order SISO continuous time interval systems.
	- (a) The reduced order model is stable.
	- (b) Give a good 'overall' frequency response matching.

(c) Direct Routh approximation method and Direct truncation method are extended to MIMO continuous time interval systems.

(2) In Chapter 5, new techniques are proposed for the order reduction of

higher order continuous time interval systems.

- (a) Overcome the drawback of dependency property.
- (b) Do not require determination of eigenvalues/ eigenvectors of the original system.
- (c) Both steady state and transient response are well matched.
- (d) Retain stable reduced order systems.
- (e) It is mathematically simple and does not have any computational burden.
- (3) In chapter 6, Modified differentiation method extended to discrete time interval systems using linear transformation and bilinear transformation. Mixed method based Alpha Routh approximation applied on discrete time interval systems using linear transformation. Both these methods obtain accurate results and an example is included to show its validity.

In this thesis we have tried to solve some of the difficulties associated with frequency domain reduction methods for interval systems. It is hoped that contributions of this thesis will become a useful working tool for the control community.