CHAPTER 1
INTRODUCTION

1.1 MOTIVATION FOR MODEL REDUCTION

Every physical system can be interpreted into mathematical models. The mathematical procedure of system modeling often leads to a comprehensive description of a process in the form of high order differential equations which are difficult to use either for analysis or controller synthesis. It is hence useful, and sometimes necessary, to find the possibility of finding some equation of the same type but of a lower order that may be considered to adequately reflect the dominant characteristics of the system under consideration. Some of the reasons for using reduced order models of higher order linear systems could be:

(a) To have better understanding of the system:

A system of uncomfortably high order poses difficulties in its analysis, synthesis or identification. An obvious method of dealing with such type of system is to approximate it by a low order system which reflects the characteristics of the original system such as time constant, damping ratio, natural frequency etc.

(b) To reduce computational complexity:

The development of state-space methods and optimal control techniques have been made the design of control system for high order multivariable system quite feasible. When the order of the system become high, special numerical techniques are required to permit the calculation to be done at a reasonable cost on fast digital computers. This saves both time and memory required by the computer.

(c) To reduce Hardware complexity:

A control system design for a high order system is likely to be very complicated and of a high order itself. This is particularly true for controllers based on optimal control theory. Controller designed on the basis of low order model will be more reliable, less costly and easy to implement and maintain.

1.2 MODEL ORDER REDUCTION: HISTORICAL ASPECT

In recent years, this field has been extended to the numerical analysis. This shows a very good competitive effect in the field of MOR. Excellent textbooks [1-9] have elaborated the theoretical developments of the MOR techniques. Different reduction techniques have been proposed and algorithms of various computational complexities have been developed for different class of systems. In the literature, several classifications are divided based on the techniques used in the field of model order reduction. One main classification on the basis of time domain and frequency domain techniques. When the model reduction techniques are applied to reduce the large state space models they are called time domain reduction methods whereas when applied to the transfer function models of the system they are called frequency domain reduction methods.

There are several time domain methods are available, some of the popular techniques are discussed. Modal analysis approach is subdivided into two methods, both retain the dominant eigenvalues in the reduced system. First method was introduced by Davison [10], The Davison technique proposes that a large scale system can be reduced to low order model by taking into account the effects of the most dominant eigenvalues only. The basic principle of the method is to considering eigenvalues of the original system that are nearest from the origin and retain only dominant eigenvalues and hence dominant time constants of the original system in the reduced order model. The drawback of Davison technique is it fails to obtain accurate steady state response. To overcome this problem Davison and Chidambara [11-13] proposes alternative methods to acquire low order model through Davision technique. Davison and Chidambara proposed an approach for model reduction. In order to nullify the steady state error by ignoring the left out states of the transient response and considering only the steady state contribution of these states. Davison and Chidambara method has a limitation, it produce steady-state error of the system response. To overcome this limitation Davison [14] proposes improved technique based on his previous methods. Second method had proposed by Marshall [15], this is an alternative method compared to the Chidambara technique, however these methods are quite similar to each other for obtaining reduced order state equation. The main limitation of the Marshall technique is the difficulty in determining the dominant poles of the original system, which should be retained in the reduced order models.

One of the popular methods in the time domain techniques for modelling of higher order system is aggregation method [16], the aggregation concept in model simplification and control of large scale dynamic systems was pioneered by Aoki. Later, based on aggregated from Chen and Shieh [17] introduced continued fraction technique and extended by many other researchers. An improved technique had introduced by Gruca and Bertrand [18], to improve the quality of simplified aggregated models of the system without increasing order of the state differential equations. It consisted of introduction of delay in the output vector of aggregated modal to minimize a quality index function of the output vector of aggregated model to minimize a quality index function of the output vector. However, the numerical difficulties and the absence of guidelines for selecting the weighting matrices in performance index of this method were well observed by the researchers. Inooka and Obinata [19] proposed a method based on combining the method of aggregation and integral square error. An important variation of dominant eigen value concept was proposed by Gopal and Mehta [20] where in the original order system is replaced by three models, successfully representing the initial, intermediate and final stage of the transient response.

An alternative approach to the problem of order reduction is based on the construction of a best invariant sub-space in the state space that the projection error is minimal. The geometrical approach is developed by the Mitra [21]. The Krylov subspace-based order reduction is developed using projection based method that was initially applied in the field of large-scale linear mathematical problems and later it has been developed in the field of control engineering and model order reduction [22-26]. Krylon subspace has been extended to multi-input-multi-output (MIMO) systems for model order reduction [27]. Krylon subspace iteration methods were motioned among the “top 10 algorithms of the 20th century” [28-29]. Krylov subspaces are essential in this method to estimate projection matrices that are shown to form bases of these subspaces when aiming at moment and/or Markov parameter matching. To this end, numerically efficient and iterative methods known as Lanczos [30], the Arnoldi [31] and the two sided Arnoldi [32-33] algorithms are developed.

The perception of model order reduction was changed after developing the new methods such as balanced truncation [34] and optimal Hankel norm approximation [35-36]. The advantage of these techniques is obtaining stable reduced order models and their solutions are of closed form, and they have a priori frequency response bounds.

Balanced truncation technique based on balanced realization theory was proposed by Moore [34], this technique has drawn a great attention of many researchers. The extension of Moore work were done by Pernebo and Silverman [37] and provided a proof for Moores work and successfully applied to MIMO systems and also extended the technique to discrete time systems. Later, Ferando and Nicholson [38-39] applied the balanced realization with singular perturbation theory [40] for model order reduction and it is later discussed by Liu and Anderson [41] with several properties in the time domain and frequency domains.

Adamjan et al. [42] proposed a closed-form optimal solution for reduction techniques with respect to Hankel norm criterion for the scalar case. Kung et al. [43] later extended Adamjan's approach to MIMO systems by using closed-form optimal Hankel norm solution and developed minimum degree approximation algorithm. Later, Enns [44] introduced frequency weighted balanced truncation, this technique is a modification of balanced truncation technique to include frequency weightings. Drawback of Enns technique is unable to obtain stable reduced order models.

To overcome the limitations of the Enns technique instability, Lin and Chiu [45] introduced a method with strictly proper weightings, which yields stable reduced order models in case of double sided weightings. Later, Sreeram et al. [46] generalized the technique to include proper weights. Varga and Anderson [47] proved that Lin and Chiu's method is not applicable for controller reduction applications due to non pole-zero cancellation assumption required in the method. Later many researchers further developed several techniques [48-50] based on Lin and Chiu's method.

Nagar and Singh [51-52] extended and developed algorithms based on bilinear realization and also developed tools suitable for linear time-invariant continuous/ discrete, minimal/ non-minimal, stable/unstable and with or without time delay systems. In case of discrete systems the proposed algorithms are free from bilinear transformation and hence the possibility of two-fold error occurring discrete to continuous and back is completely eliminated. The preservation of DC gain for non-minimal system also investigated. Later, Deepak and Nagar [53-57] extended the methods based on frequency weighted model reduction techniques. In addition to this a new frequency weighted balanced model reduction technique is investigated based on the combination of parameterized unweighted balancing approach and modified frequency weighted partial fraction expansion techniques.

Basically frequency domain methods have been sub-divided into three subgroups.

(i) Classical reduction methods (CRM)

(ii) Stability preservation methods (SPM)

(iii) Stability criterion method (SCM)

(i) Classical reduction methods:

The first group is classical reduction method (CRM) which is based on classical theories of mathematical approximation or mathematical concepts such as Continued fraction expansion [58], Pade approximation [59] and time moment matching [60]. These methods are algebraic in nature and it can be proved that CRM approaches are equivalent to each other. The problems such as instability, non-minimum phase behavior and low accuracy in the middle and high-frequency range of reduced order model limits the applications of CRM.

Wall introduced continued fractions [58] and developed algorithms which are used for synthesize ladder network driving-point impedances. Later, several researchers contributed towards the development of continued fractions [61-84]. Chen and Shieh [61] developed a block diagram corresponds to continued fraction expansion. Based on the state- space formulation, an algorithmic method for inverting a continued fraction to a rational fraction of two polynomials is developed. It is an easy algorithm for convert a continued fraction into a rational fraction of two polynomials. The second form is not only important in RC network synthesis but also plays a significant role in control systems analysis. A major drawback of using Cauer continued- fraction expansion about $s = 0$ to derive reduced models is that it does not give consideration to the initial transient response and can, in some cases, yields unstable models. Chen and Shieh proposed continued- fraction method [62], it recognised as one of the best techniques in the field of frequency domain techniques for linear system reduction. Continued fraction method is smile and effective, and it also preserves the property of the moment-matching and for different step and polynomial inputs it retains good steady- state approximation. The major drawback of this method is it fails to obtain stable reduced order models. To achieve a better approximation to the initial transient response of the system, Chuang [63] and Shieh and Goldman [66] modified the continued fractions by combining expansions about $s = 0$ and $s = \infty$ alternately.

Later, several forms of two-point continued-fraction expansion, which include the expansion about an arbitrary point $s = a$ (a is real) have been suggested [72, 78, 80, 81] to derive reduced order models having good responses. Katsube et al. [79] proposed a multipoint continued fraction expansion about arbitrary points on the real frequency axis, to drive reduced models for continuous-time systems. However, this method involves operations of complex values, and it is thus inconvenient for computation.

Hwang and Chen [83] proposed a multipoint continued-fraction expansion (MCFE) about arbitrary points on the real axis for reduced order modelling of linear time-invariant systems. Computerized algorithms for the expansion and inversion operations of MCFE are derived. A new realization for the MCFE of a transfer function has proposed and the corresponding state equation has established. Also similarity transformation matrix has constructed to transform a state-space model in the phase variable canonical form to one in the MCFE canonical form.

Time moment matching [60, 86] is based on matching the time moments of the full models impulse response to those of the reduced model. The number of matched time moments determines in turn the order of the specified/ reduced model. The more the number of time moments are matched the more accurate will be the simplified transfer function. However, the increase in its order will diminish the usefulness of simplification. The time moment matching technique is equivalent to continued fraction expansion (CFE) approach. While CFE technique always gives the reduced order models in which the degree of the numerator is either equal or one less than the degree of denominator, such a restriction does not exist in the time moment matching method. The extension of moment matching method to the reduction of multivariable systems can be found in [85].

In the field of model reduction another interesting algorithm to procure an approximate continuous model via Markov parameter estimation from input-output data is a well-known method [87]. However, G.P. Rao [88] extended these Markov parameters for system identification and model reduction. Later, G. P. Rao [89] proposed a Markov parameter bias for model quality assessment and model reduction.

Another popular method is Pade approximation. The Pade approximation was introduced by Pade [90]. Later, Pade approximation [91-95] is used successfully to obtain a reduced order model for SISO systems. In Pade approximation method, Taylor series expansion about $s = 0$ for the higher order original systems and reduced order systems are matched up to the maximum number terms. Pade approximation is used especially when the original systems does not have any dominant poles. Pade approximation has three basic drawbacks.

- The reduced model may be unstable although the original model is stable.
- The Pade approximant often shows non-minimum phase characteristics.
- The reduced model often shows poor matching in the transient zone although the steady state values are same as for the original model.

Later, Pade approximation has been extended to multivariable systems [97-98]. To overcome the limitations of the Pade approximation an alternative approach has been proposed by Pal and Ray [96], which is known as improved Pade approximation. The improved Pade approximation removes the possibility that the reduced order model may occasionally exhibit non-minimum phase characteristics, which are actually not shown by the original system. To overcome the limitations of the classical reduction method (CRM), a new class of reduction techniques has been developed which is known as stability preservation methods (SPM).

(ii) Stability Preservation Methods (SPM):

The second subgroup is a development of CRM and includes stability preservation methods such as Routh approximation [99], $\alpha-\beta$ approximation [100], Hurwitz polynomial approximation [105], Schwarz approximation [106], Dominant Pole retention [110], stability equation method [111], the method of truncation [113], Mihailov criterion [114], differentiation method [115], factor division method [117] and $\gamma-\delta$ approximation [119].

$\alpha-\beta$ approximation [100] is based on Routh convergent, which a table is formed for determining the stability of a linear system from the characteristic equation that does not entail computing the poles. The $\alpha-\beta$ approximation has three basic steps

(i) Reciprocal transformation

(ii) α,β expansion and its truncation

(iii) Second reciprocal transformation

Later, a simple method has been developed without using the reciprocal transformation [101]. Further, the development of $\alpha-\beta$ approximation has been proposed by Krishnamurthy and Sheshadri [102-104]. The Hurwitz polynomial approximation proposed by Appiah [105] is an equivalent method to the Routh approximation method. It is based on using Pade approximants of Hurwitz polynomials. It should be noted that Hurwitz polynomial approximation gives same reduced characteristic polynomial as due to Routh approximation of Hutton and Friedland [100]. Therefore Hurwitz polynomial approximation is only a special case of Routh approximation method by which even and odd order polynomials can be reduced to even order or odd order ones. The coefficients of numerator of simplified model are computed so as to fit initial time moments.

Schwarz approximation [106-108] has been emerged has one of the important method in the frequency domain techniques for SISO systems. Later, Schwarz approximation has been extended to state space [109]. The dominant pole retention [110] the transfer function is expanded in partial fraction and the approximation retains those terms whose poles are closest to the imaginary axis. One of the problems with this method is that the poles of the system must be determined by solving the roots of the characteristic equation which is difficult task for very high order systems (say for 150^{th} order system). Another problem is that the properties of approximations obtained by this method may not be understood.

Stability equation method [111], the transfer function of the reduced orders is obtained directly from the pole-zero patterns of the stability equations of the original transfer function. Thus order of the stability equations of transfer function is first reduced and then the order of the original transfer function can be reduced. It may be noted that poles and zeros with smaller magnitudes and more dominant than those poles or zeros with larger magnitudes. The poles or zeros with larger magnitudes are discarded in this technique. A tabular approach to the stability equation has also been developed by Lucas [112] to avoid the problem of calculating the roots stability equations. Later, the truncation method was suggested by Shamash [113] where successively lower order models are produced by neglecting progressively higher order terms form numerator and denominator of higher order system until the reduced order model with satisfactory performance is achieved. The truncation method which requires no computation is as good as other methods. Bai WU Wan introduced an interesting method which is known as Mihailov criterion [114]. The Mihailov stability criterion is to improve the Pade approximation method, to the general case. Several reduced models which are obtained depend upon the different values of the constant in the original model. And bring the Mihailov frequency characteristic of the reduced model to approximate that of the original system at the low-frequency region.

Differentiation method was introduced by Gutman et al. [115]. The method is based on differentiation of polynomials. The reciprocals of the numerator and denominator polynomial of the higher order transfer functions are differentiated successively many times yield the coefficients of the reduced order transfer function. The reduced polynomials are reciprocated back and normalized. The straight forward differentiation is discarded because it has a drawback that zero with large modulus tend to better approximated than those with a small modulus. Later, Lucas [116] modified the differentiation method to remove the reciprocal transformation. Lucas introduced factor division method [117] as an alternative approach for linear system reduction by Pade approximation to allow retention of dominant modes. It avoids calculation of system time moments and solution of Pade equation by simply dividing out the unwanted pole factors. Later, Lucas [118] further developed factor division algorithm for conceptually simple. This algorithm show that factor division method may also used to retain initial Markov parameters as well as time moments in the reduced order model. A multipoint Routh canonical continued-fraction expansion [119] for the transfer function of a linear system is derived. A multi-frequency Routh approximant to the system has derived by selecting the expansion points on the imaginary axis and truncating the resulting continued-fraction expansion. A connection between the stability preservation property of the multi-frequency Routh approximants and the expansion points on the imaginary axis has developed. Thus the multi-frequency Routh approximation algorithm is flexible in deriving stable reduced order models and retaining the time moments and /or Markov parameters of the impulse response of the system. Model order reduction in frequency domain techniques has also been developed by Pal [120] and Prasad [121]. The SPM suffers from a serious drawback of lack of flexibility when the reduced model does not produce good enough approximation.

(iii) Stability Criterion Method:

The third group includes the mixed methods [122-139] and can be called as stability criterion method (SCM) in which the denominator of the reduced model is derived by one of SPM, while the numerator coefficients are obtained by one of CRM. All through this improves the degree of accuracy in the low frequency range. However, absolute stability of SCM is achieved only at the cost of a serious loss of accuracy. It is noted that the use of mixed methods is superior to the use of simple method. In particular a very few methods are discussed.

(i) Pade approximation and dominant mode retention [122].

(ii) Pade approximation and Routh stability criterion [123].

(iii) Pade approximation and Routh Hurwitz array [124].

(iv) Modified Cauer continued fraction and Stability equation method [125].

(v) Pade approximation and Mihaailov criterion [127].

(vi) Pade approximation and Routh approximation for multivariable systems [128].

(vii) Improved Pade- Routh approximation [129].

(viii) Cauer Second form and Eigen spectrum [130].

(ix) Factor division method and Eigen spectrum [130].

(x) GA and Stability equation method [132].

(xi) Pade approximation and Pole clustering method [133].

(xii) Cauer second form, GA and Mihaailov criterion [134].

(xiii) GA for continues and discrete time systems are applied [135-136].

(xiv) Big Bang Big Crunch optimization and Routh Approximation [137].

(xv) Big Bang Big Crunch optimization and Stability equation method [138].

(xvi) Factor division method and Stability equation method [139].

There are many mixed methods but in this thesis only few of them are considered which covers most of the important mixed methods.

A large number of methods have been developed for reducing the order of discrete systems using few different techniques. Model order reduction techniques for discrete systems have been divided into two groups. The first group transforms the high-order transfer function $G_n(z)$ into another one $G_n(w)$ using the bilinear transformation $z=\frac{(1+w)}{(1-w)}$ [149] or linear transformation $z=w+1$ or homographic transformation $z=\frac{w}{A+Bw}$ [140-144], where A and B are constants, n is the order of the original system and w is a new variable. Then one the of the well known methods for continuous systems is applied to obtain reduced order approximant $R_k(w)$ of $G_n(z)$. Finally, the desired reduced order of the system $R_k(z)$ is obtained by the inverse transformation of $R_k(w)$, where k is the reduced order of the system. The second group methods obtain reduced order model $R_k(z)$ directly from $G_n(z)$, i.e., without using any transformation into w domain [145, 148, 149]. Another type of techniques is based on step response matching [146] and frequency response matching [147]. Chung et al., [146] proposed a method based on simplification and identification of discrete systems via step-response matching. The advantages of this method are, the reduced order model obtain very close bandwidth region for both the time response and frequency response of the original system and in case of system identification, the identified model is similar to the original system. Later, this step response matching has been extended to MIMO systems by Mukherjee et al., [151]. However, bilinear transformation is useful as tool to verify the Schur stability via the Hurwitz stability; but, its application is restricted by a pathological case. Bilinear transformation technique is not applicable to discrete-time polynomials with one or more roots at $z = 1$. This is the major drawback of using bilinear transformation for discrete time systems.

1.3 MODEL ORDER REDUCTION OF INTERVAL SYSTEMS

Recently, the model order reduction techniques are applied for interval systems by using interval arithmetic rules.

Consider

$$e = \left[e^-, e^+\right] \equiv \left\{e \middle| e\left[e^-, e^+\right]\right\},\ f = \left[f^-, f^+\right] \equiv \left\{f \middle| f \in \left[f^-, f^+\right]\right\}$$

be real intervals. o is one of the fundamental procedure ‘addition’, ‘subtraction’, ‘multiplication’ and ‘division’, severally, for actual numbers, that is $o \in \{+, -, \times, \div\}$. The corresponding operations for intervals $[e]$ and $[f]$ by

$$[e]o[f] = \{eof \,/\, e \in [e], f \in [f]\} \tag{1.1}$$

Interval arithmetic rules [152-153] are defined as follows

Addition:

$$\left[e^-, e^+\right] + \left[f^-, f^+\right] = \left[e^- + f^-, e^+ + f^+\right] \tag{1.2}$$

Subtraction:

$$\left[e^-, e^+\right] - \left[f^-, f^+\right] = \left[e^- - f^+, e^+ - f^-\right] \tag{1.3}$$

Multiplication:

$$\left[e^-, e^+\right] \times \left[f^-, f^+\right] = \left[\min(e^- f^-, e^- f^+, e^+ f^-, e^+ f^+), \max(e^- f^-, e^- f^+, e^+ f^-, e^+ f^+)\right] \tag{1.4}$$

Division:

$$\left[e^{-},e^{+}\right]\div\left[f^{-},f^{+}\right]=\left[e^{-},e^{+}\right]\times\left[\frac{1}{f^{+}},\frac{1}{f^{-}}\right] \quad (1.5)$$

The entire range of operating conditions of many practical systems has parametric uncertainty. The model reduction techniques and design of interval systems [156]-[192] have received a great deal of attention. Bandyopadhyay et al., [156] has extended Routh-Pade approximation to interval systems for reducing higher-order continuous interval systems. The reduced order denominator polynomial is obtained by direct truncation of the Routh table, and the lower-order numerator polynomial is obtained by matching the coefficients of the power series expansions of the interval systems. Later the concept of γ-δ Routh approximation has been extended to continuous interval systems by Bandyopadhyay et al., [157]. The following is the limitations of above two Routh based approximations claimed by Hwang and Yang [159]: (1) Interval Routh extension formula cannot guarantee the successes in generating a full interval Routh array. (2) Some interval Routh approximation may not be robustly stable, even if the original interval system is stable. To reduce the computational effort, γ table formulation [161] has been introduced, instead of γ-δ table formulation [157]. However, the limitation of this method is that, they obtained reduced interval model may be unstable for the stable original interval model. Later, Dolgin and Zeheb [162] have proven that generalized Routh algorithm to interval systems does not guarantee the stability of the reduced order system. To overcome this problem, Dolgin and Zeheb [162] modified the generalized Routh array and claimed that this method could guarantee the stability of the reduced order system. Later, Yang [165] proved that Dolgin and Zeheb [162] method does not guarantee the stability of the reduced order interval system. To overcome this problem, Dolgin [166] has proposed a modified method of Routh algorithm for obtaining stable reduced order models. It is noted that there is a limitation in this method, that the interval arithmetic subtraction rule has been changed to obtain stable reduced order models. To overcome the limitation of the existing methods [156]-[157],[162], [166] Bandyopadhyay et al., [170] introduced a new method based on stable $\gamma-\delta$ Routh approximation of interval systems using Khartitonov polynomials, which guarantee the stability of the reduced order systems. However, this method does not require any interval arithmetic rules. Another alternative method has been proposed to overcome the limitation of the $\gamma - \delta$ Routh approximation, which is based on stable Routh-Pade approximation [171]. Ismail [157] and Shingare [167] extended some fixed model reduction techniques to interval systems. The above methods give us motivation to propose new techniques for reduction of interval systems.

In recent years many researchers are focusing on mixed methods reduction techniques of interval systems. Saraswathi et al., [168] proposes a method based on Eigen spectrum for the reduction of denominator coefficients and for numerator reduction Pade approximation is used. The eigen values of reduced order interval system are obtained using Eigen spectrum by preserving some of the characteristics such as centroid and stiffness of the original interval system. The numerator polynomial is obtained using Pade approximation by preserving some of the time moments and Markov parameters. In this method, the eigen values of the interval systems are obtained by using the results of [154]. Later, Selvaganesan [169] introduced a mixed method by using generalized Routh table for determining the denominator polynomial and numerator polynomial reduction is obtained by using factor division method and gain factor is used for minimizing the steady state error. The main drawback of this method uses generalized Routh table for reduction of denominator polynomial which already proves that it fails to produce stable reduced order models. Recently, Saini and Prasad [172] applied genetic algorithm technique to interval systems but the denominator reduction polynomial is reduced by generalized Routh array, which fails to obtain stable reduced order models, proved by Dolgin and Zeheb [162]. Later, Yan Zhe et al., [174] extended genetic algorithm for reduction of MIMO interval systems. Recently a simple and direct method of reduction technique for linear interval systems using Kharitonov's theorem are presented in [173], [174]-[176] to obtain stable reduced order models.

A few literatures are available for discrete interval systems reduction techniques [177]-[184]. The method of reduction for discrete interval systems using Pade approximation to allow the retention of dominant poles has been introduced by Ismail et al., [177]. The denominator polynomial of the reduced order model is formed by retaining the dominant poles of the original system, while the numerator is obtained by matching interval time moments. Later, Choo [180] proposed an alternative method for retention of dominant poles. Recently, Singh [182]-[183] proposed a reduction of discrete system is proposed, where the denominator is obtained by retaining the dominant poles thus preserving stability and numerator is obtained is obtained by matching the time moments of the system. Instead of using retention of dominant poles, Pappa and Babu [181] extended differentiation technique to discrete interval systems. Finally, Kiran and Sastry [184] applied least square method for reduction of discrete time interval systems.

Classical control system design used for fixed plant transfer functions are well known for engineers from past few decades. A major contribution on systems with uncertain parameters was introduced by the Russian mathematician Kharitionov [192], who applied Routh stability criterion to interval polynomials that is where the polynomial coefficients may be used to lie within a specific range rather than being fixed. Later these ideas have been extended to frequency response representations such as Bode and Nyquist plots for interval plant transfer functions. Recently, attention has been given to the formulation of P, PI and PID controllers to stabilize an interval plant [185]. Tan and Atherton [186] discussed important results developed in the field of robust control for uncertain systems and also various forms of uncertain structures of polynomials. Later, Smagina and Brewaer [187] design a modal P and PI regulator synthesis for a linear multivariable dynamical system with uncertain parameters in the state space. The designed regulator has to place all coefficients of the system characteristic polynomial within the assigned intervals. Huang and Wang [188] developed a controller to simultaneously stabilize the four Kharitonov-defined vortex polynomials. Different from the prevailing works, the controller is designed systematically and graphically through the search of a non- conservative Kharitonov region in the controller coefficient parameter plane. The region characterizes all stabilizing PID controllers that stabilize an uncertain plant. Later, Pujara and Roy [189] proposed technique based on the 'Polytope Algorithm' to compute first order and higher order stabilizing controller for SISO interval systems. Tan et al., [190] proposed a method based on plotting stability boundary locus in the $\left(K_p, K_i\right)$- plane and then computing the stabilizing values of the parameters of PI controller. The advantage of this technique is does not require sweeping over the parameters and also does not need linear programming to solve set of inequalities. Beyond stabilization, the method is used to shit all poles to a shifted half plane that guarantees a specified settling time of response. Babu and Pappa [191] used technique based on Particle Swarm assisted Bacterial Foraging Optimization (PSO- BFO) based hybrid algorithm to search PID controller parameters such as K_p, K_i and K_d. The algorithm has to obtain the best possible PID parameters with integral squared error (ISE) criterion minimization as the objective function. In this algorithm, the controller parameters are obtained for reduced order model later it has tested with the higher order model.

Robust stability analysis and the design of systems subject to parametric perturbations were largely ignored before the 1980's. This was due to the fact that there were no general theories that could be used for analyzing or designing control systems with uncertain parameters. However, after Kharitonov's theorem [192] there has been a huge increase in research involving real interval parametric. The most important reason for this is the original theorem of Kharitonov's [192] was originally published in 1978 in the Russian technical literature. The theorem was complicated and very difficult to understand for this reason it remains unknown for several years. Later, Barmish [194] introduced to the western literature; many papers have appeared providing simple proofs of the Kharitonov's theorem for both continuous and discrete interval systems [194]-[211]. An alternative method developed by Anderson et al., [193], shows that for $n = 3, 4$ and 5, the number of Kharitonov polynomials required to check the stability of interval polynomial are one, two, three, respectively, instead of four. Later, Hote et al., [207] proved that Anderson et al., [193] method cannot be applicable to relative stability analysis. The main advantage of this method is that there is no need to use time-consuming graphical and analytical methods for determining gain margin and phase margin. Finally graphical approach for investigation of robust stability for discrete interval systems has been developed [208]-[211].

Different tool box are available for the analysis of interval systems [213]-[215]. Tan and Atherton [213] developed a software package (AISTK- Analysis of Interval System Toolkit) for the analysis of interval systems. The package has been developed in the MATLAB environment with a powerful graphical user interface. INTLAB [214] developed for the analysis of interval systems which can be interface with MATLAB. Mastusu [215], introduced another software tool for algebraic design of interval systems control.

1.4 OBJECTIVE OF THE THESIS

The objective of this thesis is to develop new methods of order reduction applicable to plant which are described by continuous and discrete time interval models. This has been attempted for systems described interval transfer functions and only frequency domain-based methods have been considered. Modified methods are proposed that remove some of the inherent difficulties with present-day- established techniques of order reduction.

1.5 ORGANISATION OF THE THESIS

Including this introductory chapter, the thesis has been divided into six chapters. A brief review of the existing techniques of model reduction for fixed systems and interval systems is presented in chapter 1. Chapter 2 deals with the stability analysis of the interval systems. In addition to the Kharitonov's theorem another alternative methods are discussed briefly. In chapter 3, a review of the existing Routh based techniques are presented. Further, another alternative approaches are also discussed. In Chapter 4, proposed new mixed methods of model reduction for continuous interval systems are discussed. In Chapter 5, two modified methods are proposed based on differentiation method and Schwarz approximation method considering dependency property. Chapter 6, deals with a new methods of model order reduction of discrete time interval systems.

A survey of the contributions made in this thesis has been given in the concluding chapter.

The research area in this thesis is shown in Fig. 1.1.

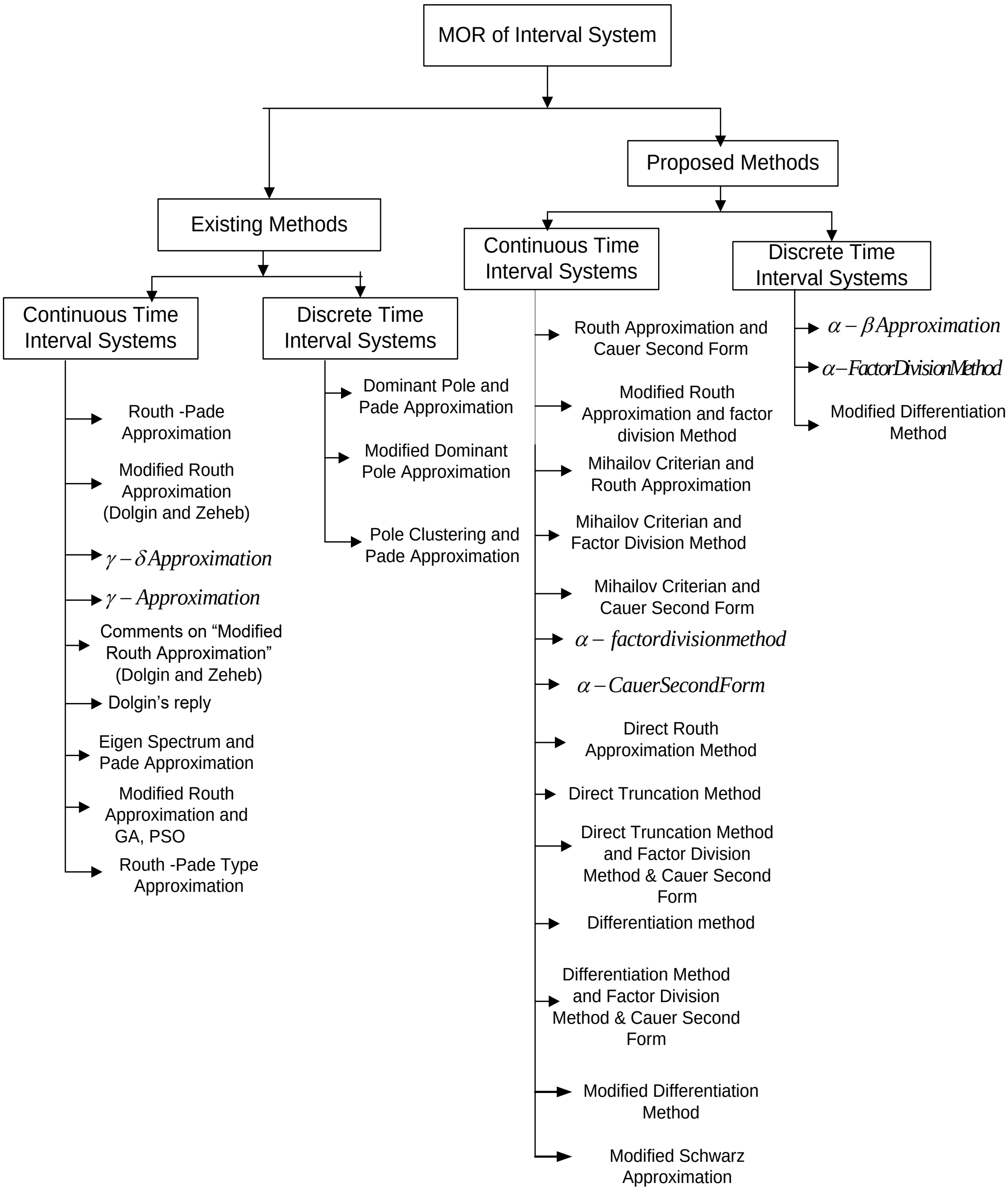

Fig. 1.1. Classification of model order reduction methods of linear interval systems**.**