

## **CHAPTER 2**

### **CONCEPTS OF SIGNAL PROCESSING & FEATURE VECTORS**

#### **EXTRACTION**

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##### **2.1 Introduction**

The unprecedented advancement in the field of signal processing over the last few decades makes it a competent tool for analyzing the behavior of any waveform. Signal processing mechanism significantly helps in identifying the critical information about the waveform by transforming the factual signal to the other domain. It allows the unfolding of main features of the waveform that are not feasible to discern in the actual domain. The domain transformation the waveform has been performed by using mathematical operations which provides different representation of the factual signal. The transient signal associated with the abnormality events in the transmission network has multiple frequency components. Hence, its analysis can be easily utilized for exploring the information regarding the abnormality events in the network. Numerous algorithms for fault events identification and categorization in the transmission network have been reported by various authors using transient signal analysis. Signal processing techniques like FT, STFT, CWT and (DWT) have been effectually applied by different authors for ascertaining the fault events in power transmission network. The transient signal processing and feature vector selection mechanism considerably helps in endowing faster time response of the relaying system. This chapter presents a brief overview on consequences of feature selection mechanism and

development of different signal processing techniques that have been comprehensively applied for transient signals analysis.

## 2.2 Fourier Transform

FT technique is one of the classical and most widely used signal processing tools for reflecting a time domain signal into frequency domain. Its concept had been introduced by French physicist and mathematician Joseph Fourier. It simply decomposes the factual time domain signal into complex exponential functions at various frequencies level. It directly helps in analyzing the different frequency content of the factual signal as the coefficients of the decomposed signal represents the contribution of every frequency level. The FT of time domain signal  $S(t)$  is given by:

$$S(\omega) = \int_{-\infty}^{\infty} S(t)e^{-j\omega t} dt \quad (2.1)$$

$t$  is time;  $S(\omega)$  represents the signal in frequency domain. Although the FT technique has been effectually utilized for fault analysis in power transmission network, but it has resolution and localization limitations. The inherent disadvantage of FT is the fact that it is incompetent in providing the simultaneous time information of the signal. Fault transients signals are usually of non-stationary nature, which restricted the use of FT for performing signal analysis as it only confer frequency information of the signal. To overcome the prime limitations of FT, another signal processing mechanism called as STFT or windowed Fourier Transform (WFT) has been introduced by Hungarian physicist Denis Gabor in 1946.

### 2.3 Short Time Fourier Transform

STFT can be termed as the revised version of FT that has the proficiency of providing both time and frequency information of any signal. STFT simply gives a solution to the problem of proper representation of non-stationary and non-periodic signals by using sliding window length mechanism. In STFT mechanism the factual signal is segmented into small sections by varying the sliding window length. Every sub segments are analyzed for its frequency components separately. The STFT of any original signal is simply the FT of the real signal multiplied by a window function. Hence, the STFT results in a two dimensional representation of the real signal. The STFT of a signal  $S(t)$  is given by:

$$STFT\{S(t)\} \equiv X(\tau, \omega) = \int_{-\infty}^{\infty} S(t)W(t-\tau)e^{-j\omega t} dt \quad (2.2)$$

$W(t)$  is used window function, may be a rectangular, Gaussian or Hamming window. It has been observed that even though STFT gives better representation than FT, but one major particularity of STFT is that the sliding window length is constant throughout the whole plane. Hence, it offers a trade-off between the time and frequency representation of the signal. If a narrow window length is picked, it gives better time resolution but provide poor frequency localization. Similarly, if wide window length is acquired than it results in better frequency resolution, but gives poor time resolution. Various strategies based on STFT have been successfully developed and transcribed in the literature for fault diagnosis in power transmission network. But STFT mechanism involves resolution issue and it is difficult to acquire the exact time-frequency representation of any waveform.

## 2.4 Wavelet Transform

For outperforming the limitations of FT (no time information) and of STFT (fixed frequency and time localization), another signal processing tool called as WT has been introduced by Grossman and Morlet in 1984. Wavelets are mathematical functions and are associated with building a model for a non-stationary signal, with a set of components that are small waves. Wavelet tool can be applied for extracting the vital information from different kinds of data set like audio signals or images. The WT can effectually represent the factual signal with a very limited number of coefficients as it have precise localization attribute of the wavelets. A mathematical function can be called as mother wavelet if it meeting the following two conditions:

$$\text{i) } \int_{-\infty}^{\infty} \psi(t) dt = 0 \quad (2.3)$$

$$\text{ii) } \int_{-\infty}^{\infty} |\psi(t)|^2 dt < \infty \quad (2.4)$$

The prime attribute of WT is its potentiality to analyze the factual signal in both domain i.e. time and frequency simultaneously. It is well competent for localized analysis of non-stationary and fast transient signals. It can efficiently provide multi resolution analysis (MRA) along with dilated windows. MRA simply allows decomposition of the factual signal in several frequency bands, which significantly helps in accurate analysis of particular frequency components along with their time of occurrence. WT decomposes the factual signal over a set of dilated and translated wavelets [62-64]. The WT had been evolved as an alternative approach to STFT by the generalization of the time window on wavelets, which helps in endowing both frequency and time characteristic information of the signal. WT allows short windows at peak frequencies and elongated window at lower

frequencies, instead of only a single window for the whole plane as in STFT (shown in Figure 2.1).

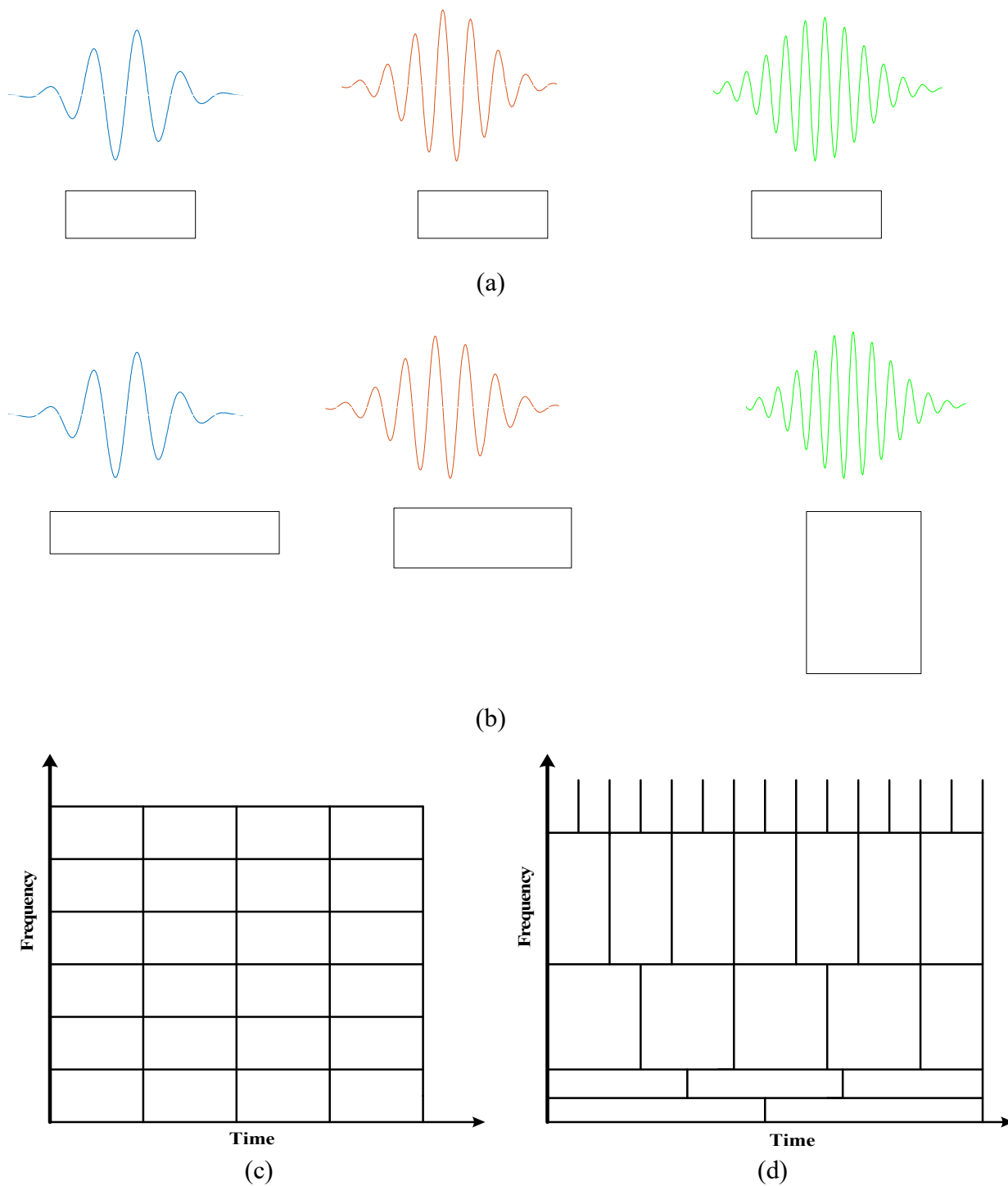


Figure 2.1 Frequency and Time resolution representation of STFT (a & c) and WT (b & d)

The wavelet transforms can be broadly categorized as-

- i. Continuous wavelet transform (CWT)
- ii. Discrete wavelet transform (DWT)

#### 2.4.1 Continuous Wavelet Transform

It can be defined as the addition of signals multiplied by a scaling factor and shifted version of the base function. Mathematically, the CWT of a factual signal  $S(t)$  is given by:

$$CWT(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} S(t) \psi^* \left( \frac{t-b}{a} \right) dt \quad (2.5)$$

where  $\psi^*(t)$  represents the specific mother wavelet;  $a$ ,  $b$  are the scale and translation factors respectively. The CWT of any signal can be performed by changing the wavelet scaling and shifting parameters i.e. 'a', 'b' and comparing it with the factual signal along the time axis. The mother wavelet i.e.  $\psi^*(t)$  must meet the essential conditions as mentioned above in equation (2.3) and (2.4). There are various kinds of mother wavelets with distinct characteristics and capability like Haar, Daubechies, Symmlet, Coiflets, Morlet, Meyer, Mexican Hat wavelet etc. The description of few of them has been given below:

##### *Haar Wavelet:*

It has been introduced by Haar in early 1910. Mathematically it can be defined as a bipolar step function (shown in Figure 2.2). The Haar wavelet  $\psi(t)$  is given by:

$$\psi(t) = \begin{cases} 1 & \text{if } 0 < t \leq 1/2 \\ -1 & \text{if } 1/2 < t \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (2.6)$$

$$\phi(t) = \begin{cases} 1 & \text{if } 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (2.7)$$

The Haar scaling function  $\phi(t)$  defined in equation (2.6) and  $\psi(t)$  has compact support lengths of 1 and have 1 vanishing moment. It is simply a local operation in the time frame of the signal. The time resolution relies on the scaling factor and if the factual signal is constant than Harr transform is zero. Haar wavelet transform has large magnitude values during the discontinuities of the signal. Due to poor localization competency of Haar transform in the frequency domain, it is very rarely applied in practical applications.

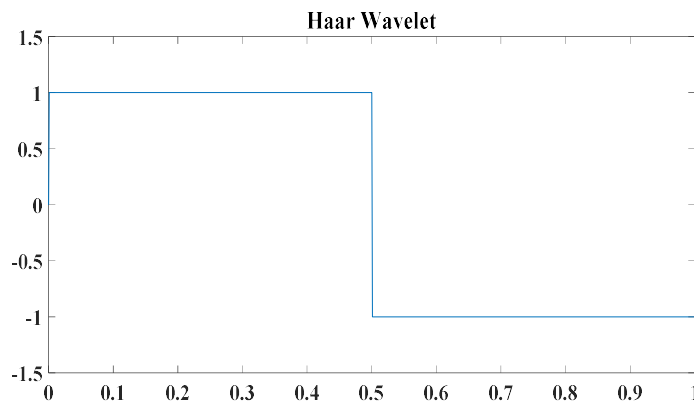


Figure 2.2 Haar wavelet function

#### *Daubechies Wavelet:*

The Daubechies (Db) wavelets are commonly denoted as dbN, where N simply represents the order. It has been introduced by an eminent mathematician Ingrid Daubechies. Db wavelets are well competent in handling the orthogonal wavelets with compact support and arbitrary regularity. Db wavelet is the most appropriate for analyzing power transient signals and has been widely applied for fault analysis applications. The scaling and wavelet function have compact support of length of  $2N$  and there are  $N$  vanishing moments present in the scaling function. The basic characteristic of Db wavelet is shown in Figure 2.3.

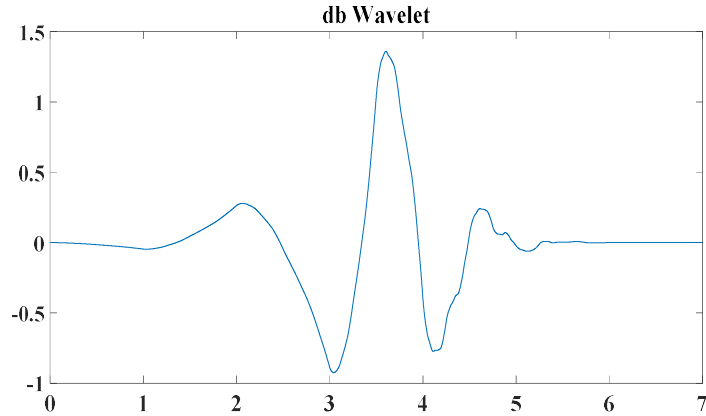


Figure 2.3 Db wavelet function

*Symlet wavelets:*

It has also been proposed by Ingrid Daubechies by transforming the construction of Db wavelets. These are normally more symmetric than that of Db wavelets, which is the prime benefit of it. It is usually denoted as symN. The scaling and wavelet function have compact support of length of  $2N$  and there are  $N$  vanishing moments present in the scaling function.

The characteristic of symlet mother wavelet is shown in Figure 2.4.

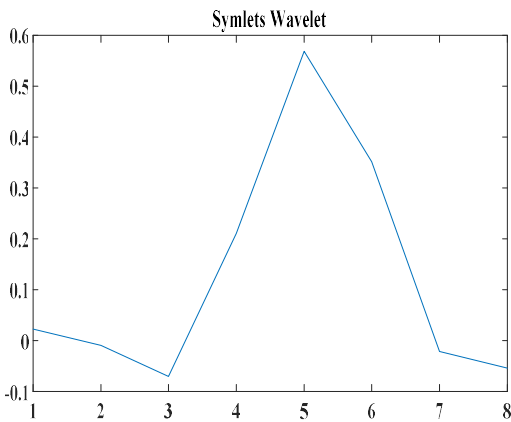


Figure 2.4 Symlet wavelet function

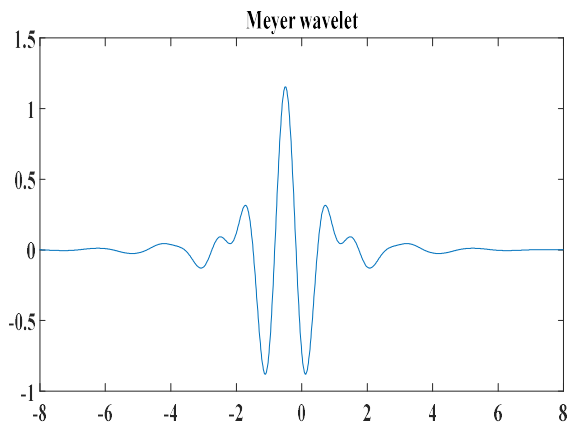


Figure 2.5 Meyer wavelet function



*Meyer wavelets:*

Meyer wavelet has been proposed by Yves Meyer. Its characteristic is shown in Figure 2.5.

The scaling and wavelet function have infinite support. The FT of the scaling function is given by

$$\phi(\omega) = \begin{cases} 1 & \text{if } |\omega| \leq \frac{2\pi}{3} \\ \cos\left[\frac{\pi}{2}v\left(\frac{3}{4\pi}|\omega|-1\right)\right] & \text{if } \frac{2\pi}{3} \leq |\omega| \leq \frac{4\pi}{3} \\ 0 & \text{otherwise} \end{cases} \quad (2.8)$$

Where  $v$  is given by

$$v(t) = \begin{cases} 0 & \text{if } t \leq 0 \\ 1 & \text{if } t \geq 1 \end{cases} \quad (2.9)$$

Similarly, the FT of the wavelet function  $\psi(t)$  is given by

$$\psi(\omega) = \begin{cases} \exp\left(\frac{i\omega}{2}\right) \sin\left(\frac{\pi}{2}v\left(\frac{3|\omega|}{2\pi}-1\right)\right) & \frac{2\pi}{3} \leq |\omega| \leq \frac{4\pi}{3} \\ \exp\left(\frac{i\omega}{2}\right) \cos\left(\frac{\pi}{2}v\left(\frac{3|\omega|}{4\pi}-1\right)\right) & \frac{4\pi}{3} \leq |\omega| \leq \frac{8\pi}{3} \end{cases} \quad (2.10)$$

#### 2.4.2 Discrete Wavelet Transforms

DWT is nothing but the execution of CWT at a discrete set of scales and translation, so as to provide a minimal representation of the factual signal without losing any information.

The CWT simply maps the one-dimensional factual time signal in to a two-dimensional time-scale representation, which causes the redundancy of data. However, the prime objective of implementation of signal processing mechanism is to represent the factual signal efficiently with least number of parameters. Hence, DWT has been widely used in

transient signal analysis because of its better competency of perfect reconstruction and non-redundant nature [65-80]. Mathematically the DWT for a signal  $S(t)$  is represented as:

$$DWT_{\psi_s(m,n)} = \int_{-\infty}^{\infty} S(t) \psi_{m,n}^*(t) dt \quad (2.11)$$

$$\psi_{m,n}(t) = \frac{1}{\sqrt{a_0^m}} \psi \left( \frac{t - nb_0 a_0^m}{a_0^m} \right) \quad (2.12)$$

Where,  $a_0^m = S_{c(scale)}$ ;  $nb_0 a_0^m = \tau_{(translation)}$ ;  $a_0$  and  $b_0$  are fixed constant

and  $a_0 > 1$ ;  $b_0 > 0$ ;  $m, n \in Z$  where  $Z$  is set of integers. For exploring the signal at multiple frequency ranges with reasonable resolutions, it decomposes the factual signal into coarse approximation (a) and detail (d) information. Decomposition of the factual signal into multiple frequency ranges is simply procured by using successive high and low pass filtering of the time domain signal. The approximations represents the high-scale, low-frequency factors of the signal, where as details are the low-scale, high frequency factors. The output acquired from the low pass filter is fed again to second set of high pass and low pass filters for getting the next detailed and approximation coefficients. This mechanism is repeated till the factual signal is decomposed to the required level of decomposition. Figure 2.6 shows wavelet decomposition tree. Out of all available mother wavelets, pertinent selection of the mother wavelet is inevitably needed for proper characterization of the signal. The kind of wavelet and order is usually dependent on the nature of particular application and symmetry between the factual signal and the mother wavelet. It has been observed that the Daubechies (Db) mother wavelet is the most appropriate for analyzing power transient signals and has been effectually applied by various researchers in the

literature [81-85]. Db5 mother wavelet is well competent in precisely classifying the magnitude of D1 components of faulty and sound phases compared with other wavelets.

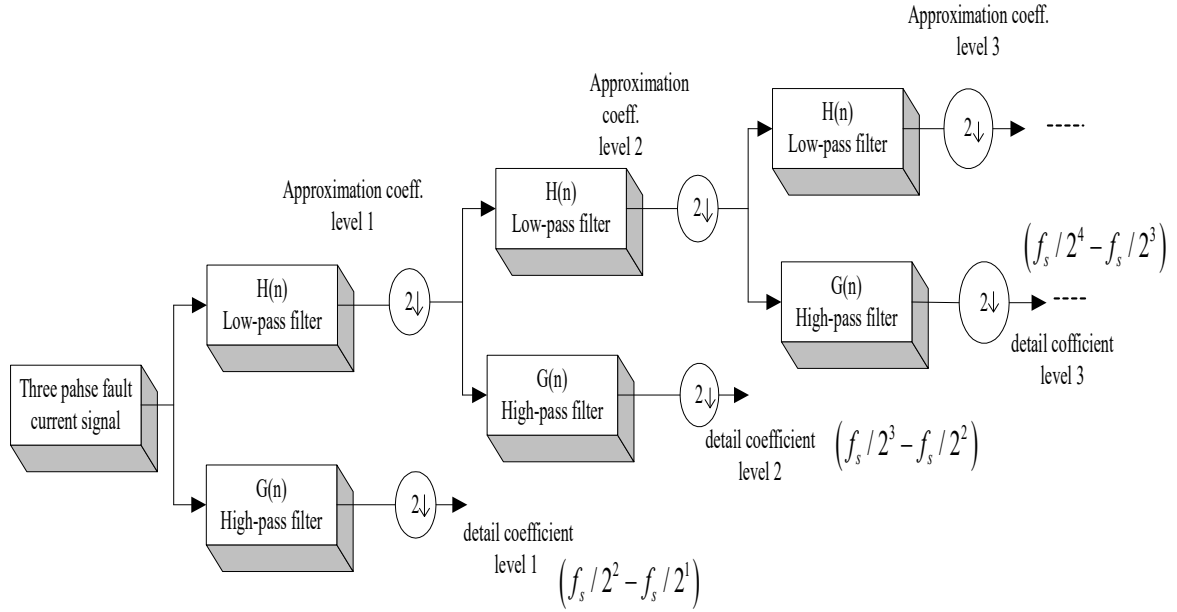


Figure 2.6 Wavelet frequency decomposition tree

After comparing the performance (in terms events classification and location accuracy acquired) of various combinations of Daubechies mother wavelet (Db1 to Db5) Db5 up to level 5 has been selected in the present work for decomposing the fault current signals. Let the actual sampling frequency is  $f_s$ , then the corresponding ranges of different decomposition level of approximation and detail coefficients are  $(0 - f_s/2^{j+1})$  Hz and  $(f_s/2^{j+1} - f_s/2^j)$  Hz respectively. Here j represents the level of decomposition of the signal. It has been observed that the applied DWT combined with intelligent computing techniques based approaches are very effectual in categorizing and locating the fault events in series compensated power network with high accuracy level. Although DWT

decomposition based protection mechanism giving very good fault events categorization and location accuracy, but the DWT decomposition technique involves some big concern like selection of applicable mother wavelet and section of decomposition level. The precision of signal decomposition and reconstruction while using DWT is incumbent of accurate selection the mother wavelet according to the symmetry of the original signal. Hence DWT based decomposition seeks proper analysis regarding the election of mother wavelet and decomposition level. For validating the competency of the proposed intelligent computing techniques one more signal processing mechanism has been applied in present work i.e. empirical mode decomposition (EMD). EMD based signal decomposition is simply free from picking of predefined mother mathematical function as in the DWT. The principle and procedure of EMD technique has been discussed in the next section.

## **2.5 Empirical Mode Decomposition**

The Empirical Mode Decomposition mechanism is an adaptive signal decomposing algorithm. Its versatility and effectuality makes it apt for extracting the prime features of the non-stationary signals. In comparison with the above mentioned classical signal processing mechanisms like FT, STFT or wavelet algorithms, it has very large extent of adaptation. Furthermore, EMD is totally free from selection of any preset mother mathematical function like the wavelet transformation. It comprehensively outperforms the major concern of wavelet transform i.e. picking of proper mother wavelet and level of decomposition. It has been significantly utilized for analysing the nonlinear and non-stationary signals by decomposing the factual signal into different mono component functions termed as intrinsic mode functions (IMFs) by applying sifting mechanism [86-

90]. The obtained IMFs contain vital information about the factual signal. The concept of IMFs was firstly introduced in year 1998 by Huang et al. There are two mandatory criteria for being the decomposed function to be IMFs are as follows:

- 1) The extreme and number of zero crossing should be same or at most differ by one.
- 2) At any point, mean of upper and lower envelopes defined by local extremal is zero.

According to the sifting mechanism initially, the lower and upper envelopes of the factual signal are figure out by determining the local extrema of the original signal. The envelopes are constructed simply by using smooth interpolation technique. Thereafter, the mean of the acquired envelopes is termed as the local mean of the factual signal. It can be set as the reference which separates the lower and highest frequency oscillations in the original signal. By subtracting the estimated local mean from the factual signal, we obtain the first set of IMFs (if it satisfies the mandatory criteria). Afterward the residue has been computed and the same sifting mechanism is repeated again for acquiring next IMF and a new residue, till the value of the residue is more than the threshold value. The EMD based segmentation of AG fault current signal is demonstrated in Figure 2.7.

The steps of EMD are given below:

- Identify the local minima (M) and maxima (m) of input current signal  $I(t)$
- Execute interpolation between M and m for acquiring envelopes  $e_{min}(t)$  and

$$e_{max}(t)$$

- Compute average of the envelopes using

$$m(t) = \frac{[e_{max}(t) + e_{min}(t)]}{2}$$

- Extract  $C_1(t) = I(t) - m(t)$
- $C_1(t)$  is an IMF if it meets the aforementioned criteria. If  $C_1(t)$  is not an IMF, than repeat steps 1 to 4 on  $C_1(t)$ , so long as new acquired  $C_1(t)$  meets the criteria.
- Compute the residue,  $r_1(t) = I(t) - C_1(t)$
- If the  $r_1(t)$  is more than a threshold value, than repeat steps for acquiring next IMF and a new residue.

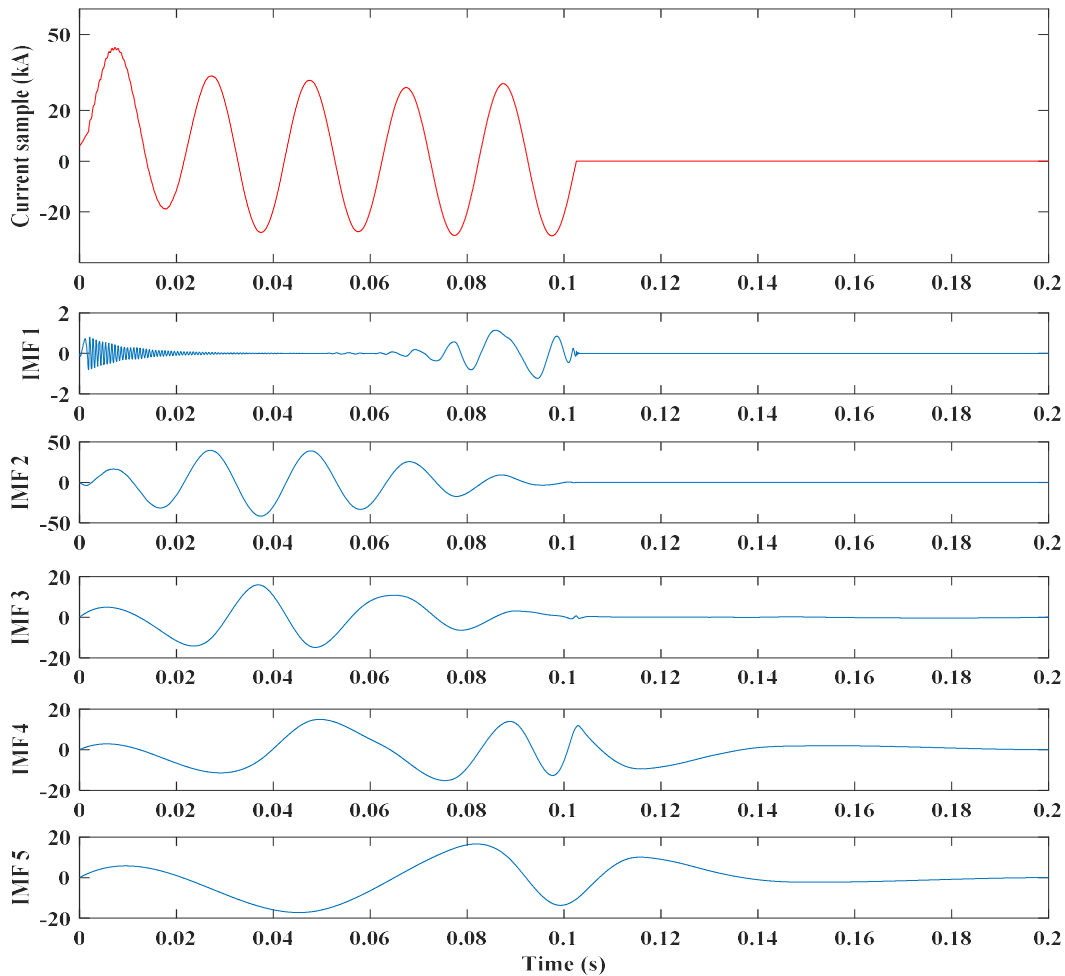


Figure 2.7 Empirical mode decomposition of post fault current signal during A-G fault in the simulated network at 50 km from sending side.

## 2.6 Feature Vectors Selection Mechanism

The procedure of acquiring the prominent characteristics patterns/signs of a signal that can significantly represent the real behaviour of the original signal is termed as feature extraction technique. The feature vectors selection phenomenon considerably slackens the dimensionality of the training and testing feature set without losing any information content. Instead of utilizing the whole transient signal for analysing the changes due to abnormality occurrence in the power network, only the selected characteristic features are utilized for ascertaining the changes. It comprehensively mitigates the complexity of training and testing of the intelligent computing based distance estimator and classifier models. It also reduces the net training and testing time period which leads to faster time of response of the relaying scheme.

### 2.6.1 Feature vector selection with DWT

As aforementioned, in the present work two different signal processing techniques i.e. DWT and EMD have been applied for processing current transient signal. The 3-phase post fault current transient signals retrieved from the MATLAB/RTDS simulation during the period of shunt abnormality events in the simulated test network are decomposed into coarse approximation (a) and detail (d) information using (Db5) mother wavelet. Once the wavelet coefficients has been computed, thereafter the characteristic feature vectors of the 3-phase current transient signals are computed in terms of entropy of the detail DWT coefficients, SD of detail coefficients (cD), and minimum and maximum value of the DWT coefficients.

Entropy has been frequently utilized as feature vector in different signal processing application. The competency of wavelet entropy measure in signal feature analysis and fault diagnosis in the power transmission network has been addressed in [93-94]. There are multiple kinds of entropy measures are available like Shannon, norm, threshold, log energy and sure entropy. In this work two of them i.e. Norm entropy of the DWT coefficients has been utilized as one of the feature vector. The norm entropy (with power p) for a signal 'S' is given by

$$e(S) = \sum_i |S_i|^p \quad (2.13)$$

The SD of the detail DWT coefficients has been also computed as a feature vector of the post fault 3-phase current signal. For estimating the SD of the detail DWT coefficients, expression shown in equation (2.14) has been utilized.

$$\sigma_a = \left( \frac{1}{n-1} \sum_{j=1}^n (cD_{ij} - \mu_a cD_i)^2 \right)^{1/2} \quad (2.14)$$

$$\mu_a cD_i = \frac{1}{n} \sum_{j=1}^n cD_{ij} \quad (2.15)$$

Where a shows the respective phase; i= 1 to 5 i.e. level of decomposition;  $\mu$  is mean sample and n is samples of wavelet detail coefficients.

### 2.6.2 Feature vector selection with EMD

While using the second decomposition technique i.e. EMD, energy level of the respective IMFs has been computed as the feature vector. The 3-phase post fault current samples are



segmented into multiple IMFs by applying EMD mechanism. After signal decomposition, the FFT mechanism has been applied on the different obtained IMFs for distinguishing the desirable IMFs level that is having the similar transient frequency band. Once, the specific IMF has been detected thereafter the fault feature vectors has been computed in terms of its energy using equation (2.16), where ‘a’ represents the particular phase.

$$Energy_{(a)} = \sum |IMF(t)|^2 \quad (2.16)$$

## 2.7 Conclusion

In this chapter, the concepts of transient signal processing and characteristic features extraction mechanism have been thoroughly described in detail. In present analysis, two different signal processing techniques have been applied (i.e. DWT and EMD) for decomposing the post fault current samples for acquiring the critical fault feature vectors. The fundamental advantages of wavelet transform mechanism over other techniques such as FFT and STFT are explicitly addressed in this chapter. The characteristic fault features are realized in terms of norm entropy, Shannon entropy of the DWT coefficients, standard deviation (SD) of detail coefficients (cD), and minimum and maximum value of the DWT coefficients. In addition, for resolving the critical concerns of wavelet transform such as selection of applicable mother wavelet and section of decomposition level empirical mode decomposition technique has been utilized. During EMD based signal decomposition, the features are extracted in terms of energy of the appropriate IMFs coefficients. In the coming chapters, these concepts have been utilized for realizing the training and testing feature vectors from the post fault current samples. Thereafter, the realized features are

applied to the ML based classifier and distance estimator models for ascertaining the categories and location of fault events in the power transmission network.