APPENDICES

APPENDIX-I

Bubble size distribution (BSD) is presented here. The table provides values of N_b in various size-ranges i.e. class of bubbles. The values of $d_{b,i}$ and $d_{b,i+1}$ are lower and higher values of bubble diameter in a class of bubbles.

Table A-1: Number of bubbles, N_b, at various bubble class for air/Water at H_s =0.20 m for various values of U_g.

$d_{b,i}$, m	$d_{b,i+1}$, m	U_g, m s^{-1}									
		0.0083	0.0125	0.0167	0.0208	0.0250	0.0292	0.0333	0.0417	0.0500	0.0583
0.000	0.002	265	403	478	581	526	644	649	647	778	738
0.002	0.003	237	538	579	746	703	754	650	597	576	547
0.003	0.004	172	295	307	450	456	534	494	366	376	288
0.004	0.005	89	146	155	191	234	258	259	203	185	170
0.005	0.006	59	77	85	106	103	120	129	93	76	88
0.006	0.007	35	37	42	41	58	58	63	57	57	45
0.007	0.008	27	21	22	22	30	38	41	36	25	21
0.008	0.009	10	11	17	16	15	19	15	27	31	16
0.009	0.010	6	13	9	5	11	12	15	18	16	25
0.010	0.011	5	5	3	4	9	12	15	11	16	11
0.011	0.012	1	6	3	7	5	4	10	11	14	6
0.012	0.013	5	2	3	3	4	3	10	5	7	9
0.013	0.014	2	1	3	1	1	6	3	7	11	5
0.014	0.015	1	2	4	4	4	2	6	5	8	7
0.015	0.016	1	1	1	1	1	0	3	6	4	4
0.016	0.017	0	0	0	3	1	1	3	3	6	3
0.017	0.018	1	1	1	2	3	1	3	0	8	1
0.018	0.019	2	0	0	1	2	1	1	5	4	4
0.019	0.020	0	0	2	1	1	1	0	1	2	1
0.020	0.021	0	0	0	1	0	0	1	4	2	4
0.021	0.022	0	0	0	2	1	0	2	0	1	1
0.022	0.023	0	0	0	2	0	1	1	1	2	1
0.023	0.024	0	0	0	0	1	0	1	1	0	0
0.024	0.025	0	0	0	0	0	1	0	2	1	1
0.025	0.026	0	0	1	0	0	0	0	2	0	1
0.026	0.027	0	0	0	1	1	0	1	0	0	1
0.027	0.028	0	0	0	0	1	0	0	1	0	0
0.028	0.029	0	0	0	0	0	1	0	1	0	1
0.029	0.030	0	1	0	0	0	0	0	0	0	2
0.030	0.031	0	0	0	0	0	1	0	0	2	0
0.031	0.032	0	0	0	0	0	0	0	0	2	0
0.032	0.033	0	0	0	0	0	0	0	0	1	0
0.033	0.034	0	0	1	0	0	1	0	0	0	1
0.034	0.035	0	0	0	0	0	0	0	0	0	2
0.035	0.036	0	0	0	0	0	1	0	0	0	2
0.036	0.037	0	0	0	0	0	0	1	0	0	1
0.037	0.038	0	0	0	0	0	0	0	1	0	1
0.039	0.040	0	0	0	0	0	0	0	0	0	1
0.043	0.044	0	0	0	0	1	0	0	0	0	0
0.047	0.048	0	0	0	0	0	0	0	0	1	1
0.060	0.061	0	0	0	0	0	0	0	0	0	1

Table A-2: Number of bubbles, N_b, at various bubble class for air/Water at H_s =0.22 m for various values of U_g.

$d_{b,i}$, m	$d_{b,i+1}$, m	U_g, m s^{-1}									
		0.0083	0.0125	0.0167	0.0208	0.0250	0.0292	0.0375	0.0417	0.0500	0.0583
0.000	0.002	457	489	563	642	690	699	755	715	906	845
0.002	0.003	559	770	712	735	664	698	726	642	592	616
0.003	0.004	377	453	545	465	482	507	481	485	325	349
0.004	0.005	185	232	244	268	260	275	245	251	219	218
0.005	0.006	71	94	115	124	126	108	150	128	108	105
0.006	0.007	32	29	46	57	61	62	81	74	59	59
0.007	0.008	20	20	24	23	43	38	41	51	35	36
0.008	0.009	14	14	20	23	19	15	25	30	28	19
0.009	0.010	11	7	9	14	17	17	23	20	28	17
0.010	0.011	5	5	6	7	7	6	15	14	17	19
0.011	0.012	3	5	4	6	7	8	16	14	9	9
0.012	0.013	0	4	3	6	3	7	7	10	9	10
0.013	0.014	5	1	5	7	4	1	5	6	3	8
0.014	0.015	2	0	1	1	6	4	8	7	6	5
0.015	0.016	0	4	2	1	3	1	0	6	6	6
0.016	0.017	1	0	2	1	2	1	1	6	8	4
0.017	0.018	0	2	1	0	1	1	2	6	2	3
0.018	0.019	1	1	1	0	0	2	0	5	1	0
0.019	0.020	0	0	1	1	1	0	3	4	4	3
0.020	0.021	1	3	1	1	0	2	1	4	0	0
0.021	0.022	0	0	0	2	0	1	2	0	1	2
0.022	0.023	0	0	1	0	0	1	1	1	2	5
0.023	0.024	0	0	0	1	2	0	0	0	3	1
0.024	0.025	1	0	0	0	0	0	0	1	1	3
0.025	0.026	0	0	1	1	1	0	0	1	2	2
0.026	0.027	0	0	0	0	0	0	0	0	1	0
0.027	0.028	0	0	0	0	0	0	0	0	1	2
0.028	0.029	0	0	1	0	0	0	1	0	1	0
0.029	0.030	0	0	0	0	0	0	1	0	0	0
0.030	0.031	0	0	0	0	0	0	1	0	0	3
0.031	0.032	0	0	0	0	0	0	0	1	0	1
0.033	0.034	0	0	0	0	1	0	0	0	0	0
0.036	0.037	0	0	0	0	0	0	0	0	1	0
0.037	0.038	0	0	0	0	0	0	0	0	1	0
0.038	0.039	0	0	0	0	0	0	0	0	1	0
0.042	0.043	0	0	0	0	0	0	0	0	0	1
0.043	0.044	0	0	0	0	1	0	0	0	1	1
0.044	0.045	0	0	0	0	0	0	0	0	0	0
0.045	0.046	0	0	0	0	0	0	0	0	0	0
0.046	0.047	0	0	0	0	0	0	0	0	0	1
0.056	0.057	0	0	0	0	0	0	0	0	1	0

Table A-3: Number of bubbles, N_b, at various bubble class for air/Water at H_s =0.23 m for various values of U_g.

$d_{b,i}$, m	$d_{b,i+1}$, m	U_g, m s^{-1}									
		0.0083	0.0125	0.0167	0.0208	0.0250	0.0292	0.0375	0.0417	0.0500	0.0583
0.000	0.002	278	477	581	628	585	706	796	693	760	818
0.002	0.003	296	621	748	795	792	703	724	654	633	612
0.003	0.004	215	365	515	473	487	468	461	468	350	343
0.004	0.005	112	176	258	224	254	236	276	282	218	191
0.005	0.006	60	80	119	107	130	106	126	138	129	108
0.006	0.007	46	48	70	56	61	70	78	54	67	66
0.007	0.008	27	37	24	30	41	33	33	38	39	42
0.008	0.009	15	7	18	17	19	15	21	25	29	28
0.009	0.010	14	8	6	8	11	14	27	19	28	12
0.010	0.011	8	2	8	10	11	12	16	9	9	18
0.011	0.012	4	5	4	4	4	4	12	7	10	9
0.012	0.013	3	1	2	2	6	7	11	7	8	12
0.013	0.014	4	4	2	3	0	3	8	6	5	9
0.014	0.015	3	2	1	2	1	2	2	5	5	6
0.015	0.016	1	0	1	3	2	4	10	9	6	10
0.016	0.017	0	2	1	2	2	1	1	2	5	5
0.017	0.018	1	0	3	2	0	0	3	2	2	1
0.018	0.019	0	0	1	1	0	0	1	1	1	2
0.019	0.020	0	0	0	0	0	2	1	0	3	3
0.020	0.021	0	0	1	0	0	0	1	1	2	0
0.021	0.022	0	0	0	1	1	1	5	1	2	2
0.022	0.023	0	0	0	1	1	0	1	2	0	2
0.023	0.024	0	1	0	1	0	0	0	1	1	0
0.024	0.025	0	0	0	0	1	0	2	2	1	0
0.025	0.026	0	0	0	0	0	0	4	0	1	0
0.026	0.027	0	0	1	0	0	0	0	0	4	0
0.027	0.028	0	0	0	0	0	0	0	1	1	1
0.028	0.029	0	0	2	0	0	0	0	0	0	2
0.029	0.030	0	0	0	0	0	0	0	2	1	0
0.030	0.031	0	1	0	0	0	0	0	1	2	0
0.031	0.032	0	0	0	0	0	0	1	0	0	2
0.032	0.033	0	0	0	0	0	1	0	0	0	1
0.034	0.035	0	0	0	0	0	1	0	1	1	0
0.037	0.038	0	0	0	0	0	0	0	0	0	1
0.041	0.042	0	0	0	0	0	0	0	0	1	0
0.043	0.044	0	0	0	0	1	0	0	0	0	0
0.044	0.045	0	0	0	0	0	0	0	1	0	0
0.052	0.053	0	0	0	0	0	1	0	0	0	0
0.053	0.054	0	0	0	0	0	0	0	0	0	1
0.063	0.064	0	0	0	0	0	0	0	0	0	1

Table A-4: Number of bubbles, N_b, at various bubble class for air/Water at H_s =0.24 m for various values of U_g.

$d_{b,i}$, m	$d_{b,i+1}$, m	U_g, m s^{-1}									
		0.0083	0.0125	0.0167	0.0208	0.0250	0.0292	0.0375	0.0417	0.0500	0.0583
0.000	0.002	315	552	528	745	615	762	852	790	699	315
0.002	0.003	288	788	693	909	803	743	807	710	675	288
0.003	0.004	198	461	463	576	506	546	504	445	424	198
0.004	0.005	114	233	245	303	267	333	270	295	230	114
0.005	0.006	68	83	101	124	116	140	144	137	112	68
0.006	0.007	37	51	53	56	63	70	65	64	56	37
0.007	0.008	22	16	31	38	40	45	50	40	49	22
0.008	0.009	15	13	9	26	11	22	33	24	26	15
0.009	0.010	11	5	9	14	10	18	13	15	11	11
0.010	0.011	7	7	12	13	7	13	15	16	11	7
0.011	0.012	2	3	3	5	3	3	14	9	9	2
0.012	0.013	3	1	4	5	10	2	10	6	7	3
0.013	0.014	3	4	4	9	2	2	9	5	6	3
0.014	0.015	3	3	1	3	1	2	0	4	0	3
0.015	0.016	1	5	0	1	1	2	4	3	4	1
0.016	0.017	1	0	2	0	2	1	5	3	6	1
0.017	0.018	0	0	0	1	1	0	4	1	0	0
0.018	0.019	0	0	1	1	1	2	1	5	1	0
0.019	0.020	1	0	1	0	0	0	1	3	2	1
0.020	0.021	0	0	1	0	0	3	0	1	4	0
0.021	0.022	0	0	0	0	0	0	2	1	2	0
0.022	0.023	0	0	1	0	1	1	0	1	0	0
0.023	0.024	0	1	0	0	2	1	2	2	0	0
0.024	0.025	0	0	1	0	1	0	0	1	1	0
0.025	0.026	0	0	0	0	0	2	2	2	0	0
0.026	0.027	0	0	0	0	0	0	0	2	1	0
0.027	0.028	0	0	0	0	0	0	1	1	1	0
0.028	0.029	0	0	0	0	0	2	0	0	3	0
0.029	0.030	0	0	0	0	1	0	0	0	1	0
0.030	0.031	0	0	0	0	0	0	0	0	1	0
0.038	0.039	0	0	0	0	0	0	0	1	1	0
0.045	0.046	0	0	0	0	0	0	0	1	0	0
0.050	0.051	0	0	0	0	0	0	0	0	1	0

Table A-5: Number of bubbles, N_b, at various bubble class for air/Water at H_s =0.26 m for various values of U_g.

$d_{b,i}$, m	$d_{b,i+1}$, m	U_g, m s^{-1}									
		0.0083	0.0125	0.0167	0.0208	0.0250	0.0292	0.0333	0.0375	0.0417	0.0458
0.000	0.002	549	589	671	645	821	758	842	856	841	936
0.002	0.003	673	803	855	781	812	831	752	772	738	775
0.003	0.004	441	716	549	532	589	572	502	551	508	483
0.004	0.005	215	352	301	258	313	298	283	304	313	309
0.005	0.006	86	132	122	119	145	153	140	177	148	154
0.006	0.007	50	67	65	72	89	70	68	90	66	68
0.007	0.008	10	26	30	21	34	45	44	45	49	49
0.008	0.009	12	13	18	19	30	33	21	30	34	30
0.009	0.010	7	8	10	9	19	9	19	16	16	28
0.010	0.011	7	6	7	9	11	15	15	7	11	12
0.011	0.012	2	6	3	7	10	5	5	14	9	12
0.012	0.013	6	6	2	2	9	6	8	10	7	7
0.013	0.014	3	0	1	1	4	4	5	8	3	5
0.014	0.015	1	1	1	4	1	2	5	4	6	5
0.015	0.016	1	4	6	0	5	0	1	3	3	4
0.016	0.017	1	0	1	0	4	2	1	1	4	1
0.017	0.018	1	1	1	1	2	1	0	1	1	1
0.018	0.019	2	0	0	2	0	0	1	3	1	1
0.019	0.020	0	0	1	1	1	0	1	0	1	2
0.020	0.021	0	0	0	0	0	5	2	2	0	1
0.021	0.022	1	0	0	0	1	0	2	1	0	1
0.022	0.023	0	0	0	0	0	2	0	1	1	0
0.023	0.024	0	1	0	0	0	0	0	0	2	1
0.024	0.025	0	0	0	1	0	1	0	0	0	2
0.025	0.026	0	0	0	1	0	0	0	1	0	0
0.026	0.027	0	0	0	0	0	0	0	0	0	1
0.027	0.028	0	0	0	0	0	1	0	0	0	0
0.028	0.029	0	0	1	0	0	0	0	0	0	0
0.031	0.032	0	0	0	0	0	0	0	0	1	0
0.032	0.033	0	0	0	0	0	0	0	1	0	0
0.033	0.034	0	0	0	0	1	0	0	0	0	0
0.034	0.035	0	0	0	0	0	0	0	1	0	0
0.035	0.036	0	0	0	0	0	0	0	0	1	0
0.036	0.037	0	0	0	0	0	0	0	0	1	0
0.039	0.040	1	0	0	0	0	0	1	0	0	0

Table A-6: Number of bubbles, N_b, at various bubble class for air/Water at H_s =0.28 m for various values of U_g.

		U_g, m s^{-1}							
$d_{b,i}$, m	$d_{b,i+1}$, m	0.0083	0.0125	0.0167	0.0208	0.0250	0.0292	0.0333	0.0417
0.000	0.002	595	690	783	801	833	843	797	782
0.002	0.003	882	984	824	914	819	857	759	739
0.003	0.004	624	634	599	654	638	620	471	514
0.004	0.005	307	306	334	370	371	369	296	338
0.005	0.006	106	126	108	187	190	165	136	170
0.006	0.007	47	46	61	89	74	85	80	85
0.007	0.008	23	23	22	56	45	49	52	49
0.008	0.009	10	15	17	24	21	24	24	37
0.009	0.010	8	9	4	16	17	17	22	25
0.010	0.011	2	7	10	8	12	14	24	14
0.011	0.012	2	4	2	9	9	6	7	22
0.012	0.013	1	4	7	3	6	7	3	11
0.013	0.014	1	4	3	1	6	4	3	10
0.014	0.015	0	1	4	6	1	1	1	3
0.015	0.016	0	1	0	3	3	4	4	1
0.016	0.017	1	0	0	1	1	1	4	7
0.017	0.018	0	1	1	0	1	1	4	3
0.018	0.019	0	0	0	0	1	0	2	2
0.019	0.020	0	1	0	0	1	2	3	0
0.020	0.021	0	0	1	1	0	2	2	0
0.021	0.022	0	0	0	0	1	0	3	2
0.022	0.023	0	0	0	0	0	1	0	1
0.023	0.024	0	0	0	0	1	2	1	1
0.024	0.025	0	0	0	1	0	0	0	0
0.025	0.026	0	0	0	0	0	2	0	0
0.026	0.027	0	0	3	0	1	0	0	0
0.028	0.029	0	0	0	0	0	2	0	0
0.029	0.030	0	0	0	0	0	0	1	0
0.030	0.031	0	0	0	0	0	1	0	0
0.031	0.032	0	0	0	1	0	0	0	0
0.032	0.033	0	0	0	0	0	0	0	1
0.033	0.034	0	0	0	0	0	0	0	1
0.041	0.042	0	0	0	0	0	0	0	1

Table A-7: Number of bubbles, N_b, at various bubble class for air/ 0.5% (w/w) CMC solution at H_s =0.23 m for various values of U_g.

$d_{b,i}$, m	$d_{b,i+1}$, m	U_g, m s^{-1}								
		0.0083	0.0125	0.0167	0.0208	0.0250	0.0292	0.0333	0.0375	0.0412
0.000	0.002	474	518	565	446	461	388	419	445	567
0.002	0.003	861	776	815	641	631	540	489	422	474
0.003	0.004	441	424	527	317	342	317	289	307	295
0.004	0.005	154	160	256	124	162	169	152	153	152
0.005	0.006	41	58	122	63	84	60	76	76	77
0.006	0.007	26	30	41	38	51	44	47	41	44
0.007	0.008	13	15	18	14	15	31	30	20	37
0.008	0.009	6	12	19	14	17	16	20	15	15
0.009	0.010	4	7	6	9	4	8	14	19	18
0.010	0.011	8	2	8	8	9	7	11	18	9
0.011	0.012	3	2	6	3	6	12	9	7	9
0.012	0.013	3	3	6	6	6	4	7	5	13
0.013	0.014	3	2	2	2	3	3	1	3	4
0.014	0.015	3	4	4	0	2	3	8	4	6
0.015	0.016	1	0	0	4	2	2	1	6	3
0.016	0.017	0	1	1	0	0	6	1	2	1
0.017	0.018	1	2	0	0	3	2	2	5	4
0.018	0.019	0	0	2	1	2	1	0	3	0
0.019	0.020	0	0	0	1	0	4	2	1	4
0.020	0.021	0	0	1	0	0	1	1	2	1
0.021	0.022	0	1	1	1	0	3	0	0	2
0.022	0.023	0	0	2	1	0	0	1	0	1
0.023	0.024	0	0	0	1	1	2	1	3	0
0.024	0.025	0	1	1	1	1	2	2	2	0
0.025	0.026	0	0	0	0	0	0	2	1	2
0.026	0.027	0	0	0	0	0	0	1	1	2
0.027	0.028	0	0	0	1	0	0	1	1	1
0.028	0.029	1	0	0	0	0	0	2	0	0
0.029	0.030	0	0	0	0	0	0	2	0	0
0.030	0.031	0	0	0	0	0	0	0	0	1
0.031	0.032	0	0	0	0	1	0	1	1	0
0.032	0.033	0	0	0	0	0	0	2	0	0
0.033	0.034	0	0	0	0	0	1	0	1	0
0.037	0.038	0	0	0	0	0	0	1	0	1
0.040	0.041	0	0	0	0	0	0	1	0	0
0.041	0.042	0	0	0	0	0	0	0	0	1
0.046	0.047	0	0	0	0	0	0	0	1	0
0.049	0.050	0	0	0	0	0	0	0	0	1
0.051	0.052	0	0	0	0	0	0	0	1	1

Table A-8: Number of bubbles, N_b, at various bubble class for air/ 1.0 % (w/w) CMC solution at H_s =0.23 m for various values of U_g.

$d_{b,i}$, m	$d_{b,i+1}$, m	U_g, m s^{-1}								
		0.0083	0.0125	0.0167	0.0208	0.0250	0.0292	0.0333	0.0375	0.0412
0.000	0.002	264	386	464	646	685	712	646	677	732
0.002	0.003	240	499	601	812	824	822	720	589	622
0.003	0.004	124	282	350	485	467	537	506	456	412
0.004	0.005	91	120	160	197	225	243	236	221	211
0.005	0.006	42	54	61	96	94	121	119	111	114
0.006	0.007	26	51	31	48	56	62	77	57	64
0.007	0.008	20	25	19	15	26	45	51	44	41
0.008	0.009	10	15	16	11	22	18	39	22	25
0.009	0.010	3	12	8	4	12	24	23	14	22
0.010	0.011	4	4	6	6	11	16	14	14	18
0.011	0.012	2	3	3	3	8	9	12	4	11
0.012	0.013	2	2	1	7	11	4	6	7	5
0.013	0.014	2	3	1	3	5	2	10	7	5
0.014	0.015	0	0	2	1	1	3	6	8	4
0.015	0.016	0	1	1	3	2	3	6	5	7
0.016	0.017	0	2	0	3	2	2	10	5	5
0.017	0.018	0	1	2	1	1	0	0	4	1
0.018	0.019	3	0	0	0	4	0	3	1	2
0.019	0.020	0	0	0	3	0	1	2	1	0
0.020	0.021	1	0	0	0	0	1	1	1	2
0.021	0.022	1	0	0	0	0	0	1	3	2
0.022	0.023	1	1	0	0	1	1	0	1	1
0.023	0.024	0	0	1	0	0	1	2	2	2
0.024	0.025	0	0	0	0	0	1	0	1	3
0.025	0.026	0	0	0	0	1	1	1	3	2
0.026	0.027	0	0	0	0	0	0	0	0	1
0.027	0.028	0	0	1	0	1	0	0	0	0
0.028	0.029	0	0	0	0	1	2	0	0	0
0.029	0.030	0	1	0	0	0	0	1	2	0
0.030	0.031	0	0	0	0	0	1	0	0	1
0.031	0.032	0	0	0	0	0	0	1	1	0
0.032	0.033	0	0	0	0	0	1	0	1	0
0.033	0.034	0	0	0	0	0	0	0	0	3
0.034	0.035	0	0	0	0	0	0	0	1	0
0.036	0.037	0	0	0	0	0	0	0	1	0
0.053	0.054	0	0	0	0	0	1	0	0	0

Table A-9: Number of bubbles, N_b, at various bubble class for air/2.0 % (w/w) CMC solution at H_s =0.23 m for various values of U_g.

$d_{b,i}$, m	$d_{b,i+1}$, m	U_g, m s^{-1}								
		0.0083	0.0125	0.0167	0.0208	0.0250	0.0292	0.0333	0.0375	0.0412
0.000	0.002	310	646	612	598	420	574	538	567	555
0.002	0.003	264	619	674	599	488	619	569	561	490
0.003	0.004	138	252	336	321	322	415	422	442	351
0.004	0.005	66	79	119	162	155	234	203	254	215
0.005	0.006	34	57	49	78	72	99	111	114	101
0.006	0.007	18	20	32	43	27	49	58	57	61
0.007	0.008	8	11	15	24	27	22	36	40	30
0.008	0.009	8	6	9	14	13	15	19	23	29
0.009	0.010	6	4	7	4	14	9	12	14	12
0.010	0.011	6	4	4	11	5	14	7	10	8
0.011	0.012	3	0	3	4	6	10	8	11	7
0.012	0.013	0	1	1	3	4	6	5	6	4
0.013	0.014	0	1	2	4	2	3	5	5	9
0.014	0.015	1	3	4	1	2	1	5	3	7
0.015	0.016	0	0	2	1	2	1	3	9	4
0.016	0.017	0	0	3	1	3	3	5	5	5
0.017	0.018	0	1	0	2	2	0	2	4	1
0.018	0.019	0	0	0	0	4	3	1	0	2
0.019	0.020	1	0	1	1	1	1	1	3	2
0.020	0.021	0	2	1	2	1	2	0	2	1
0.021	0.022	0	0	0	0	1	2	2	2	0
0.022	0.023	0	0	0	1	1	1	1	1	0
0.023	0.024	0	0	0	0	1	1	1	1	5
0.024	0.025	0	0	1	0	0	0	0	2	1
0.025	0.026	0	0	0	0	1	0	0	0	2
0.026	0.027	0	0	0	0	1	0	0	0	1
0.027	0.028	0	0	0	1	0	0	1	1	0
0.028	0.029	0	0	0	0	0	0	1	1	2
0.029	0.030	0	0	1	0	0	1	1	0	1
0.030	0.031	0	0	0	1	0	0	1	0	1
0.031	0.032	0	0	0	0	0	0	0	0	1
0.032	0.033	0	0	0	0	0	0	0	2	1
0.033	0.034	0	0	0	0	1	0	0	0	0
0.034	0.035	0	0	0	2	0	0	0	0	0
0.035	0.036	0	0	0	0	1	0	0	0	0
0.043	0.044	0	0	0	0	0	1	0	0	0
0.056	0.057	0	0	0	0	0	1	0	0	0

Table A-10: Number of bubbles, N_b, at various bubble class for air/3.0 % (w/w) CMC solution at H_s =0.23 m for various values of U_g.

$d_{b,i}$, m	$d_{b,i+1}$, m	U_g, m s^{-1}								
		0.0083	0.0125	0.0167	0.0208	0.0250	0.0292	0.0333	0.0375	0.0412
0.000	0.002	815	800	850	817	780	762	640	719	635
0.002	0.003	1151	1147	937	859	771	617	548	545	560
0.003	0.004	553	605	466	491	454	355	389	341	296
0.004	0.005	202	264	179	194	202	145	179	167	165
0.005	0.006	73	120	85	105	98	73	89	83	70
0.006	0.007	35	38	38	48	49	50	50	47	45
0.007	0.008	20	25	27	27	33	26	34	38	33
0.008	0.009	15	14	16	19	21	21	23	23	25
0.009	0.010	9	11	8	11	18	13	19	12	21
0.010	0.011	4	12	9	5	10	16	20	9	16
0.011	0.012	5	5	4	7	7	7	8	10	9
0.012	0.013	8	2	5	8	8	3	5	5	6
0.013	0.014	4	1	7	4	3	5	5	9	9
0.014	0.015	3	3	0	4	4	6	9	8	4
0.015	0.016	2	2	4	1	4	5	3	3	2
0.016	0.017	1	2	0	5	8	3	3	5	7
0.017	0.018	1	2	3	2	1	2	4	3	4
0.018	0.019	0	1	2	2	2	1	2	4	6
0.019	0.020	2	0	0	4	1	2	2	4	2
0.020	0.021	1	0	0	0	1	1	1	5	4
0.021	0.022	0	0	2	0	1	2	2	2	2
0.022	0.023	0	0	0	1	2	3	0	7	3
0.023	0.024	0	0	0	2	0	2	1	1	1
0.024	0.025	1	1	0	1	1	1	0	2	2
0.025	0.026	0	0	0	0	2	1	1	2	0
0.026	0.027	0	0	0	1	2	0	1	0	1
0.027	0.028	0	1	0	0	0	1	1	0	0
0.028	0.029	0	0	0	0	0	0	0	0	0
0.029	0.030	0	0	2	0	0	1	0	1	2
0.030	0.031	0	1	0	0	0	0	0	2	1
0.031	0.032	0	0	1	0	0	1	0	2	0
0.032	0.033	0	0	1	0	0	0	1	1	2
0.033	0.034	0	0	0	0	1	0	1	0	2
0.034	0.035	0	0	0	0	0	0	1	0	0
0.035	0.036	0	0	0	0	1	0	1	1	0
0.036	0.037	0	0	0	0	0	0	0	2	1
0.037	0.038	0	0	0	0	0	0	0	0	1
0.038	0.039	0	0	0	0	0	0	0	0	1
0.039	0.040	0	0	0	0	0	0	1	0	0
0.040	0.041	0	0	0	0	0	1	0	0	0
0.041	0.042	0	0	0	0	0	0	0	1	1
0.042	0.043	0	0	0	1	1	0	0	0	1
0.043	0.044	0	0	0	0	0	0	0	1	0
0.047	0.048	0	0	0	0	0	1	0	0	0
0.049	0.050	0	0	0	0	0	0	1	0	0
0.051	0.052	0	0	0	0	0	1	0	0	0
0.057	0.058	0	0	0	0	0	1	0	0	0
0.061	0.062	0	0	0	0	0	0	1	0	0

APPENDIX-II

Table B-1: Values of H_s, U_g, ε , d_{32} and a_i for air/water

H_s, m	U_g, ms^{-1}	ε , -	d_{32}, m	a_i ,m^2/m^3
0.2	0.0083	0.0072	0.088	73.3
0.2	0.0125	0.0068	0.120	105.5
0.2	0.0167	0.0080	0.160	120.6
0.2	0.0208	0.0078	0.198	152.6
0.2	0.0250	0.0094	0.266	169.2
0.2	0.0292	0.0090	0.281	187.1
0.2	0.0333	0.0089	0.294	197.8
0.2	0.0417	0.0110	0.366	199.2
0.2	0.0500	0.0131	0.492	224.9
0.2	0.0583	0.0197	0.794	242.1
0.22	0.0083	0.0066	0.115	104.7
0.22	0.0125	0.0065	0.137	126.4
0.22	0.0167	0.0074	0.182	148.5
0.22	0.0208	0.0072	0.185	154.4
0.22	0.0250	0.0094	0.269	171.2
0.22	0.0292	0.0070	0.182	157.0
0.22	0.0375	0.0088	0.284	194.8
0.22	0.0417	0.0111	0.412	222.4
0.22	0.0500	0.0158	0.601	228.8
0.22	0.0583	0.0151	0.574	228.1
0.23	0.0083	0.0070	0.091	78.1
0.23	0.0125	0.0069	0.123	106.5
0.23	0.0167	0.0074	0.178	144.4
0.23	0.0208	0.0069	0.159	138.8
0.23	0.0250	0.0082	0.207	151.5
0.23	0.0292	0.0108	0.286	158.7
0.23	0.0375	0.0099	0.333	202.6
0.23	0.0417	0.0115	0.376	195.5
0.23	0.0500	0.0126	0.429	204.1
0.23	0.0583	0.0171	0.615	216.1
0.23	0.0083	0.0070	0.091	78.1
0.24	0.0083	0.0070	0.083	71.0
0.24	0.0125	0.0061	0.117	115.6
0.24	0.0167	0.0068	0.142	125.1
0.24	0.0208	0.0061	0.159	155.8
0.24	0.0250	0.0075	0.181	145.6
0.24	0.0292	0.0080	0.228	169.8
0.24	0.0375	0.0085	0.265	186.8
0.24	0.0417	0.0121	0.403	199.5

H_s, m	U_g, ms^{-1}	ε, -	d_{32}, m	a_i ,m^2/m^3
0.24	0.0500	0.0131	0.407	185.9
0.24	0.0583	0.0124	0.404	195.0
0.26	0.0083	0.0078	0.138	106.3
0.26	0.0125	0.0058	0.136	139.4
0.26	0.0167	0.0064	0.142	133.1
0.26	0.0208	0.0066	0.142	128.2
0.26	0.0250	0.0074	0.202	164.2
0.26	0.0292	0.0075	0.201	159.7
0.26	0.0333	0.0081	0.206	153.4
0.26	0.0375	0.0086	0.253	177.8
0.26	0.0417	0.0092	0.259	169.6
0.26	0.0458	0.0080	0.225	169.6
0.28	0.0083	0.0049	0.088	107.6
0.28	0.0125	0.0055	0.114	123.1
0.28	0.0167	0.0068	0.144	128.1
0.28	0.0208	0.0067	0.179	159.4
0.28	0.0250	0.0069	0.180	157.2
0.28	0.0292	0.0084	0.240	170.7
0.28	0.0333	0.0084	0.219	156.2
0.28	0.0375	0.0097	0.294	181.7

Table B-2: Values of CMC con., U_g, ε , d_{32} and a_i for air/CMC solution (H_s = 0.23 m)

Con. CMC %(w/w)	U_g, ms^{-1}	**ε , -**	d_{32}, m	a_i, m^2/m^3
0.5	0.0083	0.091	0.00608	90.0
0.5	0.0125	0.097	0.00627	92.9
0.5	0.0167	0.150	0.00699	129.2
0.5	0.0208	0.130	0.00823	94.5
0.5	0.0250	0.143	0.00819	104.6
0.5	0.0292	0.215	0.01051	122.8
0.5	0.0333	0.387	0.01520	152.7
0.5	0.0375	0.411	0.01622	152.0
0.5	0.0417	0.470	0.01723	163.5
1.0	0.0083	0.073	0.00828	52.6
1.0	0.0125	0.101	0.00742	81.6
1.0	0.0167	0.097	0.00674	86.1
1.0	0.0208	0.120	0.00634	113.5
1.0	0.0250	0.189	0.00805	141.3
1.0	0.0292	0.337	0.01166	173.1
1.0	0.0333	0.289	0.00940	184.3
1.0	0.0375	0.383	0.01253	183.4
1.0	0.0417	0.355	0.01175	181.4
2.0	0.0083	0.040	0.00596	40.2
2.0	0.0125	0.063	0.00576	65.8
2.0	0.0167	0.112	0.00755	89.1
2.0	0.0208	0.192	0.01030	111.5
2.0	0.0250	0.212	0.01122	113.2
2.0	0.0292	0.333	0.01363	146.5
2.0	0.0333	0.229	0.00973	141.5
2.0	0.0375	0.302	0.01080	167.9
2.0	0.0417	0.340	0.01247	163.7
3.0	0.0083	0.136	0.00629	129.4
3.0	0.0125	0.166	0.00683	145.9
3.0	0.0167	0.204	0.00897	136.5
3.0	0.0208	0.245	0.00972	151.1
3.0	0.0250	0.325	0.01166	167.4
3.0	0.0292	0.550	0.01857	177.7
3.0	0.0333	0.561	0.01793	187.7
3.0	0.0375	0.588	0.01682	209.7
3.0	0.0417	0.583	0.01719	203.6

APPENDIX-III

MATLAB codes are presented below.

A3-1: Gas holdup

```
height_allframe=[]; height=[];          % Initialising values
[filename,path]=uigetfile('*.mov','*.*'); myfile=[path filename];
FBed=VideoReader(myfile); numFrames = FBed.NumberOfFrames;
for nframe=1:numFrames;
myimage=read(FBed,nframe);
I1 = imcrop(myimage,[481.5 45.5 290 564]);
K = rangefilt(I1);
I = rgb2gray(K);
background = imopen(I,strel('disk',5));
% Display the Background Approximation as a Surface
I2 = I - background;
I5 = medfilt2(I2);
I3 = imadjust(I5);
I4 = adapthisteq(I3);
level = graythresh(I4);
bw = im2bw(I4,level);
Q = sum(double(bw),2);
x = find(Q>15,1);
h = 37-(x-1)*37/536;
for N = 1
  height(1,N)=nframe/120;
  height(2,N)=h/100;
end
height_allframe=[height_allframe height];
end
%% Writing the results in a Excel file
% New worksheet has the same name as the name of the avi file
wsname=filename(1:length(filename));
xlswrite('height_calculation.xls', height_allframe', wsname);

disp('Data stored in height_calculation.xls')
% The programme ends - *****
```

A3-2 Bubble characteristics

```
clear all
close all                         % Free memory
for x=1:1:8
    sheet=x;
    s=[]; r=[];    r1=[];       % Initialising values
```

```
srcFiles = dir('D:\1 \*.jpg');  % the folder in which your images exists
filename = strcat('D:\1 \',srcFiles(x).name);
myimage=imread(filename);
ro = imrotate(myimage,-90);
g = rgb2gray(ro);
c = auto_crop(g);
mn = min(min(c)); mx = max(max(c)); avg = (mn + mx)/2;
[m2,n2] = size(c);
for i = 1:1:m2
   for j = 1:1:n2
   if c(i,j) >= avg;
      c(i,j)=mx;
   else c(i,j)=c(i,j);
   end
   end
end
a = adapthisteq(c,'Distribution','exponential');
bw = divide_7_4_mod_bw(a);
bw1 = bwareaopen(bw,200);
s  = regionprops(bw1,'Centroid');
r = regionprops(bw1, 'MajorAxisLength', 'MinorAxisLength');
r1 = regionprops(bw1,'Orientation', 'Eccentricity');
s1 = cat(1, s.Centroid);
s2 = struct2cell(r);
s3 = cell2mat(s2);
s4 = cat(2,s1,s3')*0.00009;
s5 = struct2cell(r1);
s6 = cell2mat(s5);
s7 = cat(2,s4,s6');
xlswrite('hs.xls', s7, sheet);
end
```

A3-3 'auto_crop' function

```
function croppedImage = auto_crop(I)
I1 = I;
K = rangefilt(I1);
grayImage = im2bw(K,graythresh(K));
% Get the dimensions of the image.
[rows columns numberOfColorBands] = size(grayImage);
% Get all rows and columns where the image is nonzero
[nonZeroRows nonZeroColumns] = find(grayImage);
% Get the cropping parameters
Q = sum(double(grayImage),2);
x = find(Q>200,1);
topRow = min(x);
```

```
bottomRow = max(nonZeroRows(:));
leftColumn = min(nonZeroColumns(:));
rightColumn = max(nonZeroColumns(:));
% Extract a cropped image from the original.
croppedImage = I1(topRow:bottomRow, leftColumn:rightColumn);
```

A3-4 To divide the image into 7x4 parts and join after processing 'divide_7_4_mod_bw' function

```
function newImg_f = divide_7_4_mod_bw(I)
[rows, columns, numberOfColorChannels] = size(I);
r = int32(rows/7);
c = int32(columns/4);
% Extract the 28 images.
image1 = I(1:r, 1:c);
image1a = mod_bwimage(image1);
image2 = I(1:r, c+1:2*c);
image2a = mod_bwimage(image2);
image3 = I(1:r, 2*c+1:3*c);
image3a = mod_bwimage(image3);
image4 = I(1:r, 3*c+1:end);
image4a = mod_bwimage(image4);
image5 = I(r+1:2*r, 1:c);
image5a = mod_bwimage(image5);
image6 = I(r+1:2*r, c+1:2*c);
image6a = mod_bwimage(image6);
image7 = I(r+1:2*r, 2*c+1:3*c);
image7a = mod_bwimage(image7);
image8 = I(r+1:2*r, 3*c+1:end);
image8a = mod_bwimage(image8);
image9 = I(2*r+1:3*r, 1:c);
image9a = mod_bwimage(image9);
image10 = I(2*r+1:3*r, c+1:2*c);
image10a = mod_bwimage(image10);
image11 = I(2*r+1:3*r, 2*c+1:3*c);
image11a = mod_bwimage(image11);
image12 = I(2*r+1:3*r, 3*c+1:end);
image12a = mod_bwimage(image12);
image13 = I(3*r+1:4*r, 1:c);
image13a = mod_bwimage(image13);
image14 = I(3*r+1:4*r, c+1:2*c);
image14a = mod_bwimage(image14);
image15 = I(3*r+1:4*r, 2*c+1:3*c);
image15a = mod_bwimage(image15);
image16 = I(3*r+1:4*r, 3*c+1:end);
image16a = mod_bwimage(image16);
```

```
image17 = I(4*r+1:5*r, 1:c);
image17a = mod_bwimage(image17);
image18 = I(4*r+1:5*r, c+1:2*c);
image18a = mod_bwimage(image18);
image19 = I(4*r+1:5*r, 2*c+1:3*c);
image19a = mod_bwimage(image19);
image20 = I(4*r+1:5*r, 3*c+1:end);
image20a = mod_bwimage(image20);
image21 = I(5*r+1:6*r, 1:c);
image21a = mod_bwimage(image21);
image22 = I(5*r+1:6*r, c+1:2*c);
image22a = mod_bwimage(image22);
image23 = I(5*r+1:6*r, 2*c+1:3*c);
image23a = mod_bwimage(image23);
image24 = I(5*r+1:6*r, 3*c+1:end);
image24a = mod_bwimage(image24);
image25 = I(6*r+1:end, 1:c);
image25a = mod_bwimage(image25);
image26 = I(6*r+1:end, c+1:2*c);
image26a = mod_bwimage(image26);
image27 = I(6*r+1:end, 2*c+1:3*c);
image27a = mod_bwimage(image27);
image28 = I(6*r+1:end, 3*c+1:end);
image28a = mod_bwimage(image28);
% Joining images
newImg = cat(2,image1a,image2a);
newImg1 = cat(2,newImg,image3a);
newImg2 = cat(2,newImg1,image4a);
newImg3 = cat(2,image5a,image6a);
newImg4 = cat(2,newImg3,image7a);
newImg5 = cat(2,newImg4,image8a);
newImg6 = cat(2,image9a,image10a);
newImg7 = cat(2,newImg6,image11a);
newImg8 = cat(2,newImg7,image12a);
newImg9 = cat(2,image13a,image14a);
newImg10 = cat(2,newImg9,image15a);
newImg11 = cat(2,newImg10,image16a);
newImg12 = cat(2,image17a,image18a);
newImg13 = cat(2,newImg12,image19a);
newImg14 = cat(2,newImg13,image20a);
newImg15 = cat(2,image21a,image22a);
newImg16 = cat(2,newImg15,image23a);
newImg17 = cat(2,newImg16,image24a);
newImg18 = cat(2,image25a,image26a);
newImg19 = cat(2,newImg18,image27a);
newImg20 = cat(2,newImg19,image28a);
```

```
newImg21 = cat(1,newImg2,newImg5);
newImg22 = cat(1,newImg21,newImg8);
newImg23 = cat(1,newImg22,newImg11);
newImg24 = cat(1,newImg23,newImg14);
newImg25 = cat(1,newImg24,newImg17);
newImg_f = cat(1,newImg25,newImg20);
```

A3-5 Converting to binary image
'mod_bwimage' function

```
function im = mod_bwimage(I)
mat = MinimaxAT(I);
cm = imcomplement(mat);
bw = bwareaopen(cm,50);
f = imfill(bw,'holes');
D = bwdist(~f,'chessboard');
D = -D;
D(~f) = Inf;
W = watershed(D);
im=f;
im(W==0)=0;
```

A3-6: Applying threshhold
'MinimaxAT' function

```
function [B,T] = MinimaxAT(I)
% I: input image
% B: thresholded image
% T: Threshold surface
% implements a line search
Iterations = 100; % increase if it is inadequate
epsilon=1e-7; % for termination condition, decrease if it is inadequate
pow=1; % although 1 is a pretty good value,
    % you may need to play with this parameter, pow takes POSITIVE real values
I = double(I);
% linearly map I to the range [0, 1]
I(:) = (I-min(I(:)))/(max(abs(I(:)))-min(I(:)));
% display I
%figure(1),imagesc(I),colormap(gray);axis image; drawnow;
[h,w] = size(I);
rowC = 1:h;       rowN = [1 1:h-1];    rowS = [2:h h];
colC = 1:w;       colE = [1 1:w-1];    colW = [2:w w];
Ix = (I(rowC,colW) - I(rowC,colE))/2; Ix(:,1) = 0; Ix(:,end) = 0;
Iy = (I(rowS,colC) - I(rowN,colC))/2; Iy(1,:) = 0; Iy(end,:) = 0;
% [Ix,Iy] = gradient(I);
g = (Ix.^2+Iy.^2).^(pow/2);
```

```
g(:) = g/max(g(:));
% initialize threhold surface
T=zeros(h,w);
Tx = T;
Ty = T;
delT=T;
delTx=T;
delTy=T;
%figure(2),set(gcf,'doublebuffer','on');
old_alpha=10;
for n=1:Iterations,
   % compute alpha
   Tx(:) = (T(rowC,colW) - T(rowC,colE))/2; Tx(:,1) = 0; Tx(:,end) = 0;
   Ty(:) = (T(rowS,colC) - T(rowN,colC))/2; Ty(1,:) = 0; Ty(end,:) = 0;
%    [Tx,Ty] = gradient(T);
   en1 = 0.3*sum(sum(g.*(T-I).^2));
   en2 = 0.3*sum(sum(Tx.^2+Ty.^2));
   alpha = en2/sqrt(en1^2+en2^2);
   % line search
   delT(:) = sqrt(1-alpha^2)*g.*(I-T) +
alpha*(T(rowC,colW)+T(rowC,colE)+T(rowN,colC)+T(rowS,colC)-4*T);
   delTx(:) = (delT(rowC,colW) - delT(rowC,colE))/2; delTx(:,1) = 0; delTx(:,end) = 0;
   delTy(:) = (delT(rowS,colC) - delT(rowN,colC))/2; delTy(1,:) = 0; delTy(end,:) = 0;
%    [delTx,delTy] = gradient(delT);
   e2 = sqrt(1-alpha^2)*sum(sum(g.*(I-T).*delT)) - alpha*sum(sum(Tx.*delTx + Ty.*delTy));
   e3 = sqrt(1-alpha^2)*sum(sum(g.*delT.*delT)) + alpha*sum(sum(delTx.*delTx +
delTy.*delTy));
   % delt should at least be 0.25
   % minimize a quadratic expression: e1 - delt*e2 + 0.5*delt*delt*e3, see
   % the journal paper
   delt = max(0.25,e2/e3);
   T(:) = T+delt*delT;
   % show
   %figure(2),imagesc(I>T),colormap(gray);axis image;
   %title(['Iteration: ' num2str(n) ' alpha: ' num2str(alpha)]);
   %drawnow;
   if abs(old_alpha-alpha)<=epsilon,
      break;
   end
   old_alpha=alpha;
end
B = I>T;
% figure(3),imagesc(T),colormap(gray);axis image;
```