Preface

This thesis is devoted to design and analysis of robust domain decomposition methods for certain classes of singularly perturbed parabolic problems. The solution of these problems exhibits boundary layers, that is, narrow regions where solution and its derivatives change rapidly. Due to this reason, standard numerical methods are not adequate for solving these problems. So, special approaches are needed to obtain good numerical approximations. This has led to the development of the so called parameter-robust or uniformly convergent numerical methods.

We first consider a class of singularly perturbed parabolic reaction-diffusion problems with time delay. We establish a maximum principle and derive a priori bounds on the solution and its derivatives. We treat directly the parabolic problem and decompose the original computational domain into three overlapping subdomains. We place a uniform mesh in both time and spatial directions and consider the backward Euler scheme in time direction and central differencing in spatial direction on each subdomain. We analyze the method with the help of some auxiliary problems and a new discrete maximum principle that we establish. We prove that the method yields uniformly convergent numerical approximations of almost second order in space and first order in time. More notably, we prove that the desired accuracy is achieved in only one iteration when perturbation parameter is small.

We next consider a class singularly perturbed semilinear parabolic reaction-diffusion problems. We directly treat the time dependent problem and developed a domain decomposition method. The error analysis is given based on the auxiliary problems and linearised version of the discrete operator. We prove that the method is uniformly convergent and that the method converges much faster for small values of perturbation parameter.

The next class of problem considered is a class of coupled systems of singularly perturbed parabolic reaction-diffusion problems with distinct small positive parameters. The solution exhibits overlapping layers of different widths which makes the construction of robust numerical methods and their analysis quite difficult. The method splits the original domain into five overlapping subdomains. On each subdomain we discretize the problem using a finite difference scheme comprising of the backward Euler scheme on a uniform mesh in time direction and central differencing on a uniform mesh in spatial direction. We provide convergence analysis based on some auxiliary problems and using suitable barrier functions. The numerical approximations generated by the method are proved to be almost second order convergent in space and first order in time, independently of the perturbation parameters.

We then turn our attention to a class of coupled systems of singularly perturbed parabolic reaction-diffusion problems with time delay. We establish continuous maximum principle and drive a priori bounds on the solution and its derivatives. We also provide a decomposition of the solution into regular and layer parts. We designed a domain decomposition method by extending our ideas as above. The approximations generated by the method are proved to be almost second order accurate in space and first order accurate in time.

At the end we again consider a class of coupled systems of singularly perturbed parabolic reaction-diffusion problems. We introduce an improved domain decomposition method, which is based on the decomposition of the problem domain into three overlapping subdomains: two finely meshed layer subdomains and one coarsely meshed regular subdomain. In spatial direction we consider a non-uniform mesh in the layer subdomains and a uniform mesh in the regular subdomain. In time direction we consider a uniform mesh on each subdomain. We discretize the problem on each subdomain by using the backward Euler scheme in time direction and central differencing in spatial direction. We prove that the numerical approximations generated from the method are robust with respect to the perturbation parameters. Moreover, we demonstrate the benefits of the method in terms of number of iterations required to get the desired accuracy.

Numerical results are presented on some standard test examples to validate the theoretical error bounds and to demonstrate the effectiveness of the developed methods.