Chapter 8

Overall conclusion and future work

8.1 Overall conclusion

In this thesis work, several goals are achieved. To analyze the movement of solute in a homogeneous aquifer, firstly, the one-dimensional nonlinear solute transport model is considered which is simply an advection-diffusion equation having source/sink term with given first type source boundary conditions and initial condition. To describe the movement of solute concerning the column length, the considered problem is solved numerically using FDM. The solution profile has been drawn for both conservative and non-conservative systems for different particular cases. A comparison of the trend of numerical solution with the existing analytical solution and thereby validating our considered numerical technique and stability analysis of the problem are the main contribution of the work. Secondly, numerical solution of the time-fractional order linear advection-reaction-diffusion model has been obtained. The striking feature of the work is the damping effect of the field variable on the solution profile when the system approaches fractional-order from the integer order for specified values of the parameters of the system which greatly describes the physical phenomenon where the rate of transportation is much faster than the usual one.

To understand the physical problems in more accurate way the space and time-fractional order one-dimensional solute transport models have been considered with prescribed initial and boundary conditions. Initially, the aquifer is considered solute free, and a time-dependent input-concentration is considered at inlet boundary. To draw the solution profile, the considered model is solved using shifted Legendre collocation method where the operational matrix has been used for fractional-order derivative. The numerical computations and graphical presentations exhibit that the method is effective and reliable during the solution of the physical model with complicated boundary conditions even in the presence of the reaction term.

One-dimensional Burgers-Fisher and Burgers-Huxley equations have importance for describing different mechanisms in several fields of science and engineering. Therefore, my work is extended to solve one-dimensional Burgers-Fisher and Burgers-Huxley equations for space-time fractional-order system using shifted Legendre collocation method. The solution profiles of considered problems have been shown through graphs for different particular cases. The estimation of the error is analyzed for shifted Legendre polynomial approximation. Also the order of convergence for both the equations are found using the proposed method.

lastly, A drive has been taken to use the shifted Legendre collocation method for solving nonlinear two-dimensional fractional-order partial differential equations. The method used contains shifted Legendre polynomial, operational matrix and Kronecker product. The efficiency and effectiveness of the method have been validated through applying it to an existing nonlinear partial differential equation having exact solution and show the superlinearly convergence rate of the proposed method. In particular the considered method has been used to solve solute transport model which is nonlinear (2+1)-dimensional space-time fractional-order reaction-advection-diffusion equation with prescribed initial and boundary conditions. The main focus is concerned with the effect of fractional order parameters, advection term and reaction term on the solution profile.

A two-dimensional nonconservative time-fractional order aerosol transport equation model as aerosol transport equation which is directly linked to climate change has been studied and analyzed. The solution profiles of the considered problem which is solved by shifted Legendre collocation method shown through graphical plots which clearly exhibit the effect of aerosol particle size and variation of mass concentration for different fractional-order time derivative. The variations of normalized mass concentration at different time levels for both integer and fractional-order systems are also shown.

8.2 Future work

The multidimensional fractional-order linear problems do not have a precise analytics solution. Especially it is hard to get for nonlinear equations in fractional-order systems. Approximate analytical methods and numerical methods are very useful for solving these types of equations. Achieving computationally efficient solutions of those evolution equations for different particular cases are very challenging jobs. The mathematical software viz., MATHEMATICA, MATLAB, MAPLE are needed during numerical computations. Due to physical relevance and important applications, there is still plenty of scope for the researchers to explore nonlinear FRADE subject to different types of initial and boundary conditions viz., Dirichlet boundary conditions, Neumann boundary conditions and Robin-type boundary condition, mainly in multidimensional cases. Therefore a number of mathematical model in multi-dimensional cases having physical relevances and engineering applications may be proposed in near future. The effect of nonlinearity appeared in reaction term as well as in the dispersion/diffusion term on the solution profiles will be considered. The solute transport equations (porous medium equation) considered in homogeneous medium may be extended for heterogeneous medium.

There are many physical and chemical phenomena in various branches of science and engineering which are described by fractional as well as variable-order nonlinear differential equations. In particular, nonlinear RADEs are generally found mathematical models for groundwater hydrology which are specially used to model the transportation of passive tracers carried by fluid flow in a porous medium. Many challenging nonlinear RADEs are trending in Industrial problems and Applied Mathematics related to groundwater contamination which predict the movement and behaviour of the concentrations of contaminants. There is plenty of rooms to extend the existing physical problems of the integer-order system to fractional-order as well as variable-order systems.

Recently, many researchers are involved in the field of aerosol transportation as it is direct by linked with the climate change. Many models on aerosol transport have been developed viz., aerosol model for atomic spectrometry, a model in one-dimension for simulation of aerosol transport and deposition in human lung, the aerosol in human lung in two and three-dimension, motion of inertial spheroidal particles in a shear flow near a solid wall with special application to aerosol transport in microgravity, aerosol transport in sequentially bifurcating airways, the effect of electrohydrodynamic flows and turbulence on aerosol transport, an aerosol transport model for coastal area and calculated effect of surf produced aerosols on processes in the marine atmospheric boundary layer, a model for Micro-particle transport and deposition in a human oral airway, a model of aerosol microscopic module for large scale, a model to validate the size-resolved particle dry depositions scheme for application in aerosol transport model etc. But to best of my knowledge the multi-dimensional space-time fractional-order reaction-advection–dispersion equation in the finite domain with arbitrary initial and boundary conditions have not been studied by any researcher. Therefore an attempt can be taken to solve the model in two/three dimensional cases using shifted Legendre collocation method or through developing more efficient and appropriate numerical tools.

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