## List of Figures

| 1.1 | Sources of groundwater contamination  | 4           |
|-----|---|-------------|
| 1.2 | Control volume for solute transport through porous media                            | 9           |
| 1.3 | A partition of the $(x,t)$ – plane into uniform cells of size $h \times k$          | 27          |
| 1.4 | Comparison between Caputo fractional derivative and shifted Legen-                  |             |
|     | dre operational matrix  | 35          |
| 2.1 | Comparison between analytical and numerical results at $t = 2.5$ , 10               |             |
|     | when $\lambda = 0$  | 49          |
| 2.2 | Plots of $u(x,t)/u_0$ vs. $x$ at $t = 2.5, 5, 10, 15, 20, D = 0.6$ and $V = 0.6$    | 40          |
| 2.3 | when $\lambda = 0$  | 49          |
| 2.5 | when $\lambda \neq 0$   | 50          |
| 2.4 | Plots of $u(x,t)/u_0$ vs. $x$ at $t = 2.5, 5, 10, D = 0.6$ and $V = 0$ when         | 00          |
|     | $\lambda = 0.$  | 50          |
| 2.5 | Plots of $u(x,t)/u_0$ vs. x at $t = 2.5, 5, 10, D = 0.6$ and $V = 0$ when           |             |
|     | $\lambda  eq 0$   | 51          |
| 2.6 | Plots of $u(x,t)/u_0$ vs. $x$ at $t = 2.5, 5, 10, 15, 20, D = 1$ and $V = 1$        |             |
|     | when $\lambda = 0$  | 51          |
| 2.7 | Plots of $u(x,t)/u_0$ vs. $x$ at $t = 2.5, 5, 10, 15, 20, D = 1$ and $V = 1$        |             |
|     | when $\lambda \neq 0$   | 52          |
| 3.1 | Comparison of variation of $u(x,t)/u_0$ vs.t with analytical result for             |             |
|     | $u_0 = x = 1$ and $V = \lambda = 0$ at $\alpha = 1$                                 | 63          |
| 3.2 | Plots of $u(x,t)/u_0$ vs. t at $\alpha = 0.7, 0.8, 0.9, 1.0, V = 0.6$ for conserva- |             |
|     | tion case $(\lambda = 0)$   | 64          |
| 3.3 | Plots of $u(x,t)/u_0$ vs. $t$ at $\alpha = 0.7, 0.8, 0.9, 1.0, V = 0.6$ for non-    | 0.4         |
|     | conservation case $(\lambda = 0.6)$   | 64          |
| 4.1 | Plot of the error function $ u_{exact}(x,1)-u_{numerical}(x,1) $ vs. $x$ for Fisher | <del></del> |
| 1.0 | equation  | 74          |
| 4.2 | first type source boundary condition when $V = 0.6$ in/h and $\beta =$              |             |
|     | 1.6, 1.7, 1.8, 1.9 and 2 at fixed $\alpha = 1$                                      | 74          |
|     |   |             |

List of Figures

| 4.3          | Plots of normalised concentration factor vs. column length with first type source boundary condition when $V=0$ in/h and $\beta=1.6,1.7,1.8,1.9$ and 2 at fixed $\alpha=1.$   |
|--------------|---|
| 4.4          | Plots of normalised concentration factor vs. column length with first type source boundary condition when $V=0.6$ in/h and $\alpha=0.6,0.7,0.8,0.9$ and 1 at fixed $\beta=2.$   |
| 4.5          | Plots of normalised concentration factor vs. column length with first type source boundary condition when $V=0$ in/h and $\alpha=0.6,0.7,0.8,0.9$ and 1 at fixed $\beta=2.$   |
| 5.1          | Plots of normalised concentration factor vs. column length with first type source boundary condition for space-time fractional-order BFE  |
| 5.2          | when $\alpha = 0.6, 0.7, 0.8, 0.9$ and 1 at fixed $\beta = 2$ 93. Plots of normalised concentration factor vs. column length with first type source boundary condition for space-time fractional-order BFE              |
| 5.3          | when $\beta = 1.6, 1.7, 1.8, 1.9$ and 2 at fixed $\alpha = 1, \dots, 9^2$ . Plots of normalised concentration factor vs. column length with first type source boundary condition for space-time fractional-order BHE    |
| 5.4          | when $\alpha = 0.6, 0.7, 0.8, 0.9$ and 1 at fixed $\beta = 2, \dots, 94$<br>Plots of normalised concentration factor vs. column length with first<br>type source boundary condition for space-time fractional-order BHE |
| 5.5          | when $\beta = 1.6, 1.7, 1.8, 1.9$ and 2 at fixed $\alpha = 1, \dots, 98$ . Plots of normalised concentration factor $u(x, 1)$ vs. column length   |
|              | with first type source boundary condition for BFE, BHE and $\lambda = 0$ when $\alpha = 1, \beta = 2.$  |
| 6.1          | The plot of the error function $u_{exact}(x, y, 1) - u_{6,6,6}(x, y, 1)$ for the Fishers equation   |
| 6.2          | Plots of normalised concentration factor $u(x, y, 0.5)/u_0$ vs. column length for $\lambda = -1, 0, 1$ when $D = 25, V = 50, \alpha = 1, \beta = 2$   |
| 6.3          | Plots of normalised concentration factor $u(x,y,t)/u_0$ vs. column length for time $t=0.2,0.4,0.6,0.8$ when $D=25,V=50,\alpha=1,\beta=2.10$   |
| 6.4          | Plots of normalised concentration factor $u(x, y, 0.5)/u_0$ vs. column length for different $\alpha = 0.4, 0.6, 0.8$ and 1 when $\beta = 2, V = 50$ 108   |
| 6.5          | Plots of normalised concentration factor $u(x, y, 0.5)/u_0$ vs. column length for different $\alpha = 0.4, 0.6, 0.8$ and 1 when $\beta = 2, V = 0$ 108  |
| 6.6          | Plots of normalised concentration factor $u(x, y, 0.5)/u_0$ vs. column length for different $\beta = 1.4, 1.6, 1.8$ and 2 when $\alpha = 1, V = 50109$  |
| 6.7          | Plots of normalised concentration factor $u(x, y, 0.5)/u_0$ vs. column length for different $\beta = 1.4, 1.6, 1.8$ and 2 when $\alpha = 1, V = 0, \ldots$ 109  |
| 7.1<br>7.2   | Plots of the error function $ C_{exact}(x,y,1) - C(x,y,1) $ vs. $x$ and $y$ . 120 Plots of normalized mass concentration at a particular time $t=0.5$   |
| <del>-</del> | for different particle size $s=0.1$ and $s=0.2$ for $\alpha=1,\ldots,120$   |

List of Figures \_\_\_\_\_ xi

| 7.3 | Plots of normalized mass concentration at a particular time $t=0.5$               |     |
|-----|---|-----|
|     | for different fractional time derivative $\alpha = 0.4, 0.6, 0.8, \text{ and } 1$ | 121 |
| 7.4 | Plots of the normalized mass concentration vs. $x$ and $y$ for various            |     |
|     | time at particle size $s = 0.1$ for $\alpha = 1$                                  | 121 |
| 7.5 | Plots of the normalized mass concentration vs. $x$ and $y$ for various            |     |
|     | time at particle size $s = 0.1$ for $\alpha = 0.4$                                | 122 |