Preface

This thesis consists of five chapters. Chapter 1 is the introductory chapter in which some definitions of well known polynomials like Lagrange polynomials, Legendre polynomials, and Euler polynomials have been introduced. Also, this chapter includes the history of ordinary differential equations, partial differential equations, operational matrices and fractional derivatives.

In chapter 2, two schemes based on Lagrange polynomials are constructed to find the numerical solutions of linear and non-linear initial value problems. In scheme-I, the roots of Legendre polynomial are used as node points for Lagrange polynomials and we have taken random node points in the domain [0,1] and orthogonalize the resulting Lagrange polynomials using the Gram-Schmidt orthogonalization process in scheme-II. The function approximations by using generating interpolating scaling functions (ISF) and orthonormal Lagrangian basis functions (OLBF) over the space $L^2[0,1]$ are introduced and then the operational matrices of integration and product based on newly designed approximations namely ISF and OLBF are constructed. Operational matrices convert given linear or non-linear initial value problems into the associated system of algebraic equations. Moreover, the error bounds for the function approximation of both schemes have been established. The efficiency of proposed schemes is also confirmed in this chapter via several examples. These schemes are simple, efficient and produce very accurate numerical results for a small number of basis functions.

In chapter 3, the similar two schemes as discussed in chapter 2, have been applied to find the numerical solution of second order two-dimensional hyperbolic telegraph equation (TDTE) with the Dirichlet boundary conditions. The main equation is converted into partial integro-differential equations (PIDEs) by using initial and boundary conditions. The operational matrices of differentiation and integration are used to transform the PIDEs into algebraic generalized Sylvester equation. The results obtained by the proposed schemes are compared with the Bernoulli matrix method and B-spline differential quadrature method to show that the proposed schemes are accurate for small number of basis functions.

An efficient matrix approach by using Euler polynomials to solve two-dimensional diffusion and telegraph equations subject to the Dirichlet boundary conditions is discussed in chapter 4. By using initial and boundary conditions, the main equation is converted into PIDEs. The operational matrices of differentiation and integration of Euler polynomials are used to transform that PIDEs into algebraic generalized Sylvester equation. Also, some examples are included to predict the accuracy and effectiveness of the scheme. Moreover, obtained results are compared with the numerical results of [1], [2].

Chapter 5 is devoted to the semi-discretization technique combined with operational matrix approach to solve the fractional order wave equation that arises in a dielectric medium. In this approach, the Caputo derivative terms of order α and β are approximated by difference scheme of order $\mathcal{O}(\tau^{3-\alpha})$ and $\mathcal{O}(\tau^{3-\beta})$, $1 < \beta < \alpha < 2$, respectively, to transform the proposed fractional order wave equation into system of second order ordinary differential equations (ODEs). To solve the ODEs, operational matrix method is used which has several advantages over the several ODE solvers. The convergence of the approximation taken in the spatial direction at k^{th} level of time is established. Moreover, the scheme is unconditionally stable with the rate of convergence $\mathcal{O}(\tau^{3-\alpha})$ and $\mathcal{O}\left(\frac{1}{2^{N+1}(N+1)!}\right)$ in the time and spatial directions, respectively. Finally, test examples are included to show the efficiency and accuracy of the proposed method and to support the theoretical results.