APPENDIX

A. 1 Derivation of the expression (5.1)

The force equation can be represented as:

$$F = eE_z \tag{A.1}$$

$$\Rightarrow \quad \frac{dp}{dt} = eE_z$$

$$\Rightarrow \quad \frac{d(\gamma_0 mv)}{dt} = eE_z$$

$$\gamma_0 m \frac{d(v)}{dt} = eE_z \tag{A.2}$$

Using convective derivative in (A.2)

$$\gamma_0 m \left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial z} \right) v = eE_z \tag{A.3}$$

When the potential depression starts, the kinetic energy drop to the point where $\gamma = \gamma_0^{1/3}$

and the expression (A.3) becomes:

$$\gamma^3 \left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial z} \right) v = \frac{e}{m} E_z \tag{A.4}$$

A. 2 Derivation of the expression (5.4)

The relativistic mass factor is given as:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{A.5}$$

Rearranging the (A.5) provides:

$$\gamma^2 = \frac{1}{1 - \frac{v^2}{c^2}} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2} \Rightarrow \frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2}$$

$$\Rightarrow \quad \frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2}$$

$$v^2 = c^2\left(1 - \frac{1}{\gamma^2}\right) \tag{A.6}$$

Differentiate the (A.6) with respect to z :

$$\frac{\partial v^2}{\partial z} = \frac{\partial}{\partial z}\left(c^2\left(1 - \frac{1}{\gamma^2}\right)\right) \tag{A.7}$$

$$\Rightarrow \quad v\frac{\partial v}{\partial z} = \frac{c^2}{\gamma^3}\frac{\partial \gamma}{\partial z}$$

$$\gamma^3 v\frac{\partial v}{\partial z} = c^2\frac{\partial \gamma}{\partial z} \tag{A.8}$$

A. 3 Derivation of the expression (5.8)

From conservation of energy

$$\gamma mc^2 + e\phi = \gamma_{inj} mc^2$$

$$\Rightarrow \quad \gamma_{inj} = \gamma + \frac{e\phi}{mc^2} \tag{A.9}$$

The propagation constant can be written as:

$$\beta = \frac{v}{c} \Rightarrow v = \beta \times c \tag{A.10}$$

Assume that,

$$\sigma v = \text{const}$$

$$\Rightarrow \quad \sigma = \frac{const}{\beta \times c} \tag{A.11}$$

The term σ can be expressed as:

$$\sigma = \frac{\varepsilon_0}{r_c \ln(r_m / r_c)} \phi$$

$$\Rightarrow \quad \phi = \sigma \times \frac{r_c \ln(r_m / r_c)}{\varepsilon_0} \tag{A.12}$$

Substituting (A.11) in (A.12) provides:

$$\phi = \frac{const}{\beta \times c} \times \frac{r_c \ln(r_m / r_c)}{\varepsilon_0} \tag{A.13}$$

Now substitute (A.13) in (A.9) gives:

$$\gamma_{inj} = \gamma + \frac{e}{mc^2} \times \frac{const}{\beta \times c} \times \frac{r_c \ln(r_m / r_c)}{\varepsilon_0} \tag{A.14}$$

The above expression can be expressed as:

$$\gamma_{inj} = \gamma + \frac{1}{\beta} \times \frac{e \ln(r_m / r_c)}{2\pi\varepsilon_0 mc^3} \times 2\pi r_c \times const$$

$$\Rightarrow \quad \gamma_{inj} = \gamma + \frac{I_0}{I_s \beta} \tag{A.15}$$

where $I_0 = 2\pi r_c \times const$ and $I_s = \dfrac{2\pi\varepsilon_0 mc^3}{e \ln(r_m / r_c)}$. The expression (A.15) represent the equation (5.8).