Appendix-A

Stability Theorems

The stability analysis of a nonlinear system involves several theorems as given in reference [76], few of which have been used for nonlinear stability analysis of DC microgrid system in Section 5.4 and have been stated here for ease of reader.

Theorem 1:

Let x=0 be an equilibrium point for the nonlinear system

$\dot{x} = f(x)$ where f: D→ R^n is continuously differentiable and D is a neighbourhood of the origin. Let

$$\boldsymbol{A} = \frac{\partial f}{\partial x}(x)\bigg|_{x=0}$$

Then,

- The origin is asymptotically stable if Re $\lambda_i < 0$ for all eigenvalues of $\boldsymbol{A}$.
- The origin is unstable if Re $\lambda_i > 0$ for one or more of the eigenvalues of $\boldsymbol{A}$.

Theorem 2:

If g_1 and g_2 are twice continuously differentiable and satisfy (38), all eigenvalues of A_1 have zero real parts, and all eigenvalues of A_2 have negative real parts, then there exist a constant $\delta > 0$ and a continuously differentiable function h(y), defined for all $||y|| > \delta$, such that $z = h(y)$ is a center manifold for (39).

Theorem 3:

Under the assumptions of theorem 3, if the origin y=0 of the reduced order system is asymptotically stable then the origin of the full system is also asymptotically stable.

Theorem 4:

Let V: $[0,\infty] \times R^n \rightarrow R$ be a continuously differentiable function such that

$$\alpha_1(\|x\|) \leq V(t,x) \leq \alpha_2(\|x\|)$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t,x,u) \leq -W_3(x), \forall \|x\| \geq \rho(\|u\|) > 0$$

$$\forall (t,x,u) \in [0,\infty) \times R^n \times R^m,$$

where $\alpha_1, \alpha_2$are class K_∞functions, $\rho$is a class K function, and $W_3(x)$is a continuous positive definite function on R^n. Then system is input-to-state stable with $\gamma = \alpha_1^{-1} \circ \alpha_2 \circ \rho$.