Chapter 5

A study of edge cracks in an interfacial orthotropic composite material under the normal load

5.1 Introduction

Due to the less possibility of crack formation and crack propagation in composite materials, these materials are very much used in aero-space ships, wings and several types of ships, structures, etc. Engineers and Scientists are very curious about the composite structures. They have found several types of physical quantities like stress intensity factors, energy release rate, thermal stress intensity factors which are used in ship cruise, aerospace structures, and buildings to use these structures in a proper

The contents of this chapter have been communicated for the publication.

way. Orthotropic composite materials are more beneficial and eco-friendly than the other anisotropic materials.

The problem concerned with the two edge cracks in the finite orthotropic strip under the normal point loading had been solved by Das *et al*. (2008) in which they have found the expressions of stress intensity factors. The problem related to the edge crack under the point loading, which is orthogonal to the surface of an orthotropic strip of finite width whereas another face of this strip is bonded to other orthotropic half plane had been solved by Das *et al*. (2011a). Das *et al*. (2010) have found the stress intensity factor with the help of weight function, and the solving method was the Hilbert transform method. The problem concerned with edge crack under the normal point loading in the finite composite orthotropic strip has been dealt by Das *et al*. (2011b), where the singular integral equations have been solved using Hilbert transform technique. Banks-Sills (2018) has studied the interface fracture mechanics problem with an asymptotic expression for the stress and displacement fields in the vicinity of the interfacial crack tip between two homogeneous isotropic, linear elastic materials. Qu *et al*. (2018) have studied the interface cracks in anisotropic biomaterials in which the authors found stress intensity factors and the energy release rate in terms of explicit expressions for the crack tip field. Rice (1988) has studied a crack at the interface of dissimilar solids in which the author finds complex stress intensity factors K_I and K_{II} for Mode-1 and Mode-2 with a critical failure locus in the complex plane. The problem considered with the edge crack in the infinite orthotropic strip which is solved by Wang *et al*. (1996) with the aid of an integral transform technique. Mishra *et al*. (2017) have studied the cruciform crack in infinite orthotropic elastic media under the thermal loadings in which they have considered Boron-epoxy composite as the orthotropic material.

In the current chapter, the study is concerned about the elastostatic double edge crack problem under the tensile loadings at the edges of two bounded semi-infinite orthotropic strips, each having finite depth h. Steel-Mylar and Boron-Epoxy composite materials have been used for numerical computations as a particular case. The problem is reduced into the singular integral equations of the first kind with Cauchy-type singularities, which have been solved using Chebyshev polynomials. The expressions of the stress intensity factors (SIFs) are found at the cracks' tips.

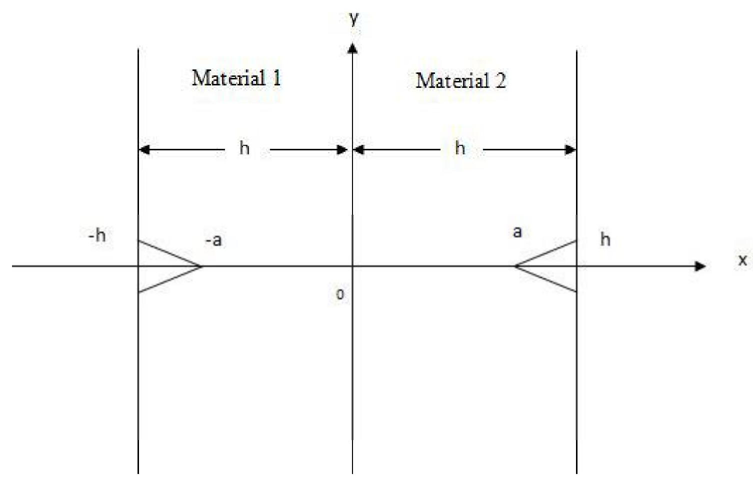

Figure 5.1: Geomerty of the Problem

5.2 Mathematical formulation

Let us consider an elastostatic problem of a composite material with two bonded semi-infinite orthotropic strips each of depth h $(-h \leq x \leq 0, -\infty < y < \infty;\ 0 \leq x \leq h, -\infty < y < \infty)$ and two symmetrically located edge cracks defined by $a \leq \mid x \mid \leq h$ under the normal traction $p(x)$ (Figure 5.1).

The equations of equilibrium in the absence of any type of force in the forms of displacements are expressed through the equations (3.1) and (3.2), where $u = u(x, y)$ and $v = v(x, y)$ are displacement components in x and y directions and $C_{jk}{}^{(i)}$'s are elastic constants of the orthotropic materials. Here the superscripts $i = 1$ and $i = 2$ represent the material 1 and material 2 respectively. The concerned boundary conditions are given as follows.

For material 1,

$$\sigma_{yy}{}^{(1)}(x, 0) = p(x), \qquad -h \le x \le -a, \tag{5.1}$$

$$v^{(1)}(x, 0) = 0, \qquad -a \le x \le 0, \tag{5.2}$$

$$\sigma_{xy}{}^{(1)}(x, 0) = 0, \qquad -h \le x \le 0, \tag{5.3}$$

$$\sigma_{xx}{}^{(1)}(-h, y) = 0, \qquad -\infty < y < \infty, \tag{5.4}$$

$$\sigma_{xy}{}^{(1)}(-h, y) = 0, \qquad -\infty < y < \infty. \tag{5.5}$$

For material 2,

$$\sigma_{yy}{}^{(2)}(x, 0) = p(x), \qquad a \le x \le h, \tag{5.6}$$

$$v^{(2)}(x, 0) = 0, \qquad 0 \le x \le a, \tag{5.7}$$

$$\sigma_{xy}{}^{(2)}(x, 0) = 0, \qquad 0 \le x \le h, \tag{5.8}$$

$$\sigma_{xx}{}^{(2)}(h, y) = 0, \qquad -\infty < y < \infty, \tag{5.9}$$

$$\sigma_{xy}{}^{(2)}(h, y) = 0, \qquad -\infty < y < \infty. \tag{5.10}$$

At the interface of two materials i.e., at $x = 0$,

$$u^{(1)}(0, y) = u^{(2)}(0, y), \qquad -\infty < y < \infty, \tag{5.11}$$

$$v^{(1)}(0, y) = v^{(2)}(0, y), \qquad -\infty < y < \infty, \tag{5.12}$$

$$\sigma_{xx}{}^{(1)}(0, y) = \sigma_{xx}{}^{(2)}(0, y), \qquad -\infty < y < \infty, \tag{5.13}$$

$$\sigma_{xy}{}^{(1)}(0, y) = \sigma_{xy}{}^{(2)}(0, y), \qquad -\infty < y < \infty. \tag{5.14}$$

All the components of stresses and displacements vanish at the remote distances from the cracks.

5.3 Solution of the Problem

The displacements and stresses can be expressed in terms of two harmonic functions as

$$u^{(i)} = \frac{\partial {\phi_1}^{(i)}}{\partial x} + \frac{\partial {\phi_2}^{(i)}}{\partial x}, \tag{5.15}$$

$$v^{(i)} = {\lambda_1}^{(i)} \frac{\partial {\phi_1}^{(i)}}{\partial y} + {\lambda_2}^{(i)} \frac{\partial {\phi_2}^{(i)}}{\partial y}, \tag{5.16}$$

$$\frac{{\sigma_{xx}}^{(i)}}{C_{66}} = -[(1+{\lambda_1}^{(i)}) \frac{\partial^2 {\phi_1}^{(i)}}{\partial y^2} + (1+{\lambda_2}^{(i)}) \frac{\partial^2 {\phi_2}^{(i)}}{\partial y^2}], \tag{5.17}$$

$$\frac{{\sigma_{yy}}^{(i)}}{C_{66}} = (1+{\lambda_1}^{(i)})({\gamma_1}^{(i)})^2 \frac{\partial^2 {\phi_1}^{(i)}}{\partial y^2} + (1+{\lambda_2}^{(i)})({\gamma_2}^{(i)})^2 \frac{\partial^2 {\phi_2}^{(i)}}{\partial y^2}, \tag{5.18}$$

$$\frac{{\sigma_{xy}}^{(i)}}{C_{66}} = (1+\lambda_1^{(i)}) \frac{\partial^2 {\phi_1}^{(i)}}{\partial x \partial y} + (1+{\lambda_2}^{(i)}) \frac{\partial^2 {\phi_2}^{(i)}}{\partial x \partial y}, \tag{5.19}$$

where ${\phi_j}^{(i)}(x, y)$ satisfy the following partial differential equation as

$$\left(\frac{\partial^2}{\partial x^2} + ({\gamma_j}^{(i)})^2 \frac{\partial^2}{\partial y^2} \right) \phi_j^{(i)}(x, y) = 0, i = 1, 2, j = 1, 2, \tag{5.20}$$

where $({\gamma_j}^{(i)})^2$, $(j = 1, 2)$ are the positive real roots of the above equations

$${C_{11}}^{(i)}{C_{66}}^{(i)}(\gamma_j^i)^4 + (({C_{12}}^{(i)})^2 + 2{C_{12}}^{(i)}{C_{66}}^{(i)} - {C_{11}}^{(i)}{C_{22}}^{(i)})(\gamma_j^i)^2 + {C_{22}}^{(i)}{C_{66}}^{(i)} = 0 \tag{5.21}$$

and

$${\lambda_j}^{(i)} = \frac{{C_{11}}^{(i)}({\gamma_j}^{(i)})^2 - {C_{66}}^{(i)}}{{C_{66}}^{(i)} + {C_{12}}^{(i)}}, j = 1, 2, \tag{5.22}$$

where $C_{jk}^{(i)}$'s are the elastic constants of the orthotropic material 1 and material 2 represented by $i = 1$ and $i = 2$.

The Harmonic functions for material 1 and material 2 are given by

$$\phi_j{}^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty A_j(\alpha)[e^{-\gamma_j{}^{(1)}\alpha x} + e^{\gamma_j{}^{(1)}\alpha x}]\cos\ \alpha y\ d\alpha + \frac{2}{\pi}\int_0^\infty B_j(\alpha)e^{-\frac{\alpha y}{\gamma_j{}^{(1)}}}\cos\ \alpha x\ d\alpha,\ j = 1,2, \quad (5.23)$$

$$\phi_j{}^{(2)}(x,y) = \frac{2}{\pi}\int_0^\infty C_j(\alpha)[e^{-\gamma_j{}^{(2)}\alpha x} + e^{\gamma_j{}^{(2)}\alpha x}]\cos\ \alpha y\ d\alpha + \frac{2}{\pi}\int_0^\infty D_j(\alpha)e^{-\frac{\alpha y}{\gamma_j{}^{(2)}}}\cos\ \alpha x\ d\alpha,\ j = 1,2, \quad (5.24)$$

where $A_1(\alpha)$, $B_1(\alpha)$, $A_2(\alpha)$, $B_2(\alpha)$ and $C_1(\alpha)$, $C_2(\alpha)$, $D_1(\alpha)$, $D_2(\alpha)$ are undetermined arbitrary functions of the material 1 and material 2 respectively.

Our aim is to reduce the materials' constants by using continuity conditions (5.11)-(5.14) and the conditions on the two strips (5.4)-(5.5) and (5.9)-(5.10), and then use the materials' constants to calculate our desired expressions of SIFs at the edge cracks' tips using conditions (5.1)-(5.3) and (5.6)-(5.8).

Applying the boundary conditions (5.3) and (5.8), we get

$$B_1(\alpha) = -\frac{\gamma_1^{(1)}}{\gamma_2^{(1)}}\frac{(1+\lambda_2{}^{(1)})}{(1+\lambda_1{}^{(1)})}B_2(\alpha), \quad (5.25)$$

$$D_1(\alpha) = -\frac{\gamma_1^{(2)}}{\gamma_2^{(2)}}\frac{(1+\lambda_2{}^{(2)})}{(1+\lambda_1{}^{(2)})}D_2(\alpha). \quad (5.26)$$

Applying the boundary conditions (5.2) and (5.7), we get

$$\int_0^\infty B_2(\alpha)\ \alpha\ cos\ \alpha x\ d\alpha = 0, -a \leq x \leq 0, \quad (5.27)$$

$$\int_0^\infty D_2(\alpha)\ \alpha\ cos\ \alpha x\ d\alpha = 0, 0 \leq x \leq a. \quad (5.28)$$

The boundary conditions (5.4) − (5.5) with the aid of equation (5.25) give rise to

$$\int_0^\infty \{A_1(\alpha)a_1^{(1)}(\alpha h) + A_2(\alpha)a_2^{(1)}(\alpha h)\}\alpha^2 \cos\,(\alpha y)\;d\alpha + \frac{(1+\lambda_2{}^{(1)})}{\gamma_2{}^{(1)}}\int_0^\infty \Big\{\frac{1}{\gamma_1^{(1)}}e^{-\frac{\alpha y}{\gamma_1^{(1)}}} - \frac{1}{\gamma_2^{(1)}}e^{-\frac{\alpha y}{\gamma_2^{(1)}}}\Big\}B_2(\alpha)\alpha^2 \cos\,(\alpha h)\;d\alpha = 0, -\infty < y < \infty, \quad (5.29)$$

$$\int_0^\infty \{A_1(\alpha)b_1^{(1)}(\alpha h) + A_2(\alpha)b_2^{(1)}(\alpha h)\}\alpha^2 \sin\,(\alpha y)\;d\alpha + \frac{(1+\lambda_2^{(1)})}{\gamma_2^{(1)}}\int_0^\infty \Big\{-e^{-\frac{\alpha y}{\gamma_1^{(1)}}} + e^{-\frac{\alpha y}{\gamma_2^{(1)}}}\Big\}B_2(\alpha)\alpha^2 \sin\,(\alpha h)\,d\alpha = 0, -\infty < y < \infty, \quad (5.30)$$

where

$$\begin{aligned}
a_1^{(1)}(\alpha h) =& (1+\lambda_1^{(1)})[e^{\gamma_1^{(1)}\alpha h} + e^{-\gamma_1^{(1)}\alpha h}],\\
b_1^{(1)}(\alpha h) =& -(1+\lambda_1^{(1)})\gamma_1^{(1)}[e^{\gamma_1^{(1)}\alpha h} - e^{-\gamma_1^{(1)}\alpha h}],\\
a_2^{(1)}(\alpha h) =& (1+\lambda_2^{(1)})[e^{\gamma_2^{(1)}\alpha h} + e^{-\gamma_2^{(1)}\alpha h}],\\
b_2^{(1)}(\alpha h) =& -(1+\lambda_2^{(1)})\gamma_2^{(1)}[e^{\gamma_2^{(1)}\alpha h} - e^{-\gamma_2^{(1)}\alpha h}].
\end{aligned}$$

The Boundary conditions (5.9) − (5.10) with the aid of equation (5.26) give rise to

$$\int_0^\infty \{C_1(\alpha)a_1^{(2)}(\alpha h) + C_2(\alpha)a_2^{(2)}(\alpha h)\}\alpha^2 \cos\,(\alpha y)\;d\alpha + \frac{(1+\lambda_2^{(2)})}{\gamma_2^{(2)}}\int_0^\infty \Big\{\frac{1}{\gamma_1^{(2)}}e^{-\frac{\alpha y}{\gamma_1^{(2)}}} - \frac{1}{\gamma_2^{(2)}}e^{-\frac{\alpha y}{\gamma_2^{(2)}}}\Big\}D_2(\alpha)\alpha^2 \cos\,(\alpha h)\;d\alpha, -\infty < y < \infty, \quad (5.31)$$

$$\int_0^\infty \{C_1(\alpha)b_1^{(2)}(\alpha h) + C_2(\alpha)b_2^{(2)}(\alpha h)\}\alpha^2 \sin(\alpha y)\, d\alpha + \frac{(1+\lambda_2^{(2)})}{\gamma_2^{(2)}}\int_0^\infty \Big\{ - e^{-\frac{\alpha y}{\gamma_1^{(2)}}} + e^{-\frac{\alpha y}{\gamma_2^{(2)}}}\Big\} D_2(\alpha)\alpha^2 \sin(\alpha h)\, d\alpha = 0, -\infty < y < \infty, \quad (5.32)$$

where

$$\begin{aligned}
a_1^{(2)}(\alpha h) =& (1+\lambda_1^{(2)})[e^{\gamma_1^{(2)}\alpha h} + e^{-\gamma_1^{(2)}\alpha h}],\\
b_1^{(2)}(\alpha h) =& -(1+\lambda_1^{(2)})\gamma_1^{(2)}[e^{\gamma_1^{(2)}\alpha h} - e^{-\gamma_1^{(2)}\alpha h}],\\
a_2^{(2)}(\alpha h) =& (1+\lambda_2^{(2)})[e^{\gamma_2^{(2)}\alpha h} + e^{-\gamma_2^{(2)}\alpha h}],\\
b_2^{(2)}(\alpha h) =& -(1+\lambda_2^{(2)})\gamma_2^{(2)}[e^{\gamma_2^{(2)}\alpha h} - e^{-\gamma_2^{(2)}\alpha h}].
\end{aligned}$$

Now from equations (5.29) and (5.30), $A_1(s)$ and $A_2(s)$ are calculated in terms of $B_2(s)$ as

$$A_1(\alpha) = \left[\frac{-e^{-\frac{\alpha h}{\gamma_1^{(1)}}}\left(a_2^{(1)}(\alpha h) + \frac{b_2^{(1)}(\alpha h)}{\gamma_1^{(1)}}\right) + e^{-\frac{\alpha h}{\gamma_2^{(1)}}}\left(a_2^{(1)}(\alpha h) + \frac{b_2^{(1)}(\alpha h)}{\gamma_2^{(1)}}\right)}{a_1^{(1)}(\alpha h)b_2^{(1)}(\alpha h) - a_2^{(1)}(\alpha h)b_1^{(1)}(\alpha h)}\right]\frac{(1+\lambda_2^{(1)})}{\gamma_2^{(1)}}B_2(\alpha), \quad (5.33)$$

$$A_2(\alpha) = \left[\frac{e^{-\frac{\alpha h}{\gamma_1^{(1)}}}\left(\frac{b_1^{(1)}(\alpha h)}{\gamma_1^{(1)}} + a_1^{(1)}(\alpha h)\right) - e^{-\frac{\alpha h}{\gamma_2^{(1)}}}\left(\frac{b_1^{(1)}(\alpha h)}{\gamma_2^{(1)}} + a_1^{(1)}(\alpha h)\right)}{a_1^{(1)}(\alpha h)b_2^{(1)}(\alpha h) - a_2^{(1)}(\alpha h)b_1^{(1)}(\alpha h)}\right]\frac{(1+\lambda_2^{(1)})}{\gamma_2^{(1)}}B_2(\alpha), \quad (5.34)$$

Now from equations (5.31) and (5.32), $C_1(s)$ and $C_2(s)$ are calculated in terms of $D_2(s)$ as

$$C_1(\alpha) = \left[\frac{-e^{-\frac{\alpha h}{\gamma_1^{(2)}}}\left(a_2^{(2)}(\alpha h) + \frac{b_2^{(2)}(\alpha h)}{\gamma_1^{(2)}}\right) + e^{-\frac{\alpha h}{\gamma_2^{(2)}}}\left(a_2^{(2)}(\alpha h) + \frac{b_2^{(2)}(\alpha h)}{\gamma_2^{(2)}}\right)}{a_1^{(2)}(\alpha h)b_2^{(2)}(\alpha h) - a_2^{(2)}(\alpha h)b_1^{(2)}(\alpha h)}\right]\frac{(1+\lambda_2^{(2)})}{\gamma_2^{(2)}}D_2(\alpha), \tag{5.35}$$

$$C_2(\alpha) = \left[\frac{e^{-\frac{\alpha h}{\gamma_1^{(2)}}}\left(\frac{b_1^{(2)}(\alpha h)}{\gamma_1^{(2)}} + a_1^{(2)}(\alpha h)\right) - e^{-\frac{\alpha h}{\gamma_2^{(2)}}}\left(\frac{b_1^{(2)}(\alpha h)}{\gamma_2^{(2)}} + a_1^{(2)}(\alpha h)\right)}{a_1^{(2)}(\alpha h)b_2^{(2)}(\alpha h) - a_2^{(2)}(\alpha h)b_1^{(2)}(\alpha h)}\right]\frac{(1+\lambda_2^{(2)})}{\gamma_2^{(2)}}D_2(\alpha), \tag{5.36}$$

Setting,

$$B_2(\alpha) = \frac{1}{\alpha^2}\int_{-h}^{-a} f_1(t^2)\sin\ \alpha t\ dt, \tag{5.37}$$

$$D_2(\alpha) = \frac{1}{\alpha^2}\int_{a}^{h} f_2(t^2)\sin\ \alpha t\ dt, \tag{5.38}$$

the equations (5.27) and (5.28) are satisfied if

$$\int_{-h}^{-a} f_1(t^2)\ dt = 0 \quad \text{and} \quad \int_{a}^{h} f_2(t^2)\ dt = 0. \tag{5.39}$$

The boundary conditions (5.1) and (5.6), with the equations (5.37) and (5.38) yield the following singular integral equations

$$\int_{-h}^{-a} g_1(t^2)\left(\frac{2t}{t^2 - x^2}\right)\ dt - \int_{-h}^{-a} g_1(t^2)\ k_1(x,t)\ dt = \frac{\pi}{2\ C_{66}}p(x), \quad -h \le x \le -a, \tag{5.40}$$

$$\int_{a}^{h} g_2(t^2)\left(\frac{2t}{t^2 - x^2}\right)\ dt - \int_{a}^{h} g_2(t^2)\ k_2(x,t)\ dt = \frac{\pi}{2\ C_{66}}p(x), a \le x \le h, \tag{5.41}$$

where

$$
\begin{aligned}
g_1(t^2) =& (\gamma_2^{(1)} - \gamma_1^{(1)}) \frac{(1+\lambda_2^{(1)})}{\gamma_2^{(1)}} f_1(t^2), \\
g_2(t^2) =& (\gamma_2^{(2)} - \gamma_1^{(2)}) \frac{(1+\lambda_2^{(2)})}{\gamma_2^{(2)}} f_2(t^2),
\end{aligned} \tag{5.42}
$$

$$
\begin{aligned}
k_1(x,t) = & \frac{1}{(\gamma_2^{(1)} - \gamma_1^{(1)})} \int_0^\infty \\
& \Bigg[\left(\frac{-(\gamma_1^{(1)})^2 a_1^{(1)}(\alpha x) b_2^{(1)}(sh) + (\gamma_2^{(1)})^2 a_2^{(1)}(\alpha x) b_1^{(1)}(\alpha h)}{a_1^{(1)}(\alpha h) b_2^{(1)}(\alpha h) - a_2^{(1)}(\alpha h) b_1^{(1)}(\alpha h)} \right) \left(\frac{e^{-\frac{\alpha h}{\gamma_1^{(1)}}}}{\gamma_1^{(1)}} - \frac{e^{-\frac{\alpha h}{\gamma_2^{(1)}}}}{\gamma_2^{(1)}} \right) \\
& + \left(\frac{-(\gamma_1^{(1)})^2 a_1^{(1)}(\alpha x) a_2^{(1)}(\alpha h) + (\gamma_2^{(1)})^2 a_2^{(1)}(\alpha x) a_1^{(1)}(\alpha h)}{a_1^{(1)}(\alpha h) b_2^{(1)}(\alpha h) - a_2^{(1)}(\alpha h) b_1^{(1)}(\alpha h)} \right) \\
& \left(e^{-\frac{\alpha h}{\gamma_1^{(1)}}} - e^{-\frac{\alpha h}{\gamma_2^{(1)}}} \right) \Bigg] \sin \alpha t \; d\alpha,
\end{aligned} \tag{5.43}
$$

$$
\begin{aligned}
k_2(x,t) = & \frac{1}{(\gamma_2^{(2)} - \gamma_1^{(2)})} \int_0^\infty \\
& \Bigg[\left(\frac{-(\gamma_1^{(2)})^2 a_1^{(2)}(\alpha x) b_2^{(2)}(\alpha h) + (\gamma_2^{(2)})^2 a_2^{(2)}(\alpha x) b_1^{(2)}(\alpha h)}{a_1^{(2)}(\alpha h) b_2^{(2)}(\alpha h) - a_2^{(2)}(\alpha h) b_1^{(2)}(\alpha h)} \right) \left(\frac{e^{-\frac{\alpha h}{(\gamma_1^{(2)})}}}{\gamma_1^{(2)}} - \frac{e^{-\frac{\alpha h}{\gamma_2^{(2)}}}}{\gamma_2^{(2)}} \right) \\
& + \left(\frac{-(\gamma_1^{(2)})^2 a_1^{(2)}(\alpha x) a_2^{(2)}(\alpha h) + (\gamma_2^{(2)})^2 a_2^{(2)}(\alpha x) a_1^{(2)}(\alpha h)}{a_1^{(2)}(\alpha h) b_2^{(2)}(\alpha h) - a_2^{(2)}(\alpha h) b_1^{(2)}(\alpha h)} \right) \\
& \left(e^{-\frac{\alpha h}{\gamma_1^{(2)}}} - e^{-\frac{\alpha h}{\gamma_2^{(2)}}} \right) \Bigg] \sin \alpha t \; d\alpha,
\end{aligned} \tag{5.44}
$$

The singular integral equation (5.40) finally reduces to the following equation for the case of large h as

$$
\int_{-h}^{-a} g_1(t^2) \left(\frac{2t}{t^2 - x^2} \right) \, dt - \beta_1 \int_{-h}^{-a} t \; g_1(t^2) \; dt = \frac{\pi}{C_{66}} p(x), -h \leq x \leq -a, \tag{5.45}
$$

where

$$\beta_1 = \frac{2(\gamma_1^{(1)})^2\beta_{11}^{(1)}}{((\gamma_1^{(1)})^2+1)^2} + \frac{2(\gamma_1^{(1)})^2\beta_{12}^{(1)}}{(\gamma_1^{(1)}\gamma_2^{(1)}+1)^2} + \frac{2(\gamma_2^{(1)})^2\beta_{21}^{(1)}}{(\gamma_1^{(1)}\gamma_2^{(1)}+1)^2} + \frac{2(\gamma_2^{(1)})^2\beta_{22}^{(1)}}{((\gamma_2^{(1)})^2+1)^2},$$

with

$$\beta_{11}^{(1)} = -\frac{(\gamma_1^{(1)}+\gamma_2^{(1)})}{(\gamma_1^{(1)}-\gamma_2^{(1)})^2}\gamma_1^{(1)}, \qquad \beta_{12}^{(1)} = \frac{2\gamma_2^{(1)}}{(\gamma_1^{(1)}-\gamma_2^{(1)})^2},$$

$$\beta_{21}^{(1)} = \frac{2\gamma_1^{(1)}}{(\gamma_1^{(1)}-\gamma_2^{(1)})^2}, \qquad \beta_{22}^{(1)} = -\frac{(\gamma_1^{(1)}+\gamma_2^{(1)})}{(\gamma_1^{(1)}-\gamma_2^{(1)})^2}\gamma_2^{(1)}.$$

The singular integral equation (5.41) finally reduces to the following equation for the case of large h as

$$\int_a^h g_2(t^2)\left(\frac{2t}{t^2-x^2}\right)\ dt - \beta_2\int_a^h t\ \ g_2(t^2)\ dt = \frac{\pi}{C_{66}}p(x), a \leq x \leq h, \quad (5.46)$$

where

$$\beta_2 = \frac{2(\gamma_1^{(2)})^2\beta_{11}^{(2)}}{((\gamma_1^{(2)})^2+1)^2} + \frac{2(\gamma_1^{(2)})^2\beta_{12}^{(2)}}{(\gamma_1^{(2)}\gamma_1^{(2)}+1)^2} + \frac{2(\gamma_1^{(2)})^2\beta_{21}^{(2)}}{(\gamma_1^{(2)}\gamma_1^{(2)}+1)^2} + \frac{2(\gamma_1^{(2)})^2\beta_{22}^{(2)}}{((\gamma_1^{(2)})^2+1)^2},$$

with

$$\beta_{11}^{(2)} = -\frac{(\gamma_1^{(2)}+\gamma_2^{(2)})}{(\gamma_1^{(2)}-\gamma_2^{(2)})^2}\gamma_1^{(2)}, \qquad \beta_{12}^{(2)} = \frac{2(\gamma_2^{(2)})^2}{(\gamma_1^{(2)}-\gamma_2^{(2)})^2},$$

$$\beta_{21}^{(2)} = \frac{2\gamma_1^{(2)}}{(\gamma_1^{(2)}-\gamma_2^{(2)})^2}, \qquad \beta_{22}^{(2)} = -\frac{(\gamma_1^{(2)}+\gamma_2^{(2)})}{(\gamma_1^{(2)}-\gamma_2^{(2)})^2}\gamma_2^{(2)}.$$

After some manipulations and putting $x^2 = X$ and $t^2 = T$ in the above equations (5.45) and (5.46), we get

$$-\int_{a^2}^{h^2} \frac{g_1(T)}{T-X}\,dT + \frac{\beta_1}{2}\int_{a^2}^{h^2} g_1(T)\ dT = \frac{\pi}{C_{66}}p(\sqrt{X}), a^2 \leq X \leq h^2, \qquad (5.47)$$

$$\int_{a^2}^{h^2} \frac{g_2(T)}{T-X}\, dT - \frac{\beta_2}{2}\int_{a^2}^{h^2} g_2(T)\, dT = \frac{\pi}{C_{66}} p(\sqrt{X}), a^2 \le X \le h^2. \tag{5.48}$$

To normalize the above equations (5.47) and (5.48), let us make the following substitutions as

$$T^* = \frac{2T-(a^2+h^2)}{(h^2-a^2)}, X^* = \frac{2X-(a^2+h^2)}{(h^2-a^2)}.$$

Defining,

$$g_1(T) = g_1(T^*),\ g_2(T) = g_2(T^*), \text{ and } p(\sqrt{X}) = p(\sqrt{X^*}),$$

the equations (5.47) and (5.48) reduce to

$$-\int_{-1}^{1} \frac{g_1(T^*)}{(T^*-X^*)}\, dT^* + \frac{\beta_1(h^2-a^2)}{4\,h^2}\int_{-1}^{1} g_1(T^*)\ dT^*$$
$$= \frac{\pi}{C_{66}} p(\sqrt{X^*}), -1 \le X^* \le\ 1, \tag{5.49}$$

$$\int_{-1}^{1} \frac{g_2(T^*)}{(T^*-X^*)}\, dT^* - \frac{\beta_2(h^2-a^2)}{4\,h^2}\int_{-1}^{1} g_2(T^*)\, dT^*$$
$$= \frac{\pi}{C_{66}} p(\sqrt{X^*}), -1 \le X^* \le 1. \tag{5.50}$$

with

$$\int_{-1}^{1} g_1(T^*)\ dT^* = 0 \text{ and } \int_{-1}^{1} g_2(T^*)\ dT^* = 0. \tag{5.51}$$

Now expressing the unknown functions in terms of Chebyshev polynomials of the first kind as

$$g_1(T^*) = \frac{1}{\sqrt{(1-T^{*2})}}\sum_{n=0}^{\infty} A_n T_{2n+1}(T^*), \tag{5.52}$$

$$g_2(T^*) = \frac{1}{\sqrt{(1-T^{*2})}}\sum_{n=0}^{\infty} B_n T_{2n+1}(T^*), \tag{5.53}$$

and using the result

$$\int_{-1}^{1} T_{2j+1}(z^*)(1-z^{*2})^{-\frac{1}{2}}\frac{dz^*}{(z^*-y^*)} = \begin{cases} 0, & j=0, \\ \pi\, U_{2j}(y^*), & j>0, \end{cases} \tag{5.54}$$

and the orthogonality relation

$$\int_{-1}^{1} U_n(y^*)U_m(y^*)(1-y^{*2})^{\frac{1}{2}}dy^* = \begin{cases} 0, & n\neq m, \\ \frac{\pi}{2}, & n=m, \end{cases} \tag{5.55}$$

the equations (5.49) and (5.50) become

$$\begin{aligned} -A_m\Big(\frac{\pi^2}{2}\Big) + \frac{\beta_1(h^2-a^2)}{4\,h^2}\Big(\int_{-1}^{1}\frac{1}{\sqrt{1-T^{*2}}}\sum_{n=0}^{\infty}A_nT_{2n+1}(T^*)dT^*\Big) \\ \Big(\int_{-1}^{1}U_{2n}(X^*)\sqrt{1-X^{*2}}dX^*\Big) = \frac{\pi}{C_{66}}P_m, -1\leq X^*\leq\ 1, \end{aligned} \tag{5.56}$$

$$\begin{aligned} B_m\Big(\frac{\pi^2}{2}\Big) - \frac{\beta_2(h^2-a^2)}{4\,h^2}\Big(\int_{-1}^{1}\frac{1}{\sqrt{1-T^{*2}}}\sum_{0}^{\infty}B_nT_{2n+1}(T^*)dT^*\Big) \\ \Big(\int_{-1}^{1}U_{2n}(X^*)\sqrt{1-X^{*2}}dX^*\Big) = \frac{\pi}{C_{66}}P_m, -1\leq X^*\leq\ 1, \end{aligned} \tag{5.57}$$

where

$$P_m = \int_{-1}^{1} p(\sqrt{X^*})U_{2m}(X^*)\sqrt{1-X^{*2}}dX^*. \tag{5.58}$$

The Stress intensity factors at the edge crack tip $x = -a^+$ and the another edge crack tip $x = a^-$ are calculated as

$$K_{I(-a)} = \lim_{x \to -a^+} \sqrt{2a(x+a)}\sigma_{yy}^{(1)}(x,0) = \sqrt{\frac{h^2-a^2}{2}} C_{66}\left[\sum_{n=0}^{\infty} A_n\right],$$
$$K_{I(a)} = \lim_{x \to a^-} \sqrt{2a(a-x)}\sigma_{yy}^{(2)}(x,0) = \sqrt{\frac{h^2-a^2}{2}} C_{66}\left[\sum_{n=0}^{\infty} B_n\right].$$

5.4 Results and Discussion

In this section, numerical values of SIFs at both the cracks' tips are calculated for various depths of the strips and the sizes of the cracks, which are depicted though Figures 5.2-5.5 for different particular cases. The orthotropic strips are considered as Steel- Mylar and Boron-Epoxy composite materials, whose elastic constants are given in Table 5.1 (Mishra *et al*. (2017) and Das *et al*. (2008)). The Boron-epoxy materials have a wide range of applications in the aerospace, automotive and marine industries. Also its versatile characteristic and its diversity made it useful for different industrial applications. The use of steel-mylar composite material in commercial transport aircraft is attractive because its reduced airframe weight and low operating cost. During computations, the loadings are considered as $p(x) = p$.

Table 5.1: Elastic Constants

Materials	C_{11} 10^{10} Pa	C_{12} 10^{10} Pa	C_{22} 10^{10} Pa	C_{66} 10^{10} Pa
Steel-Mylar (Material 1)	18.70	1.30	2.92	0.62
Boron-Epoxy (Material 2)	208.91	26.06	27.85	7.79

In Figures 5.2 and 5.3, the normalized SIF $K_{I(-a^+)}/p\sqrt{a}$ is calculated at the crack tip $a = -2.0, -1.8, -1.6, -1.4$ varying the value of $h = -3.0(-0.2) - 2.2$.

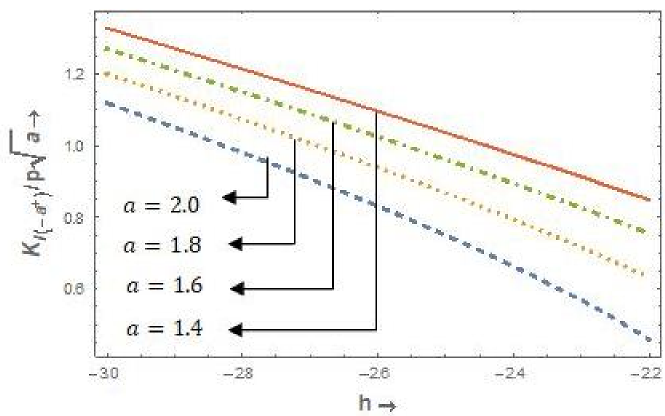

Figure 5.2: Normalized SIF at the point $x = -a^+$ versus h

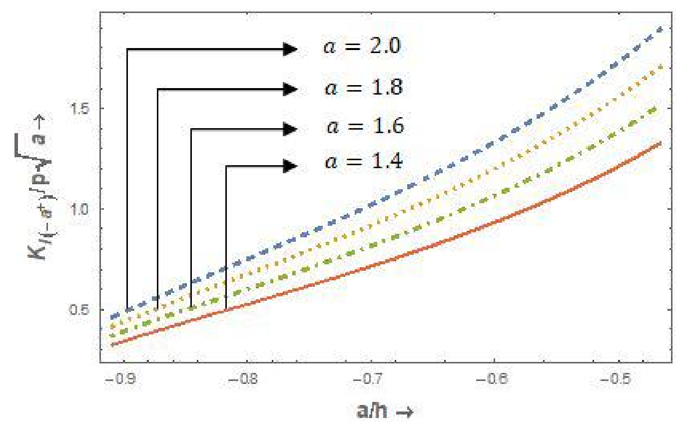

Figure 5.3: Normalized SIF at the point $x = -a^+$ versus a/h

Figure 5.2 depicts $K_{I(-a^+)}/p\sqrt{a}$ versus the width and it is seen that as the width is increasing the SIFs is decreasing. Figure 5.3 shows that the normalized SIF increases as sizes of the crack decreases.

In the Figures 5.4 and 5.5, the normalized SIFs $K_{I(a^-)}/p\sqrt{a}$ are calculated at the crack tip $a = 2.0, 1.8, 1.6, 1.4$ varying the value of $h = 2.2(0.2)3.0$. The Figures depict that the SIF decreases with the increases in h at different crack tip position and also increases as the crack sizes decreases.

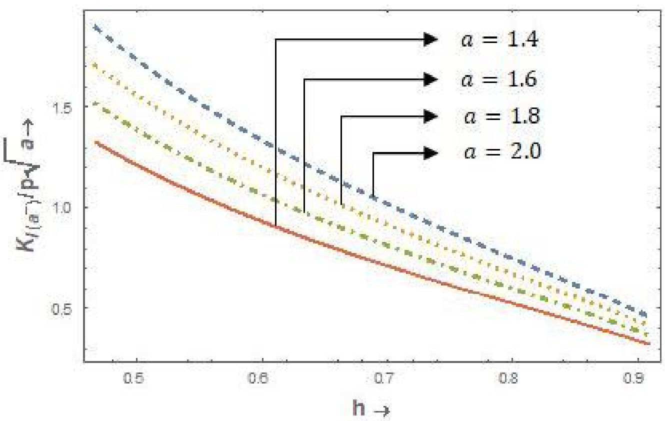

Figure 5.4: Normalized SIF at the point $x = a^-$ versus h

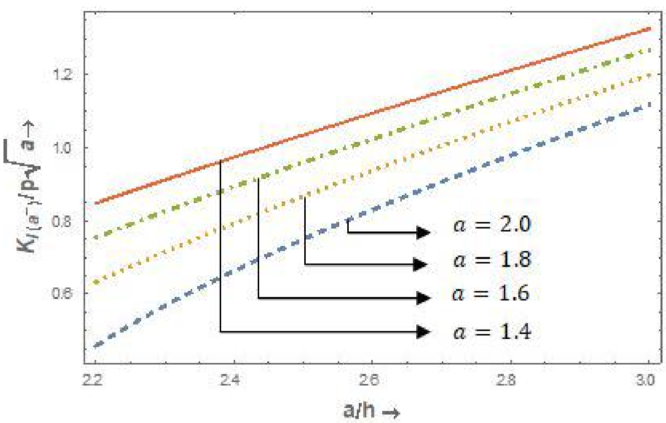

Figure 5.5: Normalized SIF at the point $x = a^-$ versus a/h

5.5 Conclusion

The aim of the chapter is to determine the stress intensity factors at the edge cracks' tips under prescribed tensile loads situated at the boundaries of two bonded orthotropic strips. the main contribution of the chapter is the graphical presentations of the variations of natures of SIFs as and when the sizes of the cracks are increasing or decreasing.

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