Chapter 4

A study of the double edge cracks and a central crack in an orthotropic strip under the tensile loads

4.1 Introduction

Edge crack or any type of crack occurs in railway track, aerospace, aeroplane wings *etc.* and this type of failure occurs due to either the thermal load or any other type of external load or stress. Edge crack problem analysis is much easier to study

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in isotropic materials than the anisotropic materials because mathematical calculations become difficult to solve for the anisotropic material. Nowadays, engineering structures are designed with the aid of the composite materials. There is less probability to grow a crack in edge crack which is occurring in the orthotropic material compared to the isotropic material which is very considerable and useful. Every metal possesses some thermal, mechanical and chemical properties. A material is said to be isotropic if its mechanical and thermal properties are the same in all directions. For anisotropic materials, they are different in each and every direction. The orthotropic material is the special case of the anisotropic material which has three mutually perpendicular planes of symmetry and their characteristics remain unchanged along their axes. Orthotropic materials' properties are the same whether a material is homogeneous or non-homogeneous.

The problems of edge crack are found few in numbers during the literature survey. The problem of symmetrical edge cracks of finite length in an orthotropic infinite strip under normal point loading is solved using Hilbert transform technique by Das *et al*. (2008). The problem of an orthotropic infinite strip with double symmetrically located edge cracks bonded to another orthotropic half plane had been solved by Das *et al*. (2011a). The problem was solved with the aid of Hilbert transform technique and weight function. The problem of single edge crack located in an orthotropic infinite strip with the finite length had been solved by Das (2010). Das *et al*. (2011b) have studied the edge crack problem in an orthotropic composite material. Gupta and Erdogan (1974) have solved the two symmetrically situated internal cracks orthogonal to the boundary by converting the boundary conditions in the form of two simultaneous singular integral equations, which are solved with the aid of numerical technique. Wang *et al*. (1996) have used the integral transform technique to solve the edge crack and internal cracks. The three-dimensional elastic problem in an orthotropic fracture specimen with an edge crack had been studied by Cruse and Vanburen (1971). Know and Lee (2000) have solved a centre crack in the finite length and width in a piezoelectric body under anti-plane shear loading, using the Fourier integral equation. The problem of a finite central crack in an infinite functionally graded piezoelectric strip under in-plane mechanical and electrical loadings had been studied by Ueda (2005). The problem in a unit circular elastic disc with internal edge crack which is applicable and considerable for circular, rotating and infinite long cylinder under thermal shock had been studied by Schneider and Danzer (1989). The problem concerned with the elastostatic axisymmetric for a long hollow cylinder possess a ring-shaped internal and edge cracks, had been solved by Erdol and Erdogan (1978) using the standard transform technique.

In the current work, the study is concerned with the elastostatic double edge cracks problem with a centre crack under tensile loadings in an infinite orthotropic elastic strip of width $2h$. The problem is reduced into the singular integral equations of the first kind with Cauchy-type singularities, which are solved using Chebyshev polynomials. The expressions of the stress intensity factors (SIFs) are found at the cracks' tips. The variations of SIF at the central crack tip keeping its length fixed and varying the edge crack for Steel-Mylar are found. Similarly, SIF at the tip of the edge cracks keeping its length fixed and varying the length of the central crack is found for the same material. The computed results are displayed graphically for different particular cases.

4.2 Problem Formulation

Let us consider an elastostatic problem of an orthotropic strip of length $2h$ with a central crack defined by $| x | \leq b$ and two symmetrically situated edge cracks defined

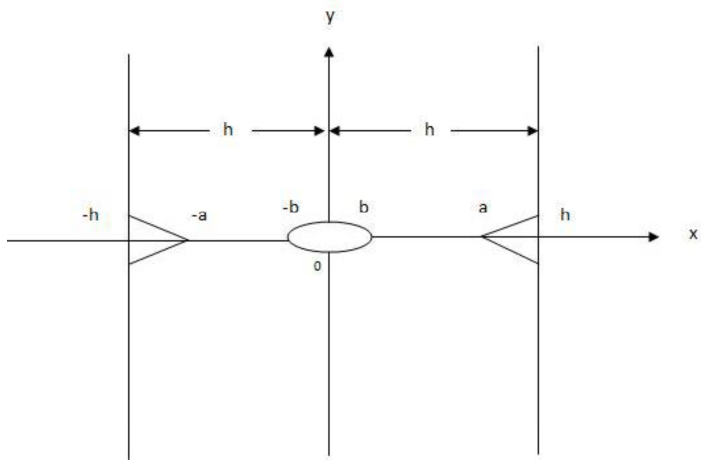

Figure 4.1: Geometry of the Problem

by $a \leq| x |\leq h$ under the normal tractions $p_1(x)$ and $p_2(x)$ respectively (Figure 4.1). The equations of equilibrium in the forms of displacements are expressed as

$$C_{11}\frac{\partial^2 u}{\partial x^2} + C_{66}\frac{\partial^2 u}{\partial y^2} + (C_{12} + C_{66})\frac{\partial^2 v}{\partial x \partial y} = 0, \tag{4.1}$$

$$C_{22}\frac{\partial^2 v}{\partial y^2} + C_{66}\frac{\partial^2 v}{\partial x^2} + (C_{12} + C_{66})\frac{\partial^2 u}{\partial x \partial y} = 0, \tag{4.2}$$

where u and v are displacements in x and y directions and C_{ij}'s are elastic constants of the orthotropic material.

Here the mathematical model is considered under symmetry with respect to y-axis and here it is sufficient to discuss the problem in the half strip $0 \leq x \leq h$.

Thus the concerned boundary conditions are given by

$$\sigma_{yy}(x,0) = p_1(x), \qquad a \le x \le h, \tag{4.3}$$

$$\sigma_{yy}(x,0) = p_2(x), \qquad 0 \le x \le b, \tag{4.4}$$

$$\sigma_{xx}(h,y) = 0, \qquad 0 \le y < \infty, \tag{4.5}$$

$$\sigma_{xy}(h,y) = 0, \qquad 0 \le y < \infty, \tag{4.6}$$

$$\sigma_{xy}(x,0) = 0, \qquad 0 \le x \le h, \tag{4.7}$$

$$v(x,0) = 0, \qquad b \le x \le a. \tag{4.8}$$

All the components of stresses and displacements vanish at the remote distances from the cracks.

4.3 Solution of the Problem

The displacement fields and components of stress are represented in the form of harmonic functions as

$$u = \frac{\partial \phi_1}{\partial x} + \frac{\partial \phi_2}{\partial x}, \tag{4.9}$$

$$v = \lambda_1 \frac{\partial \phi_1}{\partial y} + \lambda_2 \frac{\partial \phi_2}{\partial y}, \tag{4.10}$$

$$\frac{\sigma_{xx}}{C_{66}} = -\Big[(1+\lambda_1)\frac{\partial^2 \phi_1}{\partial y^2} + (1+\lambda_2)\frac{\partial^2 \phi_2}{\partial y^2}\Big], \tag{4.11}$$

$$\frac{\sigma_{yy}}{C_{66}} = (1+\lambda_1)\mu_1 \frac{\partial^2 \phi_1}{\partial y^2} + (1+\lambda_2)\mu_2 \frac{\partial^2 \phi_2}{\partial y^2}, \tag{4.12}$$

$$\frac{\sigma_{xy}}{C_{66}} = (1+\lambda_1)\frac{\partial^2 \phi_1}{\partial x \partial y} + (1+\lambda_2)\frac{\partial^2 \phi_2}{\partial x \partial y}, \tag{4.13}$$

where $\phi_i(x,y)$ satisfy the following partial differential equation as

$$\left(\frac{\partial^2}{\partial x^2}+\mu_i\frac{\partial^2}{\partial y^2}\right)\phi_i(x,y)=0, i=1,2, \tag{4.14}$$

where μ_1 and μ_2 are the positive real roots of the above equation

$$C_{11}C_{66}\mu^2+({C_{12}}^2+2C_{12}C_{66}+C_{11}C_{22})\mu+C_{22}C_{66}=0, \tag{4.15}$$

and

$$\lambda_i=\frac{C_{11}\mu_i-C_{66}}{C_{66}+C_{12}}, i=1,2. \tag{4.16}$$

The harmonic functions for an orthotropic elastic strip are given by

$$\phi_1(x,y)=\frac{2}{\pi}\int_0^\infty A_1(s)[e^{-\sqrt{\mu_1}sx}+e^{\sqrt{\mu_1}sx}]\cos\ sy\ ds \\ +\frac{2}{\pi}\int_0^\infty B_1(s)e^{-\frac{sy}{\sqrt{\mu_1}}}\cos\ sx\ ds, \tag{4.17}$$

$$\phi_2(x,y)=\frac{2}{\pi}\int_0^\infty A_2(s)[e^{-\sqrt{\mu_2}sx}+e^{\sqrt{\mu_2}sx}]\cos\ sy\ ds \\ +\frac{2}{\pi}\int_0^\infty B_2(s)e^{-\frac{sy}{\sqrt{\mu_2}}}\cos\ sx\ ds, \tag{4.18}$$

where $A_i(s)$ and $B_i(s)$ $(i=1,2)$ are undetermined arbitrary functions.

Applying the Boundary condition (4.7), we get

$$B_2(s)=-\frac{\sqrt{\mu_2}}{\sqrt{\mu_1}}\frac{(1+\lambda_1)}{(1+\lambda_2)}B_1(s), \tag{4.19}$$

Applying the Boundary condition (4.8), we get

$$\int_0^\infty B_1(s)\ s\ cos\ sx\ ds=0, b\leq x\leq a. \tag{4.20}$$

The Boundary conditions (4.5) and (4.6) with the aid of equation (4.19) give rise to

$$\int_0^\infty \{A_1(s)(a_1(sh)) + A_2(s)(a_2(sh))\}(s^2)\cos\ sy\ ds = \frac{(1+\lambda_1)}{\sqrt{\mu_1}}\int_0^\infty \Big\{ \frac{1}{\sqrt{\mu_1}}e^{-\frac{sy}{\sqrt{\mu_1}}} - \frac{1}{\sqrt{\mu_2}}e^{-\frac{sy}{\sqrt{\mu_2}}}\Big\} B_1(s)(s^2)\cos\ sh\ ds, 0 \le y < \infty, \quad (4.21)$$

$$\int_0^\infty \{A_1(s)(b_1(sh)) + A_2(s)(b_2(sh))\}(s^2)\sin\ sy\ ds + \frac{(1+\lambda_1)}{\sqrt{\mu_1}}\int_0^\infty \Big\{ e^{-\frac{sy}{\sqrt{\mu_1}}} - e^{-\frac{sy}{\sqrt{\mu_2}}}\Big\} B_1(s)(s^2)\sin\ sh\ ds = 0, 0 \le y < \infty, \quad (4.22)$$

where

$$\begin{aligned}
a_1(sh) =& (1+\lambda_1)[e^{\sqrt{\mu_1}sh} + e^{-\sqrt{\mu_1}sh}],\\
b_1(sh) =& -(1+\lambda_1)\sqrt{\mu_1}[e^{\sqrt{\mu_1}sh} - e^{-\sqrt{\mu_1}sh}],\\
a_2(sh) =& (1+\lambda_2)[e^{\sqrt{\mu_2}sh} + e^{-\sqrt{\mu_2}sh}],\\
b_2(sh) =& -(1+\lambda_2)\sqrt{\mu_2}[e^{\sqrt{\mu_2}sh} - e^{-\sqrt{\mu_2}sh}].
\end{aligned}$$

Now from equations (4.21) and (4.22), $A_1(s)$ and $A_2(s)$ are calculated in terms of $B_1(s)$ as

$$A_1(s) = \left[\frac{e^{-\frac{sh}{\sqrt{\mu_1}}}(a_2(sh) + \frac{b_2(sh)}{\sqrt{\mu_1}}) - e^{-\frac{sh}{\sqrt{\mu_2}}}(a_2(sh) + \frac{b_2(sh)}{\sqrt{\mu_2}})}{a_1(sh)b_2(sh) - a_2(sh)b_1(sh)}\right]\frac{(1+\lambda_1)}{\sqrt{\mu_1}} B_1(s), \quad (4.23)$$

$$A_2(s) = \left[\frac{-e^{-\frac{sh}{\sqrt{\mu_1}}}(\frac{b_1(sh)}{\sqrt{\mu_1}} + a_1(sh)) + e^{-\frac{sh}{\sqrt{\mu_2}}}(\frac{b_1(sh)}{\sqrt{\mu_2}} + a_1(sh))}{a_1(sh)b_2(sh) - a_2(sh)b_1(sh)}\right]\frac{(1+\lambda_1)}{\sqrt{\mu_1}} B_1(s). \quad (4.24)$$

Setting,

$$B_1(s) = \frac{1}{s^2}\int_a^h f_1(t^2)\sin\ st\ dt + \frac{1}{s^2}\int_0^b f_2(t^2)\sin\ st\ dt, \quad (4.25)$$

the equation (4.20) is satisfied if

$$\int_a^h f_1(t^2)\ dt + \int_0^b f_2(t^2)\ dt = 0. \quad (4.26)$$

The boundary conditions (4.3) and (4.4) with the aid of equation (4.25) yield the following singular integral equations as

$$\int_a^h g_1(t^2)\left(\frac{2t}{t^2-x^2}\right)\ dt + \int_0^b g_2(t^2)\left(\frac{2t}{t^2-x^2}\right)\ dt + \int_a^h [k(x,t) - k(x,-t)]\ dt + \int_0^b [k(x,t) - k(x,-t)]\ dt = \frac{\pi}{C_{66}}p_1(x), a \leq x \leq h, \quad (4.27)$$

$$\int_a^h g_1(t^2)\left(\frac{2t}{t^2-x^2}\right)\ dt + \int_0^b g_2(t^2)\left(\frac{2t}{t^2-x^2}\right)\ dt + \int_a^h [k(x,t) - k(x,-t)]\ dt + \int_0^b [k(x,t) - k(x,-t)]\ dt = \frac{\pi}{C_{66}}p_2(x), 0 \leq x \leq b, \quad (4.28)$$

where

$$\begin{aligned} g_1(t^2) =& (\sqrt{\mu_1} - \sqrt{\mu_2})\frac{(1+\lambda_1)}{\sqrt{\mu_1}} f_1(t^2), \\ g_2(t^2) =& (\sqrt{\mu_1} - \sqrt{\mu_2})\frac{(1+\lambda_1)}{\sqrt{\mu_1}} f_2(t^2). \end{aligned} \quad (4.29)$$

$$k(x,t) = \int_0^\infty \left[\frac{-\mu_1 a_1(sx) b_2(sh) + \mu_2 a_2(sx) b_1(sh)}{a_1(sh) b_2(sh) - a_2(sh) b_1(sh)} \frac{1}{(\sqrt{\mu_1} - \sqrt{\mu_2})} \right.$$

$$\left(\frac{e^{-\frac{sh}{\sqrt{\mu_1}}}}{\sqrt{\mu_1}} - \frac{e^{-\frac{sh}{\sqrt{\mu_2}}}}{\sqrt{\mu_2}} \right) + \frac{-\mu_1 a_1(sx) a_2(sh) + \mu_2 a_2(sx) a_1(sh)}{a_1(sh) b_2(sh) - a_2(sh) b_1(sh)} \frac{1}{(\sqrt{\mu_1} - \sqrt{\mu_2})}$$

$$\left. \left(e^{-\frac{sh}{\sqrt{\mu_1}}} - e^{-\frac{sh}{\sqrt{\mu_2}}} \right) \right] \sin st \; ds, \quad (4.30)$$

The singular integral equations (4.27) and (4.28) finally reduce to the following equations for the case of large h as

$$\int_a^h g_1(t^2) \left(\frac{2t}{t^2 - x^2} \right) dt + \int_0^b g_2(t^2) \left(\frac{2t}{t^2 - x^2} \right) dt + \beta \int_a^h t \; g_1(t^2) \; dt$$

$$+ \beta \int_0^b t \; g_2(t^2) \; dt = \frac{\pi}{C_{66}} p_1(x), a \leq x \leq h, \quad (4.31)$$

$$\int_a^h g_1(t^2) \left(\frac{2t}{t^2 - x^2} \right) dt + \int_0^b g_2(t^2) \left(\frac{2t}{t^2 - x^2} \right) dt + \beta \int_a^h t \; g_1(t^2) \; dt$$

$$+ \beta \int_0^b t \; g_2(t^2) \; dt = \frac{\pi}{C_{66}} p_2(x), 0 \leq x \leq b, \quad (4.32)$$

where

$$\beta = \frac{2\mu_1 \beta_{11}}{(\mu_1 + 1)^2} + \frac{2\mu_1 \beta_{12}}{(\sqrt{\mu_1}\sqrt{\mu_2} + 1)^2} + \frac{2\mu_2 \beta_{21}}{(\sqrt{\mu_1}\sqrt{\mu_2} + 1)^2} + \frac{2\mu_2 \beta_{22}}{(\mu_2 + 1)^2},$$

with

$$\beta_{11} = -\frac{(\sqrt{\mu_1} + \sqrt{\mu_2})}{(\sqrt{\mu_1} - \sqrt{\mu_2})^2} \sqrt{\mu_1}, \qquad \beta_{12} = \frac{2\mu_2}{(\sqrt{\mu_1} - \sqrt{\mu_2})^2},$$

$$\beta_{21} = \frac{2\sqrt{\mu_1}}{(\sqrt{\mu_1} - \sqrt{\mu_2})^2}, \qquad \beta_{22} = -\frac{(\sqrt{\mu_1} + \sqrt{\mu_2})}{(\sqrt{\mu_1} - \sqrt{\mu_2})^2} \sqrt{\mu_2}.$$

Putting $x^2 = X$ and $t^2 = T$, the above equations (4.31) and (4.32) become

$$\int_{a^2}^{h^2} \frac{g_1(T)}{T-X}\,dT + \int_0^{b^2} \frac{g_2(T)}{T-X}\,dT + \frac{\beta}{2}\int_{a^2}^{h^2} g_1(T)\,dT + \frac{\beta}{2}\int_0^{b^2} g_2(T)\,dT = \frac{\pi}{C_{66}} p_1(\sqrt{X}),\; a^2 \le X \le h^2, \quad (4.33)$$

$$\int_{a^2}^{h^2} \frac{g_1(T)}{T-X}\,dT + \int_0^{b^2} \frac{g_2(T)}{T-X}\,dT + \frac{\beta}{2}\int_{a^2}^{h^2} g_1(T)\,dT + \frac{\beta}{2}\int_0^{b^2} g_2(T)\,dT = \frac{\pi}{C_{66}} p_2(\sqrt{X}),\; 0 \le X \le b^2. \quad (4.34)$$

To normalize the above equations (4.33) and (4.34), let us make the following substitutions as

$$T^* = \frac{2T-(a^2+h^2)}{(h^2-a^2)}, X^* = \frac{2X-(a^2+h^2)}{(h^2-a^2)}, T^{**} = \frac{2T-b^2}{b^2}, X^{**} = \frac{2X-b^2}{b^2}.$$

Defining,

$$g_1(T) = g_1(T^*),\; g_2(T) = g_2(T^{**}),\; p_1(\sqrt{X}) = p_1(\sqrt{X^*}),\; p_2(\sqrt{X}) = p_2(\sqrt{X^{**}}),$$

the equations (4.33) and (4.34) reduce to

$$\int_{-1}^{1} \frac{g_1(T^*)}{(T^*-X^*)}\,dT^* + \int_{-1}^{1} \frac{g_2(T^{**})}{(T^{**}-X^{**})}\,dT^{**} + \frac{\beta(h^2-a^2)}{4}\int_{-1}^{1} g_1(T^*)\,dT^* + \left(\frac{\beta\, b^2}{4}\right)\int_{-1}^{1} g_2(T^{**})\,dT^{**} = \frac{\pi}{C_{66}} p_1(\sqrt{X^*}), -1 \le X^* \le 1, \quad (4.35)$$

$$\int_{-1}^{1} \frac{g_1(T^*)}{(T^* - X^*)}\, dT^* + \int_{-1}^{1} \frac{g_2(T^{**})}{(T^{**} - X^{**})}\, dT^{**} + \frac{\beta(h^2 - a^2)}{4} \int_{-1}^{1} g_1(T^*)\, dT^*$$
$$+ (\frac{\beta\, b^2}{4}) \int_{-1}^{1} g_2(T^{**})\, dT^{**} = \frac{\pi}{C_{66}} p_2(\sqrt{X^{**}}), -1 \leq X^{**} \leq 1 \quad (4.36)$$

with

$$\int_{-1}^{1} g_1(T^*)\ dT^* + \int_{-1}^{1} g_2(T^{**})\ dT^{**} = 0. \quad (4.37)$$

Now expressing the unknown functions in terms of Chebyshev polynomials of the first kind as

$$g_1(T^*) = \frac{1}{\sqrt{(1 - T^{*2})}} \sum_{n=0}^{\infty} A_n T_{2n+1}(T^*), \quad (4.38)$$
$$g_2(T^{**}) = \frac{1}{\sqrt{(1 - T^{**2})}} \sum_{n=0}^{\infty} B_n T_{2n+1}(T^{**}), \quad (4.39)$$

using the result

$$\int_{-1}^{1} T_{2j+1}(z^*)(1 - z^{*2})^{-\frac{1}{2}} \frac{dz^*}{(z^* - y^*)} = \begin{cases} 0, & j = 0, \\ \pi\, U_{2j}(y^*), & j > 0, \end{cases} \quad (4.40)$$

and the orthogonality relation

$$\int_{-1}^{1} U_n(y^*) U_m(y^*)(1 - y^{*2})^{\frac{1}{2}} dy^* = \begin{cases} 0, & n \neq m, \\ \frac{\pi}{2}, & n = m, \end{cases} \quad (4.41)$$

the equations (4.35) and (4.36) become

$$A_m(\frac{\pi^2}{2}) + \pi\sum_{n=0}^{\infty} B_n \int_{-1}^{1} U_{2n}(X^{**})U_{2m}(X^{*})\sqrt{1-X^{*2}}dX^{*} + \frac{\beta(h^2-a^2)}{4}$$
$$\left(\int_{-1}^{1}\frac{1}{\sqrt{1-T^{*2}}}\sum_{n=0}^{\infty} A_n T_{2n+1}(T^{*})dT^{*}\right)\left(\int_{-1}^{1} U_{2n}(X^{*})\sqrt{1-X^{*2}}dX^{*}\right)$$
$$+\frac{\beta\, b^2}{4}\left(\int_{-1}^{1}\frac{1}{\sqrt{1-T^{**2}}}\sum_{n=0}^{\infty} B_n T_{2n+1}(T^{**})dT^{**}\right)$$
$$\left(\int_{-1}^{1} U_{2n}(X^{*})\sqrt{1-X^{*2}}dX^{*}\right) = \frac{\pi}{C_{66}}P_{1m}, \quad (4.42)$$

$$\pi\sum_{n=0}^{\infty} A_n \int_{-1}^{1} U_{2n}(X^{*})U_{2m}(X^{**})\sqrt{1-X^{**2}}dX^{**} + B_m(\frac{\pi^2}{2}) + \frac{\beta(h^2-a^2)}{4}$$
$$\left(\int_{-1}^{1}\frac{1}{\sqrt{1-T^{*2}}}\sum_{n=0}^{\infty} A_n T_{2n+1}(T^{*})dT^{*}\right)\left(\int_{-1}^{1} U_{2n}(X^{**})\sqrt{1-X^{**2}}dX^{**}\right)$$
$$+\frac{\beta\, b^2}{4}\left(\int_{-1}^{1}\frac{1}{\sqrt{1-T^{**2}}}\sum_{n=0}^{\infty} B_n T_{2n+1}(T^{**})dT^{**}\right)$$
$$\left(\int_{-1}^{1} U_{2n}(X^{**})\sqrt{1-X^{**2}}dX^{**}\right) = \frac{\pi}{C_{66}}P_{2m}, \quad (4.43)$$

where

$$P_{1m} = \int_{-1}^{1} p_1(\sqrt{X^{*}})U_{2m}\sqrt{1-X^{*2}}dX^{*}, \quad (4.44)$$
$$P_{2m} = \int_{-1}^{1} p_2(\sqrt{X^{**}})U_{2m}\sqrt{1-X^{**2}}dX^{**}. \quad (4.45)$$

The Stress intensity factors at the edge crack tip $x = a$ and the central crack tip $x = b$ are calculated as

$$K_{Ia} = \lim_{x \to a^-} \sqrt{(a^2 - x^2)}\sigma_{yy}(x, 0) = \sqrt{\frac{h^2 - a^2}{2}}\, p\, C_{66} \left[\sum_{n=0}^{\infty} A_n\right],$$

$$K_{Ib} = \lim_{x \to b^+} \sqrt{(x^2 - b^2)}\sigma_{yy}(x, 0) = -\frac{b}{\sqrt{2}}\, p\, C_{66} \left[\sum_{n=0}^{\infty} B_n\right].$$

4.4 Results and Discussion

In this section, orthotropic material is considered as Steel-Mylar composite material whose elastic constants are given in Table 4.1 (Das *et al*. (2008).

Table 4.1: Elastic Constants

C_{11}	C_{12}	C_{22}	C_{66}
10^{10} Pa	10^{10} Pa	10^{10} Pa	10^{10} Pa
18.70	1.30	2.92	0.62

The computed normalized SIFs $K_{Ia}/p\sqrt{a}$ and $K_{Ib}/p\sqrt{b}$ at the cracks' tips $x = a$ and $x = b$ are found for the above considered material are displayed through Figure 4.2 and Figure 4.3 respectively as and when $p_1(\sqrt{X^*}) = p_2(\sqrt{X^{**}}) = p$ and $h = 4.0$ unit. In Figure 4.2, it is seen that the $K_{Ia}/p\sqrt{a}$ increases when the edge crack position is kept fixed at $a = 3.0$ unit and varying the central crack length as $b = 2.50(0.05)2.95$. That is as the central crack approaches to the edge crack, the normalized SIF at the crack tip $x = a$ increases.

Again if the central crack length is kept fixed at $b = 2.8$ and varying the edge crack location 3.8(0.1)3.1, it is seen that the SIF K_{Ib} increases (Figure 4.3). In this case also the normalized SIF at $x = b$ K_{Ib} increases when the edge crack location will be approaching close to the central crack tip.

Thus, in both cases there are possibilities of cracks' propagations.

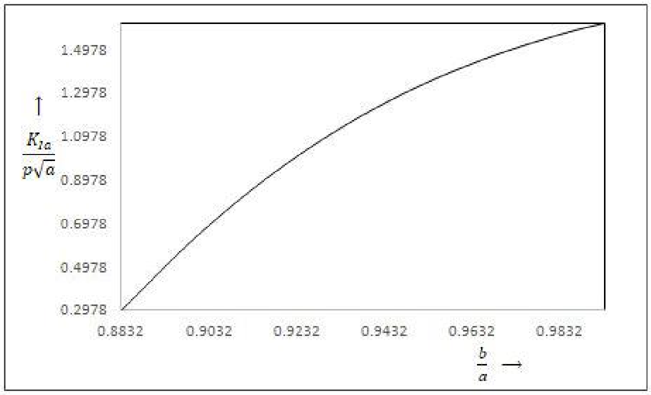

Figure 4.2: Normalized SIF at the point $a-$ versus b/a

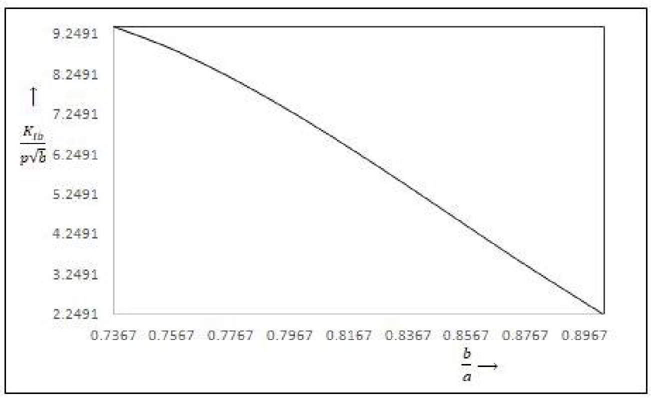

Figure 4.3: Normalized SIF at the point $b+$ versus b/a

4.5 Conclusion

In the present chapter, an endeavour has been made to find the SIFs at the cracks' tips when the orthotropic material consists of one central crack and two edge cracks. The important feature of the chapter is the graphical presentations of the increasing tendency of SIFs when the position of one crack approaches to another one.

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