PREFACE

The present thesis, embodies the results of researches carried out by me at the Department of Mathematical sciences, Indian Institute of Technology(BHU), Varanasi, during the period july 2013 to November 2018 under the supervisor of Prof. L. P. Singh. In the present work some non-linear wave propagation phenomenon have been studied. The whole thesis is divided into six chapters as fallows

Chapter one is introductory and gives a general idea of when and how a discontinuity appears. Certain terminologies commonly used in the current work have been defined. The mathematical theory and their fundamental properties have also been briefly discussed. The physical properties of non-ideal gases and dusty gas are briefly reviewed.

The second chapter concerns with the study of the Riemann problem for a quasi-linear hyperbolic system of partial differential equations governing the one dimensional isentropic dusty gas flow. The shock waves and their properties for isentropic dusty gas flow are discussed briefly. Rarefaction waves and their properties for the considered problem are investigated. Riemann invariant is defined and its properties are discussed. We also examine that how some of the properties of shock and rarefaction waves in a dusty gas flow differ from isentropic ideal gas flow. The procedure of obtaining the solution of Riemann problem for isentropic dusty gas flow is discussed in detail and verifies the validity of the solution for different initial data. The solution obtained here is consists of, at most three constant states which are separated by either a shock or a rarefaction wave. Under certain conditions, the uniqueness and existence of the solution of the Riemann problem has been proved. Finally, all possible interactions of elementary waves are discussed. The discussion of interaction of elementary waves is divided into two subsections. In first section, interactions of elementary waves belonging to different families are discussed and in second section, interactions with same family are discussed.

In the third chapter, the nature of a plane piston moving with constant velocity in a gas with dust particles in the presence of weak gravitational field is analyzed by using a combination of two methods, viz., perturbation method and similarity transformation. In perturbation method the physical variables are expanded as a series of small parameter. The solution of the problem under consideration will be determined under the assumption that shock wave is generated by impulsive motion of piston moving with constant velocity and flow variables ahead of the shock is taken to be uniform. The zeroth order result represents the uniform flow which is affected by dust particles of the mixture without gravity. The first order result shows the consequence of applied gravity in a dusty gas. The effect of dust particles present in the gas and applied gravity on the distribution of flow variables has been discussed for the case of weak and strong shock wave. The structure of the shock wave front is also discussed.

In the fourth chapter, an analytical approach is used to derive the exact solution of Euler equations governing the problem of propagation of blast wave in a one dimensional adiabatic flow of non- ideal dusty gas with generalized geometries. Here it is assumed that the density ahead of the shock front varies according to a power of the distance from the source of the explosion. The effect of dust particles and the parameter of non-idealness on the radius of blast wave are analyzed. An analytical expression for the total energy carried by blast wave in non-ideal dusty gas is derived. The effect of spherically small solid particles present in dusty gas on the total energy carried by blast wave is also discussed.

In the fifth chapter, the exact solution of quasilinear hyperbolic system of equations governing the propagation of weak shock waves in a one dimensional non-ideal adiabatic gas flow with generalized geometries is derived. Here the density ahead of the shock front is assumed to vary according to a power law of the distance. The effect of van der Waals parameter on the radius of weak shock wave is analyzed. An analytical expression for the total energy carried by weak shock wave in non-ideal gas is also derived.

In the sixth chapter, the exact solution of quasilinear hyperbolic system of equations governing the propagation of weak shock waves in a one dimensional dusty adiabatic gas flow with generalized geometries is derived. Here the density behind the shock front is assumed to vary according to a power law of the distance. An analytical expression for the total energy carried by weak shock wave in dusty gas is also derived.