Appendix A

List of Publications

A.1 Journal Papers

- **M. M. Reza**, and R. K. Srivastava, "Semi-Analytical Model for Skewed Magnet Axial Flux Machine," Progress In Electromagnetics Research M, Vol. 68, 109-117, 2018.

- **M. M. Reza** and R. K. Srivastava, "Cogging Reduction in Permanent Magnet Machines via Skewed Slot Opening and its Analytical Modeling," Progress In Electromagnetics Research M, Vol. 70, 167-176, 2018.

- P. Kumar, **M. M. Reza**, and R. K. Srivastava, "Analytical Method for Calculation of Cogging Torque Reduction Due to Slot Shifting in a Dual Stator Dual Rotor Permanent Magnet Machine with Semi-Closed Slots," Progress In Electromagnetics Research M, Vol. 70, 99-108, 2018.

- Chauhan, Avneet K., Venkata R. Vakacharla, **M. M. Reza**, M. Raghuram, and Santosh Kumar Singh, "Modied Boost Derived Hybrid Converter: Redemption Using FCM." IEEE Transactions on Industry Applications 53, no. 6 (2017): 5893-5904.

A.2 Conference Papers

- **M. M. Reza** and R. K. Srivastava, "Semi-Analytical Model for Skewed Magnet Axial Flux Machine," IEEE IAS Annual Meeting 2018.

- **M.M.Reza**, Anish Ahmad, Praveen Kumar, R.K. Srivastava, "Semi-Analytical Model for Triangular Skewed Permanent Magnet Axial ux Machine" In International Transportation Electrification Conference India (ITEC INDIA), 2017 IEEE International Conference on, pp.1-5. IEEE, 2017.

- Praveen Kumar, **M.M.Reza**,R.K. Srivastava, "Eect of Cogging Torque Minimization Techniques on Performance of an Axial Flux Permanent Magnet Machine" in International Transportation Electrication Conference India (ITEC INDIA), 2017 IEEE International Conference on, pp. 1-5. IEEE, 2017.

- **Reza, M. M.**, Avneet K. Chauhan, S. N. Mahendra, and R. K. Srivastava, "No-load magnetic field analysis of double-sided linear tubular permanent magnet synchronous machine." In Power Electronics, Drives and Energy Systems (PEDES), 2016 IEEE International Conference on, pp. 1-5. IEEE, 2016.

- Vakacharla, Venkata R., Avneet K. Chauhan, **M. M. Reza**, and Santosh K. Singh, "Boost derived hybrid converter: Problem analysis and solution." In Power Electronics, Drives and Energy Systems (PEDES), 2016 IEEE International Conference on, pp. 1-5. IEEE, 2016.

- **Reza, M. M.**, Avneet K. Chauhan, S. N. Mahendra, and R. K. Srivastava, "No load magnetic field prediction of double-sided linear permanent magnet machines" In Power Electronics, Drives and Energy Systems (PEDES), 2016 IEEE International Conference on, pp. 1-5. IEEE, 2016.

- Avneet K. Chauhan, **M. M. Reza**, M. Raghuram, and Santosh K. Singh. "High gain buck-boost matrix converter." In Power Electronics, Drives and Energy Systems (PEDES), 2016 IEEE International Conference on, pp. 1-6. IEEE, 2016.

- Praveen Kumar, **Md. Motiur Reza**, R. K. Srivastava, "Performance analysis and comparison of dual-rotor Hybrid Permanent Magnet Induction Machine topologies for electric vehicle application" In Power Electronics, Drives and Energy Systems (PEDES), 2016 IEEE International Conference on, pp. 1-5. IEEE, 2016.

- Praveen Kumar, **Md. Motiur Reza**, R. K. Srivastava, "Effect of relative phase-shift of magnetic eld axes on armature reaction and performance of Hybrid Perma-

nent Magnet Induction Machine" In Power Electronics, Drives and Energy Systems (PEDES), 2016 IEEE International Conference on, pp. 1-6. IEEE, 2016.

Appendix B

Coefficients Calculation used in Chapter 2

The integrationsB.1, and B.6 are defined, as it is required for coefficients calculation

$$\begin{array}{ll} r_1\left(n,i\right) = \int\limits_{\alpha_i}^{\alpha_i+\alpha} \cos n\theta d\theta & \mathrm{s}_1\left(n,i\right) = \int\limits_{\alpha_i}^{\alpha_i+\alpha} \sin n\theta d\theta \\ r_2\left(n,i\right) = \int\limits_{\beta_i}^{\beta_i+\beta} \cos n\theta d\theta & \mathrm{s}_2\left(n,i\right) = \int\limits_{\beta_i}^{\beta_i+\beta} \sin n\theta d\theta \end{array} \tag{B.1}$$

The above equations are further simplification as follow

$$r_1\left(n,i\right) = \left(\sin\left(n(\alpha+\alpha_i)\right) - \sin\left(n\alpha_i\right)\right)/n \tag{B.2}$$

$$s_1\left(n,i\right) = \left(-\cos\left(n(\alpha+\alpha_i)\right) + \cos\left(n\alpha_i\right)\right)/n \tag{B.3}$$

$$r_2\left(n,i\right) = \left(\sin\left(n(\beta+\beta_i)\right) - \sin\left(n\beta_i\right)\right)/n \tag{B.4}$$

$$s_2\left(n,i\right) = \left(-\cos\left(n(\beta+\beta_i)\right) + \cos\left(n\beta_i\right)\right)/n \tag{B.5}$$

The other sets of integrations are:

$$\begin{array}{ll} f_1\left(k,n,i\right) = \int\limits_{\alpha_i}^{\alpha_i+\alpha} \cos\frac{k\pi}{\beta}\left(\theta-\theta_{i.}\right) cosn\theta d\theta & g_1\left(k,n,i\right) = \int\limits_{\alpha_i}^{\alpha_i+\alpha} \cos\frac{k\pi}{\beta}\left(\theta-\theta_{.}\right)\sin n\theta d\theta \\ f_2\left(m,n,i\right) = \int\limits_{\beta_i}^{\beta_i+\beta} \cos\frac{m\pi}{\alpha}\left(\theta-\alpha_{i.}\right)\cos n\theta d\theta & g_2\left(m,n,i\right) = \int\limits_{\beta_i}^{\beta_i+\beta} \cos\frac{m\pi}{\alpha}\left(\theta-\alpha_{i.}\right)\sin n\theta d\theta \end{array} \tag{B.6}$$

the integrations B.6 are evaluated and given in equations B.7-B.10

$$f_1\left(k,n,i\right) = \begin{cases} \frac{-n\alpha^2\left((-1)^k \sin n(\alpha+\alpha_i) - \sin(n\alpha_i)\right)}{(k\pi)^2-(n\alpha)^2} & \text{for } \mathrm{k}\pi \neq \mathrm{n}\alpha \\ \frac{\alpha}{2}\left(\cos\left(n\alpha_i\right) + \frac{(\sin n(\alpha_i+2\alpha)-\sin(n\alpha_i))}{2k\pi}\right) & \text{for } \mathrm{k}\pi = \mathrm{n}\alpha \end{cases} \tag{B.7}$$

$$g_1\left(k,n,i\right) = \begin{cases} \frac{n\alpha^2\left((-1)^k \cos n(\alpha+\alpha_i) - \cos(n\alpha_i)\right)}{(k\pi)^2-(n\alpha)^2} & \text{for } \mathrm{k}\pi \neq \mathrm{n}\alpha \\ \frac{\alpha}{2}\left(\sin\left(n\alpha_i\right) - \frac{(\cos n(\alpha_i+2\alpha)-cos(n\alpha_i))}{2k\pi}\right) & \text{for } \mathrm{k}\pi = \mathrm{n}\alpha \end{cases} \tag{B.8}$$

$$f_2(m,n,i) = \begin{cases} \frac{-n\beta^2((-1)^m \sin n(\beta+\beta_i)-\sin(n\beta_i))}{(m\pi)^2-(n\beta)^2} & \text{for } m\pi \neq n\beta \\ \frac{\beta}{2}\left(\cos(n\beta_i) + \frac{(\sin n(\beta_i+2\beta)-\sin(n\beta_i))}{2m\pi}\right) & \text{for } m\pi = n\beta \end{cases} \tag{B.9}$$

and,

$$g_2(m,n,i) = \begin{cases} \frac{n\beta^2((-1)^m \cos n(\beta+\beta_i)-\cos(n\beta_i))}{(m\pi)^2-(n\beta)^2} & \text{for } m\pi \neq n\beta \\ \frac{\beta}{2}\left(\sin(n\beta_i) - \frac{(cosn(\beta_i+2\beta)-\cos(n\beta_i))}{2m\pi}\right) & \text{for } m\pi = n\beta \end{cases} \tag{B.10}$$

Now, let us define following quantities

$$\begin{array}{lll} Q_1(n) = \frac{R_2}{n}\frac{X_n(R_2,R_3)}{Y_n(R_2,R_3)}; & Q_2(n) = \frac{R_3}{n}\frac{2}{Y_n(R_3,R_2)}; & Q_3(n) = \frac{n}{R_2}\frac{Y_n(R_2,R_1)}{Y_n(R_2,R_1)} \\ Q_4(k,n,i) = \frac{f_1(k,n,i)}{\pi}; & Q_5(m,n,i) = \frac{f_2(m,n,i)}{\mu_j\pi}; & Q_6(k,n,i) = \frac{g_1(k,n,i)}{\pi} \\ Q_7(m,n,i) = \frac{g_2(m,n,i)}{\mu_j\pi}; & Q_8(i,n) = \frac{2R_2}{n\alpha}\frac{r_1(n,i)}{Y_n(R_2,R_3)}; & Q_9(i,n) = \frac{2R_3}{n\alpha}\frac{r_1(n,i)X_n(R_3,R_2)}{Y_n(R_3,R_2)} \end{array} \tag{B.11}$$

and,

$$\begin{array}{ll} Q_{10}(i,n) = \frac{2R_3}{n\alpha}\frac{s_1(n,i)}{Y_n(R_2,R_3)}; & Q_{11}(i,n) = \frac{2R_3}{n\alpha}\frac{s_1(n,i)X_n(R_3,R_2)}{Y_n(R_3,R_2)} \\ Q_{12}(i,k,n) = \frac{2R_2}{n\alpha}\frac{2f_1(k,n,i)}{x_1Y_n(R_2,R_3)}; & Q_{13}(i,k,n) = \frac{2R_3}{n\alpha}\frac{2X_n(R_3,R_2)f_1(k,n,i)}{X_1Y_n(R_3,R_2)} \\ Q_{14}(i,k,n) = \frac{2R_2}{n\alpha}\frac{2g_1(k,n,i)}{x_1Y_n(R_2,R_3)}; & Q_{15}(i,k,n) = \frac{2R_3}{n\alpha}\frac{X_n(R_3,R_2)g_1(k,n,i)}{x_1Y_n(R_3,R_2)} \end{array} \tag{B.12}$$

where, x_1, and x_2 are,

$$x_1 = \frac{\beta R_3}{k\pi}\frac{X_{\frac{k\pi}{\beta}}(R_3,R_4)}{Y_{\backslash\frac{k\pi}{\beta}}(R_3,R_4)}; \qquad x_2 = \frac{\alpha R_3}{m\pi}\frac{X_{\frac{m\pi}{\alpha}}(R_3,R_4)}{Y_{\frac{m\pi}{\alpha}}(R_3,R_4)} \tag{B.13}$$

Now equations (1.23)-(1.34) are rewritten as

$$a_{1n} = Q_1(n)\,a_{2n} + Q_2(n)\,b_{2n} \tag{B.14}$$

$$c_{1n} = Q_1(n)\,c_{2n} + Q_2(n)\,d_{2n} \tag{B.15}$$

$$Q_3(n)\,a_{1n} + F_n'(R_2)\,cos(n\delta) = \mu_r a_{2n} \tag{B.16}$$

$$Q_3(n)\,c_{1n} + F_n'(R_2)\,sin(n\delta) = \mu_r c_{2n} \tag{B.17}$$

$$b_{2n} = \sum_{i=1}^{Q}\left\{Q_4(k,n,i)\,a_{3k}^i + Q_5(m,n,i)\,a_{4m}^i\right\} \tag{B.18}$$

$$d_{2n} = \sum_{i=1}^{Q}\left\{Q_6(k,n,i)\,a_{3k}^i + Q_7(m,n,i)\,a_{4m}^i\right\} \tag{B.19}$$

$$a_0^i = \sum_{n=1,2..}\{Q_8(i,n)\,a_{2n} + Q_9(i,n)\,b_{2n} + Q_{10}(i,n)\,c_{2n} + Q_{11}(i,n)\,d_{2n}\} \tag{B.20}$$

$$a_{3k}^i = \sum_{n=1,2..}\{Q_{12}(i,k,n)\,a_{2n} + Q_{13}(i,k,n)\,b_{2n} + Q_{14}(i,k,n)\,c_{2n} + Q_{15}(i,k,n)\,d_{2n}\} \tag{B.21}$$

$$a^i_{40} = \sum_{n=1,2..} \left\{ Q_{16}\left(i,n\right) a_{2n} + Q_{17}\left(i,n\right) b_{2n} + Q_{18}\left(i,n\right) c_{2n} + Q_{19}\left(i,n\right) d_{2n} \right\} \tag{B.22}$$

and,

$$a^i_{4m} = \sum_{n=1,2..} \left\{ Q_{20}\left(i,m,n\right) a_{2n} + Q_{21}\left(i,m,n\right) b_{2n} + Q_{22}\left(i,m,n\right) c_{2n} + Q_{23}\left(i,m,n\right) d_{2n} \right\} \tag{B.23}$$

Now, the coefficients can be determined using linear simultaneous equations B.14-B.23.

Appendix C

Coefficients Calculation used in Chapter 3

The integrations C.1, and C.3 are defined, as it is required for coefficients calculation

$$\begin{array}{ll} r_1(n,i) = \int\limits_{\theta_i}^{\theta_i+\theta_s} \cos n\theta d\theta & \mathrm{s}_1(n,i) = \int\limits_{\theta_i}^{\theta_i+\theta_s} \sin n\theta d\theta \\ r_2(n,i) = \int\limits_{\beta_i}^{\beta_i+\theta_t} \cos n\theta d\theta & \mathrm{s}_2(n,i) = \int\limits_{\beta_i}^{\beta_i+\theta_t} \sin n\theta d\theta \end{array} \tag{C.1}$$

The above equations are further simplification as follow

$$\begin{aligned} r_1(n,i) &= (\sin(n(\theta_i+\theta_s)) - \sin(n\theta_i))/n \\ s_1(n,i) &= (-\cos(n(\theta_i+\theta_s)) + \cos(n\theta_i))/n \\ r_2(n,i) &= (\sin(n(\theta_t+\beta_i)) - \sin(n\beta_i))/n \\ s_2(n,i) &= (-\cos(n(\theta_t+\beta_i)) + \cos(n\beta_i))/n \end{aligned} \tag{C.2}$$

The other sets of integrations are:

$$\begin{array}{ll} f_1(m,n,i) = \int\limits_{\theta_i}^{\theta_i+\theta_s} \cos\frac{m\pi}{\theta_s}(\theta-\theta_{i.})\cos n\theta d\theta & g_1(m,n,i) = \int\limits_{\theta_i}^{\theta_i+\theta_s} \cos\frac{m\pi}{\theta_s}(\theta-\theta_{i.})\sin n\theta d\theta \\ f_2(k,n,i) = \int\limits_{\beta_i}^{\beta_i+\theta_t} cos\frac{k\pi}{\theta_t}(\theta-\beta_{i.})\cos n\theta d\theta & g_2(k,n,i) = \int\limits_{\beta_i}^{\beta_i+\theta_t} cos\frac{k\pi}{\theta_t}(\theta-\beta_{i.})\sin n\theta d\theta \end{array} \tag{C.3}$$

the integrations C.3 are evaluated and given in equations C.4-C.7

$$f_1(m,n,i) = \begin{cases} \frac{-n\theta_s^2((-1)^m \sin n(\theta_s+\theta_i) - \sin(n\theta_i))}{(m\pi)^2-(n\theta_s)^2} & \text{for m}\pi \neq \text{n}\theta_s \\ \frac{\theta_s}{2}\left(\cos(n\theta_i) + \frac{(\sin n(\theta_i+2\theta_s) - \sin(n\theta_i))}{2m\pi}\right) & \text{for m}\pi = \text{n}\theta_s \end{cases} \tag{C.4}$$

$$g_1(m,n,i) = \begin{cases} \frac{n\theta_s^2((-1)^m \cos n(\theta_s+\theta_i) - \cos(n\theta_i))}{(m\pi)^2-(n\theta_s)^2} & \text{for m}\pi \neq \text{n}\theta_s \\ \frac{\theta_s}{2}\left(\sin(n\theta_i) - \frac{(\cos n(\theta_i+2\theta_s) - \cos(n\theta_i))}{2m\pi}\right) & \text{for m}\pi = \text{n}\theta_s \end{cases} \tag{C.5}$$

$$f_2(k,n,i) = \begin{cases} \frac{-n\theta_t^2\left((-1)^k \sin n(\theta_t+\beta_i) - \sin(n\beta_i)\right)}{(k\pi)^2-(n\theta_t)^2} & \text{for } \mathrm{k}\pi \neq \mathrm{n}\theta_t \\ \frac{\theta_t}{2}\left(\cos(n\beta_i) + \frac{(\sin n(\beta_i+2\theta_t)-\sin(n\beta_i))}{2k\pi}\right) & \text{for } \mathrm{k}\pi = \mathrm{n}\theta_t \end{cases} \tag{C.6}$$

and,

$$g_2(k,n,i) = \begin{cases} \frac{n\theta_t^2\left((-1)^k \cos n(\theta_t+\beta_i) - \cos(n\beta_i)\right)}{(k\pi)^2-(n\theta_t)^2} & \text{for } \mathrm{k}\pi \neq \mathrm{n}\theta_t \\ \frac{\theta_t}{2}\left(\sin(n\beta_i) - \frac{(\cos n(\beta_i+2\theta_t)-\cos(n\beta_i))}{2k\pi}\right) & \text{for } \mathrm{k}\pi = \mathrm{n}\theta_t \end{cases} \tag{C.7}$$

The continuity of tangential flux intensity at $z = z_m$ i.e. $\frac{\partial A_{1n}}{\partial z} = \mu_r \frac{\partial A_{2n}}{\partial z}\Big|_{z=z_m}$ produces following linear equations

$$\begin{aligned} Q_1(n)\,a_{1n} &= \mu_r Q_1(n)\,a_{2n} + \mu_r Q_2(n)\,b_{2n} \\ Q_1(n)\,c_{1n} &= \mu_r Q_1(n)\,c_{2n} + \mu_r Q_2(n)\,d_{2n} \end{aligned} \tag{C.8}$$

where $Q_1(n) = \sinh\frac{nz_m}{R}$, and $Q_2(n) = \cosh\frac{nz_m}{R}$.

The other boundary condition i.e., the continuity of normal flux density at $z = z_m$ i.e. $A_{1n} = A_{2n}|_{z=z_m}$ produces linear equations

$$\begin{aligned} Q_2(n)\,a_{1n} + G_n &= Q_2(n)\,a_{2n} + Q_1(n)\,b_{2n} \\ Q_2(n)\,c_{1n} + H_n &= Q_2(n)\,c_{2n} + Q_1(n)\,d_{2n} \end{aligned} \tag{C.9}$$

Now, at $z = z_o$ two types of boundary interfaces exits namely 1) the interfaces of airgap and slots and 2) airgap and teeth. Hence, the continuity of the tangential field intensity is mathematically expressed as

$$\frac{\partial A_{2n}}{\partial z}\Big|_{z=z_0} = \begin{cases} \frac{\partial A_{3i}}{\partial z} & \forall\ \theta \in [\theta_i, \theta_i+\theta_s] \\ \frac{1}{\mu_i}\frac{\partial A_{4i}}{\partial z} & \forall\ \theta \in [\theta_i+\theta_s, \theta_i+\theta_s+\theta_t] \end{cases} \tag{C.10}$$

Expanding right hand side functions over a period of 2π and equating with left hand side expression, following equations emerged

$$\begin{aligned} a_{2n}\sinh\frac{nz_o}{R} + b_{2n}\cosh\frac{nz_o}{R} &= \frac{1}{\pi}\sum_{i=1}^{Q}\left(\int_{\theta_i}^{\theta_i+\theta_s}\frac{\partial A_{3i}}{\partial z}\Big|_{z=z_o}\cos n\theta d\theta + \int_{\beta_i}^{\beta_i+\theta_t}\frac{\partial A_{4i}}{\partial z}\Big|_{z=z_o}\cos n\theta d\theta\right) \\ c_{2n}\sinh\frac{nz_o}{R} + d_{2n}\cosh\frac{nz_o}{R} &= \frac{1}{\pi}\sum_{i=1}^{Q}\left(\int_{\theta_i}^{\theta_i+\theta_s}\frac{\partial A_{3i}}{\partial z}\Big|_{z=z_o}\sin n\theta d\theta + \int_{\beta_i}^{\beta_i+\theta_t}\frac{\partial A_{4i}}{\partial z}\Big|_{z=z_o}\sin n\theta d\theta\right) \end{aligned}$$

The above equations are rewritten as

$$\begin{aligned} Q_3(n)\,a_{2n} + Q_4(n)\,b_{2n} &= \sum_{i=1}^{Q}\left(\sum_{m=1}^{\infty} Q_5(m,n,i)\,a_{3m}^i + \sum_{k=1}^{\infty} Q_6(k,n,i)\,a_{3k}^i\right) \\ Q_3(n)\,a_{2n} + Q_4(n)\,b_{2n} &= \sum_{i=1}^{Q}\left(\sum_{m=1}^{\infty} Q_7(m,n,i)\,a_{3m}^i + \sum_{k=1}^{\infty} Q_8(k,n,i)\,a_{3k}^i\right) \end{aligned} \tag{C.11}$$

where, $Q_3(n)$, $Q_4(n)$, $Q_5(m,n,i)$, $Q_6(k,n,i)$, $Q_7(m,n,i)$, and $Q_8(k,n,i)$ are defined by

$$\begin{array}{l} Q_3(n) = \sinh \frac{nz_o}{R} \qquad Q_4(n) = \cosh \frac{nz_o}{R} \\ Q_5(m,n,i) = \frac{1}{\pi} \tanh\left(\frac{m\pi h_s}{R\theta_s}\right) f_1(m,n,i) \\ Q_6(k,n,i) = \frac{1}{\mu_i \pi} \tanh\left(\frac{k\pi h_s}{R\theta_t}\right) f_2(k,n,i) \\ Q_7(m,n,i) = \frac{1}{\pi} \tanh\left(\frac{m\pi h_s}{R\theta_s}\right) g_1(m,n,i) \\ Q_8(k,n,i) = \frac{1}{\mu_i \pi} \tanh\left(\frac{k\pi h_s}{R\theta_t}\right) g_2(k,n,i) \end{array} \tag{C.12}$$

In order to evaluate a^i_{30}, and a^i_{30} the continuity of normal flux density $A_{3i} = A_{2n}|_{z=z_0}$ is used. The Fourier expansion of the $A_{2n}|_{z=z_o}$ over the angular interval $\theta \in [\theta_i, \theta_i + \theta_s]$, gives following two equations

$$\begin{array}{l} a^i_{30} = \frac{1}{\theta_s} \int\limits_{\theta_i}^{\theta_i+\theta_s} A_{2n} d\theta \\ a^i_{3m} = \frac{2}{\theta_s} \int\limits_{\theta_i}^{\theta_i+\theta_s} A_{2n} \cos \frac{m\pi(\theta-\theta_i)}{\theta_s} d\theta \end{array}$$

further simplification of above equations lead to

$$\begin{array}{l} a^i_{30} = \sum\limits_{n=1,2,3..}^{\infty} \left(Q_9(i,n)\, a_{2n} + Q_{10}(i,n)\, b_{2n} + Q_{11}(i,n)\, c_{2n} + Q_{12}(i,n)\, d_{2n}\right) \\ a^i_{3m} = \sum\limits_{n=1,2,3..}^{\infty} \left(Q_{13}(i,n,m)\, a_{2n} + Q_{14}(i,n,m)\, b_{2n} + Q_{15}(i,n,m)\, c_{2n} + Q_{16}(i,n,m)\, d_{2n}\right) \end{array} \tag{C.13}$$

where,

$$Q_9(i,n) = \frac{R}{n\theta_s} \cosh \frac{nz_o}{R} r_1(n,i) \tag{C.14}$$

$$Q_{10}(i,n) = \frac{R}{n\theta_s} \sinh \frac{nz_o}{R} r_1(n,i) \tag{C.15}$$

$$Q_{11}(i,n) = \frac{R}{n\theta_s} \cosh \frac{nz_o}{R} s_1(n,i) \tag{C.16}$$

$$Q_{12}(i,n) = \frac{R}{n\theta_s} \sinh \frac{nz_o}{R} s_1(n,i) \tag{C.17}$$

$$Q_{13}(i,n,m) = \frac{2m\pi}{n\theta_s^2} \cosh \frac{nz_o}{R} f_1(m,n,i) \tag{C.18}$$

$$Q_{14}(i,n,m) = \frac{2m\pi}{n\theta_s^2} \sinh \frac{nz_o}{R} f_1(m,n,i) \tag{C.19}$$

$$Q_{15}(i,n,m) = \frac{2m\pi}{n\theta_s^2} \cosh \frac{nz_o}{R} g_1(m,n,i) \tag{C.20}$$

$$Q_{16}(i,n,m) = \frac{2m\pi}{n\theta_s^2} \sinh \frac{nz_o}{R} g_1(m,n,i) \tag{C.21}$$

Similarly, the continuity of normal flux density $A_{4i} = A_{2n}|_{z=z_0}$ exploited to get following two equations

$$a_{40}^{i} = \tfrac{1}{\theta_t} \int\limits_{\theta_i+\theta_s}^{\theta_i+\theta_s+\theta_t} A_{2n} d\theta$$
$$a_{4k}^{i} = \tfrac{2}{\theta_t} \int\limits_{\theta_i+\theta_s}^{\theta_i+\theta_s+\theta_t} A_{2n} \cos \tfrac{k\pi(\theta-\theta_i-\theta_s)}{\theta_t} d\theta$$

further simplification of above equations lead to

$$\begin{aligned} a_{40}^{i} &= \sum_{n=1,2,3..}^{\infty} \left(Q_{9t}\left(i,n\right)a_{2n} + Q_{10t}\left(i,n\right)b_{2n} + Q_{11t}\left(i,n\right)c_{2n} + Q_{12t}\left(i,n\right)d_{2n}\right) \\ a_{4k}^{i} &= \sum_{n=1,2,3..}^{\infty} \left(Q_{13t}\left(i,n,k\right)a_{2n} + Q_{14t}\left(i,n,k\right)b_{2n} + Q_{15t}\left(i,n,k\right)c_{2n} + Q_{16t}\left(i,n,k\right)d_{2n}\right) \end{aligned} \tag{C.22}$$

where,

$$Q_{9t}\left(i,n\right) = \frac{R}{n\theta_t} \cosh \frac{nz_o}{R} r_2\left(n,i\right) \tag{C.23}$$

$$Q_{10t}\left(i,n\right) = \frac{R}{n\theta_t} \sinh \frac{nz_o}{R} r_2\left(n,i\right) \tag{C.24}$$

$$Q_{11t}\left(i,n\right) = \frac{R}{n\theta_t} \cosh \frac{nz_o}{R} s_2\left(n,i\right) \tag{C.25}$$

$$Q_{12t}\left(i,n\right) = \frac{R}{n\theta_t} \sinh \frac{nz_o}{R} s_2\left(n,i\right) \tag{C.26}$$

$$Q_{13t}\left(i,n,m\right) = \frac{2k\pi}{n\theta_t^2} \cosh \frac{nz_o}{R} f_2\left(k,n,i\right) \tag{C.27}$$

$$Q_{14t}\left(i,n,m\right) = \frac{2k\pi}{n\theta_t^2} \sinh \frac{nz_o}{R} f_2\left(k,n,i\right) \tag{C.28}$$

$$Q_{15t}\left(i,n,m\right) = \frac{2k\pi}{n\theta_t^2} \cosh \frac{nz_o}{R} g_2\left(k,n,i\right) \tag{C.29}$$

$$Q_{16t}\left(i,n,m\right) = \frac{2k\pi}{n\theta_t^2} \sinh \frac{nz_o}{R} g_2\left(k,n,i\right) \tag{C.30}$$

The all coefficients are evaluated using solutions of linear simultaneous equations given in C.8,C.9,C.11,C.13 and C.22.

Appendix D

Determination of the Coefficients used in Chapter 4

Open Slot Modular PM Machine

The integrations D.1, and D.3 are defined, as it is required for coefficients calculation

$$
\begin{aligned}
r_1(n,i) &= \int_{\theta_i}^{\theta_i+\beta} \cos n\theta d\theta & \mathrm{s}_1(n,i) &= \int_{\theta_i}^{\theta_i+\beta} \sin n\theta d\theta \\
r_2(n,i) &= \int_{\alpha_i-\beta}^{\alpha_i} \cos n\theta d\theta & \mathrm{s}_2(n,i) &= \int_{\alpha_i-\beta}^{\alpha_i} \sin n\theta d\theta \\
r_3(n,i) &= \int_{\beta_k}^{\beta_k+\theta_o} \cos n\theta d\theta & \mathrm{s}_2(n,i) &= \int_{\beta_k}^{\beta_k+\theta_o} \sin n\theta d\theta
\end{aligned}
\tag{D.1}
$$

Above integrations gives

$$
\begin{aligned}
r_1(n,i) &= (\sin(n(\theta_i+\beta)) - \sin(n\theta_i))/n \\
s_1(n,i) &= (-\cos(n(\theta_i+\beta)) + \cos(n\theta_i))/n \\
r_2(n,i) &= (\sin(n(\alpha_i)) - \sin(n(\alpha_i-\theta_o)))/n \\
s_2(n,i) &= (-\cos(n(\alpha_i)) + \cos(n(\alpha_i-\theta_o)))/n \\
r_3(n,i) &= (\sin(n(\beta_k+\theta_o)) - \sin(n\beta_k))/n \\
s_3(n,i) &= (-\cos(n(\beta_k+\theta_o)) + \cos(n\beta_k))/n
\end{aligned}
\tag{D.2}
$$

and, the other sets of integrations are

$$
\begin{aligned}
f_1(m_1,n,i) &= \int_{\theta_i}^{\theta_i+\beta} \cos\frac{m_1\pi}{\beta}(\theta-\theta_{i.})\cos n\theta d\theta & g_1(m_1,n,i) &= \int_{\theta_i}^{\theta_i+\beta} \cos\frac{m_1\pi}{\beta}(\theta-\theta_{i.})\sin n\theta d\theta \\
f_2(m_2,n,i) &= \int_{\alpha_i-\beta}^{\alpha_i} \cos\frac{m_2\pi}{\beta}(\theta-\alpha_{i.})\cos n\theta d\theta & g_2(m_2,n,i) &= \int_{\alpha_i-\beta}^{\alpha_i} \cos\frac{m_2\pi}{\beta}(\theta-\alpha_{i.})\sin n\theta d\theta \\
f_3(m_3,n,i) &= \int_{\beta_k}^{\beta_k+\theta_o} \cos\frac{m_3\pi}{\beta_k}(\theta-\theta_{o.})\cos n\theta d\theta & g_3(m_3,n,i) &= \int_{\beta_k}^{\beta_k+\theta_o} \cos\frac{m_3\pi}{\beta_k}(\theta-\theta_{o.})\sin n\theta d\theta
\end{aligned}
\tag{D.3}
$$

The further simplification of above integration gives

$$
f_1(m_1,n,i) = \begin{cases} \frac{-n\beta^2((-1)^{m_1}\sin n(\beta+\theta_i) - \sin(n\theta_i))}{(m_1\pi)^2-(n\beta)^2} & \text{for } \mathrm{m}_1\pi \neq \mathrm{n}\beta \\ \frac{\beta}{2}\left(\cos(n\theta_i) + \frac{(\sin n(\theta_i+2\beta) - \sin(n\theta_i))}{2m_1\pi}\right) & \text{for } \mathrm{m}_1\pi = \mathrm{n}\beta \end{cases}
\tag{D.4}
$$

$$g_1\left(m_1,n,i\right)=\begin{cases}\frac{n\beta^2((-1)^{m_1}\cos n(\beta+\theta_i)-\cos(n\theta_i))}{(m_1\pi)^2-(n\beta)^2} & \text{for } \mathrm{m}_1\pi\neq \mathrm{n}\beta\\ \frac{\beta}{2}\left(\sin\left(n\theta_i\right)-\frac{(\cos n(\theta_i+2\beta)-\cos(n\theta_i))}{2m_1\pi}\right) & \text{for } \mathrm{m}_1\pi=\mathrm{n}\beta\end{cases}\tag{D.5}$$

$$f_2\left(m_2,n,i\right)=\begin{cases}\frac{-n\beta^2((-1)^{m_2}\sin n(\alpha_i-\beta)-\sin(n\alpha_i))}{(m_2\pi)^2-(n\beta)^2} & \text{for } \mathrm{m}_2\pi\neq \mathrm{n}\beta\\ \frac{\beta}{2}\left(\cos\left(n\alpha_i\right)+\frac{(\sin n(\alpha_i)-\sin(n(\theta_i-2\beta)))}{2m_2\pi}\right) & \text{for } \mathrm{m}_2\pi=\mathrm{n}\beta\end{cases}\tag{D.6}$$

$$g_2\left(m_2,n,i\right)=\begin{cases}\frac{n\beta^2((-1)^{m_2}\cos n(\alpha_i-\beta)-\cos(n\alpha_i))}{(m_2\pi)^2-(n\beta)^2} & \text{for } \mathrm{m}_2\pi\neq \mathrm{n}\beta\\ \frac{\beta}{2}\left(\sin\left(n\alpha_i\right)-\frac{(\cos n(\alpha_i)-\cos(n(\alpha_i-2\beta)))}{2m_2\pi}\right) & \text{for } \mathrm{m}_2\pi=\mathrm{n}\beta\end{cases}\tag{D.7}$$

$$f_3\left(m_3,n,i\right)=\begin{cases}\frac{-n\theta_o^2((-1)^{m_3}\sin n(\theta_o+\beta_k)-\sin(n\beta_k))}{(m_3\pi)^2-(n\theta_o)^2} & \text{for } \mathrm{m}_3\pi\neq \mathrm{n}\theta_o\\ \frac{\theta_o}{2}\left(\cos\left(n\beta_k\right)+\frac{(\sin n(\beta_k+2\theta_o)-\sin(n\beta_k))}{2m_3\pi}\right) & \text{for } \mathrm{m}_3\pi=\mathrm{n}\theta_o\end{cases}\tag{D.8}$$

$$g_3\left(m_3,n,i\right)=\begin{cases}\frac{n\theta_o^2((-1)^{m_3}\cos n(\theta_o+\beta_k)-\cos(n\beta_k))}{(m_3\pi)^2-(n\theta_o)^2} & \text{for } \mathrm{m}_3\pi\neq \mathrm{n}\theta_o\\ \frac{\theta_o}{2}\left(\sin\left(n\beta_k\right)-\frac{(\cos n(\beta_k+2\theta_o)-\cos(n\beta_k))}{2m_3\pi}\right) & \text{for } \mathrm{m}_3\pi=\mathrm{n}\theta_o\end{cases}\tag{D.9}$$

Now, let us define following quantities

$$\begin{array}{lll}
Q_1\left(n\right)=\frac{R_2}{n}\frac{X_n(R_2,R_3)}{Y_n(R_2,R_3)}; & Q_2\left(n\right)=\frac{R_2}{n}\frac{2}{Y_n(R_3,R_2)}; & Q_3\left(n\right)=\frac{n}{R_2}\frac{Y_n(R_2,R_1)}{X_n(R_2,R_1)}\\
Q_4\left(n\right)=\frac{2r_1(n,i)R_2}{n\beta Y_n(R_2,R_3)}; & Q_5\left(n\right)=\frac{r_1(n,i)R_3X_n(R_3,R_2)}{n\beta Y_n(R_3,R_2)}; & Q_6\left(n\right)=\frac{2s_1(n,i)R_2}{n\beta Y_n(R_2,R_3)}\\
Q_7\left(n\right)=\frac{s_1(n,i)R_3X_n(R_3,R_2)}{n\beta Y_n(R_3,R_2)}; & Q_8\left(n\right)=\frac{4f_1(m_1,n,i)R_2}{S_{m1}n\beta Y_n(R_2,R_3)}; & Q_9\left(n\right)=\frac{2f_1(m_1,n,i)R_3X_n(R_3,R_2)}{S_{m1}n\beta Y_n(R_3,R_2)}\\
Q_{10}\left(n\right)=\frac{4g_1(m_1,n,i)R_2}{S_{m1}n\beta Y_n(R_2,R_3)}; & Q_{11}\left(n\right)=\frac{2g_1(m_1,n,i)R_3X_n(R_3,R_2)}{S_{m1}n\beta Y_n(R_3,R2)}; & Q_{12}\left(n\right)=\frac{2r_2(n,i)R_2}{n\beta Y_n(R_2,R_3)}\\
Q_{13}\left(n\right)=\frac{r_2(n,i)R_3X_n(R_3,R_2)}{n\beta Y_n(R_3,R_2)}; & Q_{14}\left(n\right)=\frac{2s_2(n,i)R_2}{n\beta Y_n(R_2,R_3)}; & Q_{15}\left(n\right)=\frac{s_2(n,i)R_3X_n(R_3,R_2)}{n\beta Y_n(R_3,R_2)}\\
Q_{16}\left(n\right)=\frac{4f_2(m_2,n,i)R_2}{S_{m2}n\beta Y_n(R_2,R_3)}; & Q_{17}\left(n\right)=\frac{2f_2(m_2,n,i)R_3X_n(R_3,R_2)}{S_{m2}n\beta Y_n(R_3,R_2)}; & Q_{18}\left(n\right)=\frac{4g_2(m_2,n,i)R_2}{S_{m2}n\beta Y_n(R_2,R_3)}\\
Q_{19}\left(n\right)=\frac{2g_2(m_2,n,i)R_3X_n(R_3,R_2)}{S_{m2}n\beta Y_n(R_3,R_2)}; & Q_{20}\left(n\right)=\frac{2r_3(n,i)R_2}{n\theta_oY_n(R_2,R_3)}; & Q_{21}\left(n\right)=\frac{r_3(n,i)R_3X_n(R_3,R_2)}{n\theta_oY_n(R_3,R_2)}\\
Q_{22}\left(n\right)=\frac{2s_3(n,i)R_2}{n\theta_oY_n(R_2,R_3)}; & Q_{23}\left(n\right)=\frac{s_3(n,i)R_3X_n(R_3,R_2)}{n\theta_oY_n(R_3,R_2)}; & Q_{24}\left(n\right)=\frac{4f_3(m_3,n,i)R_2}{S_{m3}n\theta_oY_n(R_2,R_3)}\\
Q_{25}\left(n\right)=\frac{2f_3(m_3,n,i)R_3X_n(R_3,R_2)}{S_{m3}n\theta_oY_n(R_3,R_2)}; & Q_{26}\left(n\right)=\frac{4g_3(m_3,n,i)R_2}{S_{m3}n\theta_oY_n(R_2,R_3)}; & Q_{27}\left(n\right)=\frac{2g_3(m_3,n,i)R_3X_n(R_3,R_2)}{S_{m3}n\theta_oY_n(R_3,R_2)}
\end{array}\tag{D.10}$$

In view of all definitions, the equations (4.25), and (4.26) are rewritten as

$$\begin{aligned}a_{1n}&=Q_1\left(n\right)a_{2n}+Q_2\left(n\right)b_{2n}\\ c_{1n}&=Q_1\left(n\right)c_{2n}+Q_2\left(n\right)d_{2n}\end{aligned}\tag{D.11}$$

similarly, the expansion of equations (4.28), and (4.29) give us

$$\begin{aligned}\mu_ra_{2n}&=Q_3\left(n\right)a_{1n}+F_n^{'}\left(R_2\right)\cos\left(n\delta\right)\\ \mu_rc_{2n}&=Q_3\left(n\right)c_{1n}+F_n^{'}\left(R_2\right)\sin\left(n\delta\right)\end{aligned}\tag{D.12}$$

Now using equations (4.32) and (4.33), following linear equations are developed

$$b_{2n} = \sum_{i=1}^{Q/2} \frac{1}{\pi} \left(a_{m_1}^i f_1 (m_1, n, i) + a_{m_2}^j f_2 (m_2, n, i) - a_{m_3}^k f_3 (m_3, n, i) \frac{2}{Y_{m_2\pi/\theta_o}(R_3, R_4)} \right)$$
$$d_{2n} = \sum_{i=1}^{Q/2} \frac{1}{\pi} \left(a_{m_1}^i g_1 (m_1, n, i) + a_{m_2}^j g_2 (m_2, n, i) - a_{m_3}^k g_3 (m_3, n, i) \frac{2}{Y_{m_2\pi/\theta_o}(R_3, R_4)} \right) \tag{D.13}$$

To express the coefficients used in the slot regions (i^{th} and j^{th}) and magnetic flux gap region k^{th}, the equations (4.37)-(4.42) are used. Correspondingly, the linear equations are given as

$$a_o^i = \sum_{n=1}^{\infty} \left(Q_4 (n) a_2 n + Q_5 (n) b_2 n + Q_6 (n) c_2 n + Q_7 (n) d_2 n \right)$$
$$a_{m_1}^i = \sum_{n=1}^{\infty} \left(Q_8 (n) a_2 n + Q_9 (n) b_2 n + Q_{10} (n) c_2 n + Q_{11} (n) d_2 n \right) \tag{D.14}$$

$$a_o^j = \sum_{n=1}^{\infty} \left(Q_{12} (n) a_2 n + Q_{13} (n) b_2 n + Q_{14} (n) c_2 n + Q_{15} (n) d_2 n \right)$$
$$a_{m_2}^j = \sum_{n=1}^{\infty} \left(Q_{16} (n) a_2 n + Q_{17} (n) b_2 n + Q_{18} (n) c_2 n + Q_{19} (n) d_2 n \right) \tag{D.15}$$

and,

$$a_o^k = \sum_{n=1}^{\infty} \left(Q_{20} (n) a_2 n + Q_{21} (n) b_2 n + Q_{22} (n) c_2 n + Q_{23} (n) d_2 n \right)$$
$$a_{m_3 1}^k = \sum_{n=1}^{\infty} \left(Q_{24} (n) a_2 n + Q_{24} (n) b_2 n + Q_{26} (n) c_2 n + Q_{27} (n) d_2 n \right) \tag{D.16}$$

All the coefficients involved in the derivations is vector potential in all regions of modular PM machine with open slot are evaluated using linear simultaneous equations developed in equationsD.11-D.16.

Appendix E

Determination of the Coefficients used in Chapter 4

Semiclosed Slot Modular PM Machine

Following integration as defined and used in the determination of coefficients

$$\begin{aligned}
r_1(n,i) &= \int_{\gamma_i}^{\gamma_i+\delta} \cos n\theta d\theta & \mathrm{s}_1(n,i) &= \int_{\gamma_i}^{\gamma_i+\delta} \sin n\theta d\theta \\
r_2(n,i) &= \int_{\delta_i-\delta}^{\delta_i} \cos n\theta d\theta & \mathrm{s}_2(n,i) &= \int_{\delta_i-\delta}^{\delta_i} \sin n\theta d\theta \\
r_3(n,i) &= \int_{\theta_i}^{\theta_i+\theta_o} \cos n\theta d\theta & \mathrm{s}_2(n,i) &= \int_{\theta_i}^{\theta_i+\theta_o} \sin n\theta d\theta
\end{aligned} \tag{E.1}$$

The above equations are simplification as,

$$\begin{aligned}
r_1(n,i) &= \left(\sin\left(n(\gamma_i+\delta)\right) - \sin\left(n\gamma_i\right)\right)/n \\
s_1(n,i) &= \left(-\cos\left(n(\gamma_i+\delta)\right) + \cos\left(n\gamma_i\right)\right)/n \\
r_2(n,i) &= \left(\sin\left(n(\delta_i)\right) - \sin\left(n(\delta_i-\delta)\right)\right)/n \\
s_2(n,i) &= \left(-\cos\left(n(\delta_i)\right) + \cos\left(n(\delta_i-\delta)\right)\right)/n \\
r_3(n,i) &= \left(\sin\left(n(\theta_i+\theta_o)\right) - \sin\left(n\theta_i\right)\right)/n \\
s_3(n,i) &= \left(-\cos\left(n(\theta_i+\theta_o)\right) + \cos\left(n\theta_i\right)\right)/n
\end{aligned} \tag{E.2}$$

and, the other set of integrations are

$$\begin{aligned}
f_1(m_1,n,i) &= \int_{\gamma_i}^{\gamma_i+\delta} \cos\tfrac{m_1\pi}{\delta}\left(\theta-\gamma_{i.}\right)\cos n\theta d\theta & F_1(m_1,m_3) &= \int_{\gamma_i}^{\gamma_i+\delta} \cos\tfrac{m_1\pi}{\delta}\left(\theta-\gamma_{i.}\right)\cos\tfrac{m_3\pi}{\beta}\left(\theta-\alpha_{i.}\right)d\theta \\
g_1(m_1,n,i) &= \int_{\gamma_i}^{\gamma_i+\delta} \cos\tfrac{m_1\pi}{\delta}\left(\theta-\gamma_{i.}\right)\sin n\theta d\theta & F_2(m_2,m_4) &= \int_{\delta_i-\delta}^{\delta_i} \cos\tfrac{m_2\pi}{\delta}\left(\theta-\delta_{i.}\right)\cos\tfrac{m_4\pi}{\beta}\left(\theta-\beta_{i.}\right)d\theta \\
f_2(m_2,n,i) &= \int_{\delta_i-\delta}^{\delta_i} \cos\tfrac{m_2\pi}{\delta}\left(\theta-\delta_{i.}\right)\cos n\theta d\theta & g_2(m_2,n,i) &= \int_{\delta_i-\delta}^{\delta_i} \cos\tfrac{m_2\pi}{\delta}\left(\theta-\delta_{i.}\right)\sin n\theta d\theta \\
f_5(m_5,n,i) &= \int_{\theta_i}^{\theta_i+\theta_o} \cos\tfrac{m_5\pi}{\theta_o}\left(\theta-\theta_{i.}\right)\cos n\theta d\theta & g_5(m_5,n,i) &= \int_{\theta_i}^{\theta_i+\theta_o} \cos\tfrac{m_5\pi}{\theta_o}\left(\theta-\theta_{i.}\right)\sin n\theta d\theta
\end{aligned} \tag{E.3}$$

These integrations are evaluated and given in equations E.4-E.11

$$f_1(m_1,n,i)=\begin{cases}\frac{-n\delta^2((-1)^{m_1}\sin n(\delta+\gamma_i)-\sin(n\gamma_i))}{(m_1\pi)^2-(n\delta)^2} & \text{for } \mathrm{m}_1\pi\neq \mathrm{n}\delta\\ \frac{\delta}{2}\left(\cos(n\gamma_i)+\frac{(\sin n(\gamma_i+2\delta)-\sin(n\gamma_i))}{2m_1\pi}\right) & \text{for } \mathrm{m}_1\pi = \mathrm{n}\delta\end{cases}\tag{E.4}$$

$$g_1(m_1,n,i)=\begin{cases}\frac{n\delta^2((-1)^{m_1}\cos n(\delta+\gamma_i)-\cos(n\gamma_i))}{(m_1\pi)^2-(n\delta)^2} & \text{for } \mathrm{m}_1\pi\neq \mathrm{n}\delta\\ \frac{\delta}{2}\left(\sin(n\gamma_i)-\frac{(\cos n(\gamma_i+2\delta)-\cos(n\gamma_i))}{2m_1\pi}\right) & \text{for } \mathrm{m}_1\pi = \mathrm{n}\delta\end{cases}\tag{E.5}$$

$$f_2(m_2,n,i)=\begin{cases}\frac{n\delta^2((-1)^{m_2}\sin n(\delta_i-\delta)-\sin(n\delta_i))}{(m_2\pi)^2-(n\delta)^2} & \text{for } \mathrm{m}_2\pi\neq \mathrm{n}\delta\\ \frac{\delta}{2}\left(\cos(n\delta_i)+\frac{(\sin n(\delta_i)-\sin(n(\delta_i-2\delta)))}{2m_2\pi}\right) & \text{for } \mathrm{m}_2\pi = \mathrm{n}\delta\end{cases}\tag{E.6}$$

$$g_2(m_2,n,i)=\begin{cases}\frac{-n\delta^2((-1)^{m_2}\cos n(\delta_i-\delta)-\cos(n\delta_i))}{(m_2\pi)^2-(n\delta)^2} & \text{for } \mathrm{m}_2\pi\neq \mathrm{n}\delta\\ \frac{\delta}{2}\left(\sin(n\delta_i)-\frac{(\cos n(\delta_i)-\cos(n(\delta_i-2\delta)))}{2m_2\pi}\right) & \text{for } \mathrm{m}_2\pi = \mathrm{n}\delta\end{cases}\tag{E.7}$$

$$f_5(m_5,n,i)=\begin{cases}\frac{-n\theta_o^2((-1)^{m_5}\sin n(\theta_i+\theta_o)-\sin(n\theta_i))}{(m_5\pi)^2-(n\theta_o)^2} & \text{for } \mathrm{m}_5\pi\neq \mathrm{n}\theta_o\\ \frac{\theta_o}{2}\left(\cos(n\theta_i)+\frac{(\sin n(\theta_i+2\theta_o)-\sin(n\theta_i))}{2m_5\pi}\right) & \text{for } \mathrm{m}_5\pi = \mathrm{n}\theta_o\end{cases}\tag{E.8}$$

$$g_5(m_5,n,i)=\begin{cases}\frac{n\theta_o^2((-1)^{m_5}\cos n(\theta_i+\theta_i)-\cos(n\theta_i))}{(m_5\pi)^2-(n\theta_o)^2} & \text{for } \mathrm{m}_5\pi\neq \mathrm{n}\theta_o\\ \frac{\theta_o}{2}\left(\sin(n\theta_i)-\frac{(\cos n(\theta_i+2\theta_o)-\cos(n\theta_i))}{2m_5\pi}\right) & \text{for } \mathrm{m}_5\pi = \mathrm{n}\theta_o\end{cases}\tag{E.9}$$

$$F_1(m_1,m_3)=\begin{cases}\frac{m_3\pi}{\beta}\frac{\left((-1)^{m_1}\sin\frac{m_3\pi(\beta+\delta)}{2\beta}-\sin\frac{m_3\pi(\beta-\delta)}{2\beta}\right)}{\left(\frac{m_3\pi}{\beta}\right)^2-\left(\frac{m_1\pi}{\delta}\right)^2} & \text{for } \mathrm{m}_1\beta\neq \mathrm{m}_3\delta\\ \frac{\delta}{2}\cos\frac{m_3\pi(\beta-\delta)}{2\beta} & \text{for } \mathrm{m}_1\beta = m_3\delta\end{cases}\tag{E.10}$$

and,

$$F_2(m_2,m_4)=\begin{cases}\frac{m_4\pi}{\beta}\frac{\left((-1)^{m_2}\sin\frac{m_4\pi(\beta+\delta)}{2\beta}-\sin\frac{m_4\pi(\beta-\delta)}{2\beta}\right)}{\left(\frac{m_4\pi}{\beta}\right)^2-\left(\frac{m_2\pi}{\delta}\right)^2} & \text{for } \mathrm{m}_2\beta\neq \mathrm{m}_4\delta\\ \frac{\delta}{2}\cos\frac{m_4\pi(\beta-\delta)}{2\beta} & \text{for } \mathrm{m}_2\beta = m_4\delta\end{cases}\tag{E.11}$$

Let us define following quantities

$$\begin{aligned}
&Q_1(n)=\frac{R_2}{n}\frac{X_n(R_2,R_3)}{Y_n(R_2,R_3)}; && Q_2(n)=\frac{R_2}{n}\frac{2}{Y_n(R_3,R_2)}; && Q_3(n)=\frac{n}{R_2}\frac{Y_n(R_2,R_1)}{X_n(R_2,R_1)}\\
&Q_4(n)=\frac{2r_1(n,i)R_2}{n\delta Y_n(R_2,R_3)}; && Q_5(n)=\frac{r_1(n,i)R_3X_n(R_3,R_2)}{n\delta Y_n(R_3,R_2)}; && Q_6(n)=\frac{2s_1(n,i)R_2}{n\delta Y_n(R_2,R_3)}\\
&Q_7(n)=\frac{s_1(n,i)R_3X_n(R_3,R_2)}{n\delta Y_n(R_3,R_2)}; && Q_8(n)=\frac{4f_1(m_1,n,i)R_2}{n\delta Y_n(R_2,R_3)}; && Q_9(n)=\frac{2f_1(m_1,n,i)R_3X_n(R_3,R_2)}{n\delta Y_n(R_3,R_2)}\\
&Q_{10}(n)=\frac{4g_1(m_1,n,i)R_2}{n\delta Y_n(R_2,R_3)}; && Q_{11}(n)=\frac{2g_1(m_1,n,i)R_3X_n(R_3,R_2)}{n\delta Y_n(R_3,R_2)}; && Q_{12}(n)=\frac{2r_2(n,i)R_2}{n\delta Y_n(R_2,R_3)}\\
&Q_{13}(n)=\frac{r_2(n,i)R_3X_n(R_3,R_2)}{n\delta Y_n(R_3,R_2)}; && Q_{14}(n)=\frac{2s_2(n,i)R_2}{n\delta Y_n(R_2,R_3)}; && Q_{15}(n)=\frac{s_2(n,i)R_3X_n(R_3,R_2)}{n\delta Y_n(R_3,R_2)}\\
&Q_{16}(n)=\frac{4f_2(m_2,n,i)R_2}{n\delta Y_n(R_2,R3)}; && Q_{17}(n)=\frac{2f_2(m_2,n,i)R_3X_n(R_3,R_2)}{n\delta Y_n(R_3,R_2)}; && Q_{18}(n)=\frac{4g_2(m_2,n,i)R_2}{n\delta Y_n(R_2,R_3)}\\
&Q_{19}(n)=\frac{2g_2(m_2,n,i)R_3X_n(R_3,R_2)}{n\delta Y_n(R_3,R_2)}; && Q_{20}(n)=\frac{2r_3(n,i)R_2}{n\theta_o Y_n(R_2,R_3)}; && Q_{21}(n)=\frac{r_3(n,i)R_3X_n(R_3,R_2)}{n\theta_o Y_n(R_3,R_2)}
\end{aligned}\tag{E.12}$$

and,

$$
\begin{aligned}
&Q_{22}(n) = \tfrac{2s_3(n,i)R_2}{n\theta_o Y_n(R_2,R_3)}; && Q_{23}(n) = \tfrac{s_3(n,i)R_3 X_n(R_3,R_2)}{n\theta_o Y_n(R_3,R_2)} \\
&Q_{24}(n) = \tfrac{2f_5(m_5,n,i)R_2 Y_n(R_3,R_4)}{n\theta_o Y_n(R_2,R_3)}; && Q_{25}(n) = \tfrac{f_5(m_5,n,i)R_3 X_n(R_3,R_2) Y_n(R_3,R_4)}{n\theta_o Y_n(R_3,R_2)} \\
&Q_{26}(n) = \tfrac{2g_5(m_5,n,i)R_2 Y_n(R_3,R_4)}{n\theta_o Y_n(R_2,R_3)}; && Q_{27}(n) = \tfrac{g_5(m_5,n,i)R_3 X_n(R_3,R_2) Y_n(R_3,R_4)}{n\theta_o Y_n(R_3,R_2)}
\end{aligned}
\tag{E.13}
$$

In view of the definition given above, the coefficient equations developed for the analysis of modular PM machine with semi-closed slots are simplified into linear simultaneous equations. The development of linear equations are as described below:
The equations (4.97) and (4.98) are rewritten as

$$
\begin{aligned}
a_{1n} &= Q_1(n)\, a_{2n} + Q_2(n)\, b_{2n} \\
c_{1n} &= Q_1(n)\, c_{2n} + Q_2(n)\, d_{2n}
\end{aligned}
\tag{E.14}
$$

similarly, the equations (1.100) and (1.101) are expressed as,

$$
\begin{aligned}
\mu_r a_{2n} &= Q_3(n)\, a_{1n} + F_n^{'}(R_2)\cos(n\phi) \\
\mu_r c_{2n} &= Q_3(n)\, c_{1n} + F_n^{'}(R_2)\sin(n\phi)
\end{aligned}
\tag{E.15}
$$

Using equation (1.104), and (1.105) the coefficients b_{2n} and d_{2n} are given as

$$
\begin{aligned}
b_{2n} = &\sum_{i=1}^{Q/2} \sum_{m_1=1,2..}^{\infty} \left(a_{3m_1}^{i} \frac{X_{\frac{m_1\pi}{\delta}}(R_3,R_4)}{Y_{\frac{m_1\pi}{\delta}}(R_3,R_4)} - b_{3m_1}^{i} \frac{2}{Y_{\frac{m_1\pi}{\delta}}(R_3,R_4)} \right) \frac{m_1}{\pi R_3 \delta} f_1(m_1,n,i) \\
+ &\sum_{i=1}^{Q/2} \sum_{m_2=1,2..}^{\infty} \left(a_{3m_2}^{j} \frac{X_{\frac{m_2\pi}{\delta}}(R_3,R_4)}{Y_{\frac{m_2\pi}{\delta}}(R_3,R_4)} - b_{3m_2}^{j} \frac{2}{Y_{\frac{m_2\pi}{\delta}}(R_3,R_4)} \right) \frac{m_2}{\pi R_3 \delta} f_2(m_2,n,i) \\
- &\sum_{i=1}^{Q/2} \sum_{m_5=1,2..}^{\infty} a_{5m_5}^{i} \frac{m_5}{R_3\theta_o} \frac{2}{Y_{\frac{m_5\pi}{\theta_o}}(R_3,R_4)} f_5(m_5,n,i)
\end{aligned}
\tag{E.16}
$$

and,

$$
\begin{aligned}
d_{2n} = &\sum_{i=1}^{Q/2} \sum_{m_1=1,2..}^{\infty} \left(a_{3m_1}^{i} \frac{X_{\frac{m_1\pi}{\delta}}(R_3,R_4)}{Y_{\frac{m_1\pi}{\delta}}(R_3,R_4)} - b_{3m_1}^{i} \frac{2}{Y_{\frac{m_1\pi}{\delta}}(R_3,R_4)} \right) \frac{m_1}{\pi R_3 \delta} g_1(m_1,n,i) \\
+ &\sum_{i=1}^{Q/2} \sum_{m_2=1,2..}^{\infty} \left(a_{3m_2}^{j} \frac{X_{\frac{m_2\pi}{\delta}}(R_3,R_4)}{Y_{\frac{m_2\pi}{\delta}}(R_3,R_4)} - b_{3m_2}^{j} \frac{2}{Y_{\frac{m_2\pi}{\delta}}(R_3,R_4)} \right) \frac{m_2}{\pi R_3 \delta} g_2(m_2,n,i) \\
- &\sum_{i=1}^{Q/2} \sum_{m_5=1,2..}^{\infty} a_{5m_5}^{i} \frac{m_5}{R_3\theta_o} \frac{2}{Y_{\frac{m_5\pi}{\theta_o}}(R_3,R_4)} g_5(m_5,n,i)
\end{aligned}
\tag{E.17}
$$

The coefficients of slot opening regions i.e., a_{30}^{i}, $a_{3m_1}^{i}$, a_{30}^{j}, and $a_{m_2}^{j}$ are determined with the simplification of equations (1.109)-(1.112).

$$
\begin{aligned}
a_{30}^{i} &= \sum_{n=1}^{\infty} \left(Q_{14}(n)\, a_{2n} + Q_{15}(n)\, b_{2n} + Q_{16}(n)\, c_{2n} + Q_{17}(n)\, d_{2n} \right) \\
a_{3m_1}^{i} &= \sum_{n=1}^{\infty} \left(Q_{18}(n)\, a_{2n} + Q_{19}(n)\, b_{2n} + Q_{20}(n)\, c_{2n} + Q_{21}(n)\, d_{2n} \right)
\end{aligned}
\tag{E.18}
$$

and,

$$
\begin{aligned}
a_{30}^{j} &= \sum_{n=1}^{\infty}\left(Q_{22}(n)\,a_{2n} + Q_{23}(n)\,b_{2n} + Q_{24}(n)\,c_{2n} + Q_{25}(n)\,d_{2n}\right) \\
a_{3m_2}^{j} &= \sum_{n=1}^{\infty}\left(Q_{26}(n)\,a_{2n} + Q_{27}(n)\,b_{2n} + Q_{28}(n)\,c_{2n} + Q_{29}(n)\,d_{2n}\right)
\end{aligned}
\tag{E.19}
$$

while, the coefficients of magnetic flux gap are evaluated using equations (1.113) and (1.114),

$$
\begin{aligned}
a_{50}^{i} &= \sum_{n=1}^{\infty}\left(Q_{30}(n)\,a_{2n} + Q_{31}(n)\,b_{2n} + Q_{32}(n)\,c_{2n} + Q_{33}(n)\,d_{2n}\right) \\
a_{5m_5}^{i} &= \sum_{n=1}^{\infty}\left(Q_{34}(n)\,a_{2n} + Q_{34}(n)\,b_{2n} + Q_{36}(n)\,c_{2n} + Q_{37}(n)\,d_{2n}\right)
\end{aligned}
\tag{E.20}
$$

Other boundary conditions developed due to radial flux continuity at region 3i and region 4i interface. The corresponding linear simultaneous equations are developed using equations (1.117) and (1.118)

$$
\begin{aligned}
a_{30}^{i} &= a_{40}^{i} + \sum_{m_3=1,2..}^{\infty} a_{4m_3}^{i}\frac{2R_4}{\delta}\left(\frac{\beta}{\pi m_3}\right)^2 \frac{X_{\frac{m_3\pi}{\beta}}(R_4,R_5)}{Y_{\frac{m_3\pi}{\beta}}(R_4,R_5)} \sin\frac{m_3\pi\delta}{2\beta}\cos\frac{m_3\pi}{2} \\
a_{3m_1}^{i} &= \sum_{m_3=1,2..}^{\infty} a_{4m_3}^{i}\frac{2R_4\beta}{m_3\pi\delta}\frac{X_{\frac{m_3\pi}{\beta}}(R_4,R_5)}{Y_{\frac{m_3\pi}{\beta}}(R_4,R_5)} F_1(m_1,m_3)
\end{aligned}
\tag{E.21}
$$

Similarly, using equations (1.119) and (1.200) which ensures the radial flux continuity at region 3j and region 4j interface, following equations are developed,

$$
\begin{aligned}
a_{30}^{j} &= a_{40}^{j} + \sum_{m_4=1,2..}^{\infty} a_{4m_3}^{j}\frac{2R_4}{\delta}\left(\frac{\beta}{\pi m_4}\right)^2 \frac{X_{\frac{m_4\pi}{\beta}}(R_4,R_5)}{Y_{\frac{m_4\pi}{\beta}}(R_4,R_5)} \sin\frac{m_4\pi\delta}{2\beta}\cos\frac{m_4\pi}{2} \\
a_{3m_2}^{j} &= \sum_{m_4=1,2..}^{\infty} a_{4m_4}^{j}\frac{2R_4\beta}{m_4\pi\delta}\frac{X_{\frac{m_4\pi}{\beta}}(R_4,R_5)}{Y_{\frac{m_4\pi}{\beta}}(R_4,R_5)} F_2(m_2,m_2)
\end{aligned}
\tag{E.22}
$$

The last two linear equations are developed with the help of equations (1.201) and (1.202), and given as

$$
\begin{aligned}
a_{4m_3}^{i} &= \sum_{m_1=1,2..}^{\infty}\frac{2\pi m_1}{\beta\delta R_4}\left(a_{3m_1}^{i}\frac{2}{Y_{\frac{m_1\pi}{\delta}}(R_3,R_4)} - b_{3m_1}^{i}\frac{X_{\frac{m_1\pi}{\delta}}(R_4,R_3)}{Y_{\frac{m_1\pi}{\delta}}(R_4,R_3)}\right)F_1(m_1,m_3) \\
a_{4m_4}^{j} &= \sum_{m_1=1,2..}^{\infty}\frac{2\pi m_2}{\beta\delta R_4}\left(a_{3m_2}^{j}\frac{2}{Y_{\frac{m_2\pi}{\delta}}(R_3,R_4)} - b_{3m_2}^{j}\frac{X_{\frac{m_2\pi}{\delta}}(R_4,R_3)}{Y_{\frac{m_{12}\pi}{\delta}}(R_4,R_3)}\right)F_2(m_2,m_4)
\end{aligned}
\tag{E.23}
$$

The solution of the set of linear simultaneous equations E.14-E.23 gives the unknown coefficients.

Appendix F

FEM Model for Different Developed Machines

There are five different variants of PM machine has been developed. The developed FEM models used for analysizing the machines are shown below and their details are presented in their respective chapters.

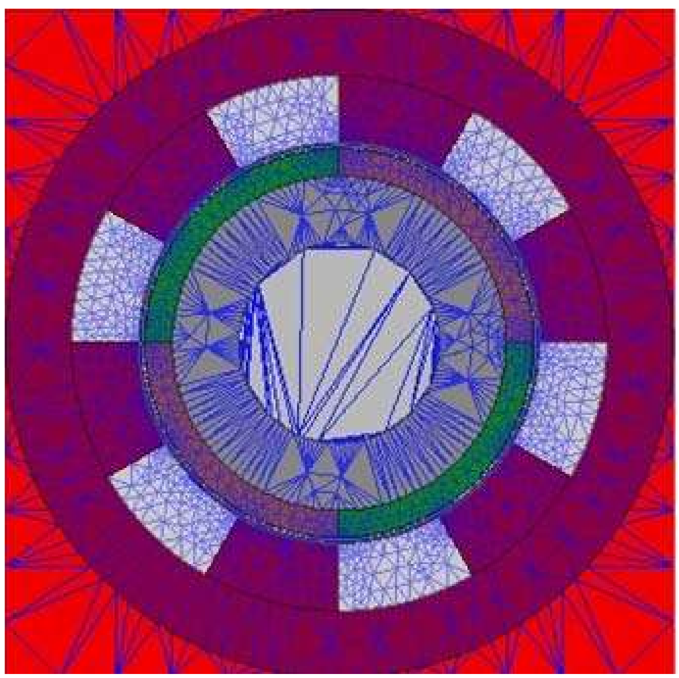
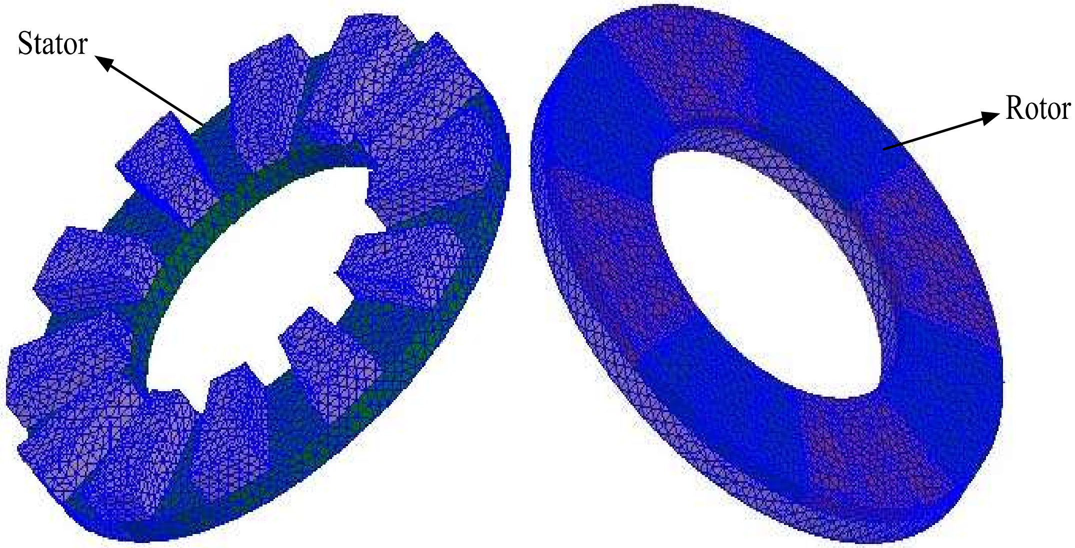


Figure F.1: FEM Model for Radial Flux Permanent Magnet Machine with Saturated Teeth



(a) Stator (b) Rotor

Figure F.2: FEM Models for Axial Flux Permanent Magnet Machine with Saturated Teeth

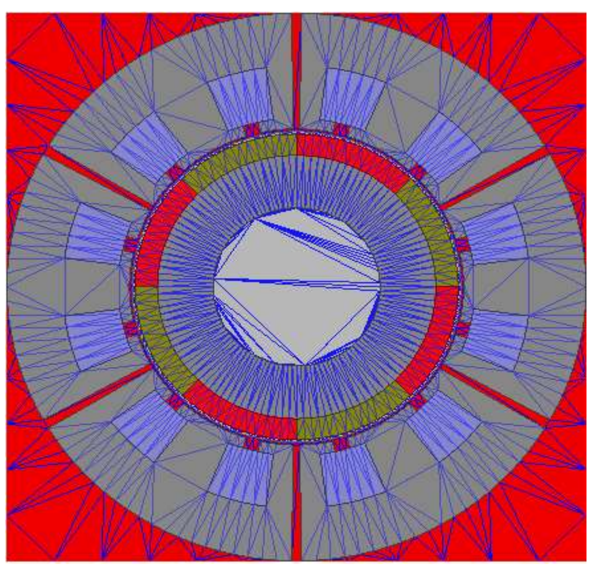
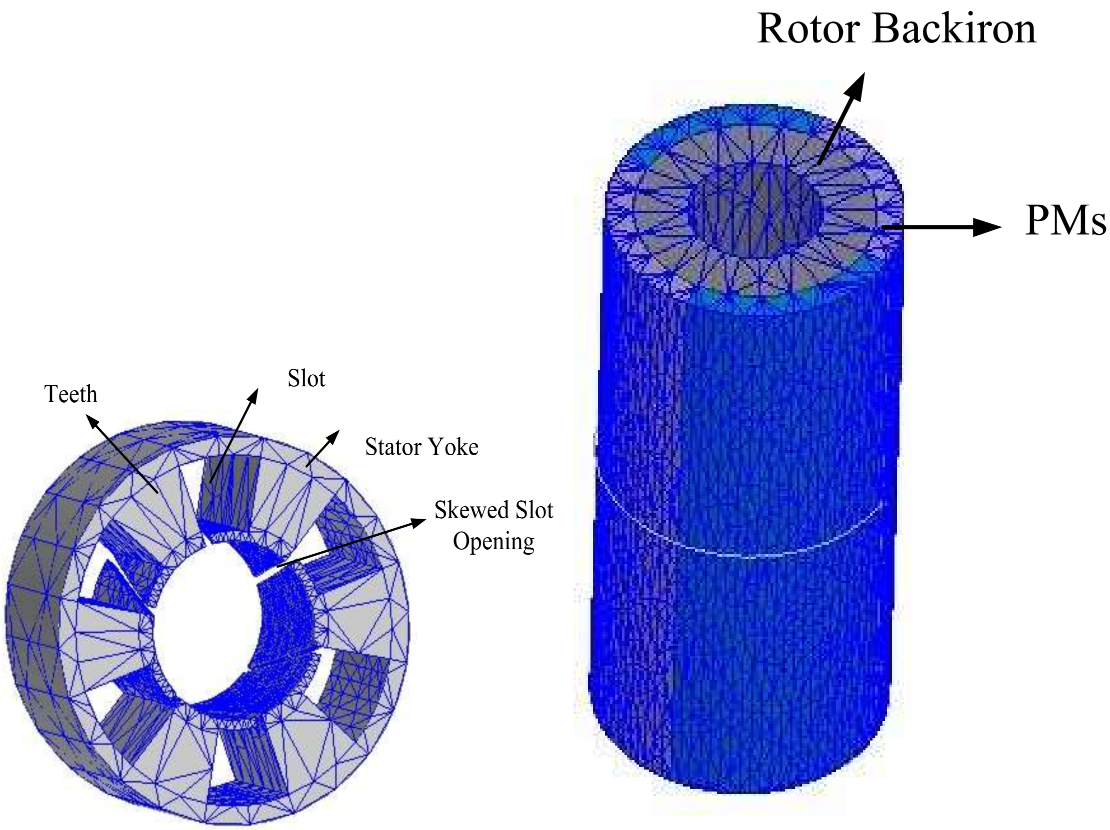


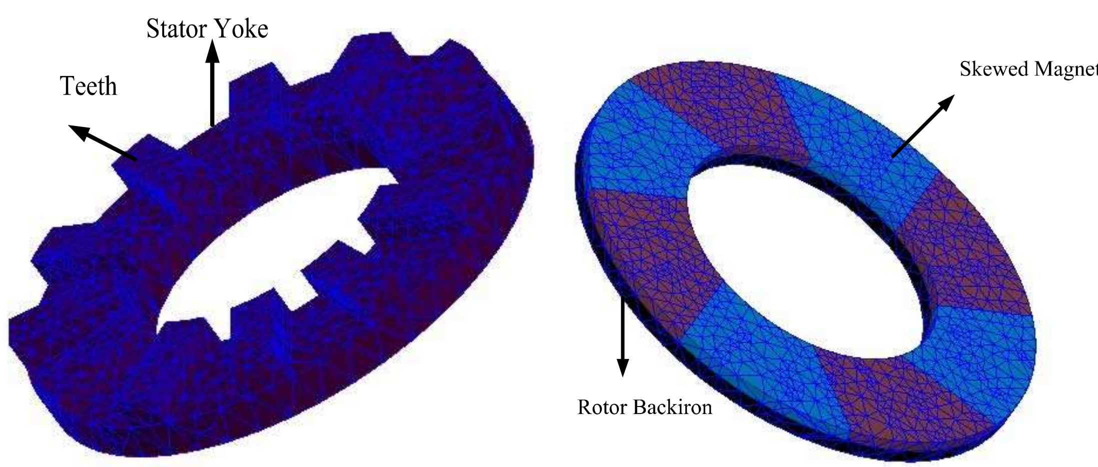
Figure F.3: FEM Model for Modular Permanent Magnet Machine



(a) Stator

(b) Rotor

Figure F.4: FEM Model for Cogging Reduction in PM Machine



(a) Stator

(b) Rotor

Figure F.5: FEM Model for Skewed Magnet Axial Flux Machine