



# Gravitational redshift and the vacuum index of refraction

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**Abstract** A physical process of the gravitational redshift was described in an earlier paper (Wilhelm and Dwivedi, *New Astron.* 31:8, 2014). This process did not require any information for the emitting atom neither on the local gravitational potential  $U$  nor on the speed of light  $c$ . Although it could be shown that the correct energy shift of the emitted photon resulted from energy and momentum conservation principles and the speed of light at the emission site, it was not obvious how this speed is controlled by the gravitational potential. The aim of this paper is to describe a physical process that can accomplish this control. We determine the local speed of light  $c$  by deducing a gravitational index of refraction  $n_G$  as a function of the potential  $U$  assuming a specific aether model, in which photons propagate as solitons. Even though an atom cannot locally sense the gravitational potential  $U$  (cf. Müller et al., *Nature* 467:E2, 2010) the gravitational redshift will nevertheless be determined by  $U$  (cf. Wolf et al., *Nature* 467:E1, 2010)—mediated by the local speed of light  $c$ .

**Keywords** Gravitation · Impact model · Aether · Gravitational index of refraction

## 1 Introduction

The study of the gravitational redshift, predicted for solar radiation by Einstein (1908), is still an important subject in modern physics and astrophysics (e.g., Kollatschny 2004; Negi 2005; Lämmerzahl 2009; Chou et al. 2010; Pasquini et al. 2011; Turyshev 2013). The displacement of metallic lines to the violet observed in the laboratory in comparison with the corresponding solar lines had first been noted by Rowland (1896) and Jewell (1896) (cf. Hentschel 1993a). Measurements of the small gravitational redshift of solar spectral lines are inherently difficult, because many processes in the atmosphere of the Sun can influence the spectrum. In particular, the high speeds of the emitting plasmas lead to line shifts due to the classical Doppler effect (cf. Hentschel 1993b). Nevertheless, early observations confirmed Einstein's prediction in general (St. John 1928; Blamont and Roddier 1961; Brault 1962, 1963; Snider 1970) (cf. Hentschel 1996). Improved observational techniques (e.g., LoPresto et al. 1980; Cacciani et al. 2006; Takeda and Ueno 2012), have established a shift of solar lines of

$$c_0 \frac{\Delta\lambda}{\lambda} \approx 600 \text{ m s}^{-1}, \quad (1)$$

where  $c_0 = 299792458 \text{ m s}^{-1}$  is the speed of light in vacuum remote from any masses and  $\lambda$  the wavelength of the electromagnetic radiation.

The gravitational potential  $U$  at a distance  $r$  from a spherical body with mass  $M$  is constraint in the weak-field approximation for non-relativistic cases (cf. Landau and Lifchitz 1972) by

$$-1 \ll \frac{U}{c_0^2} = -\frac{G_N M}{c_0^2 r} \leq 0, \quad (2)$$

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where  $G_N = 6.67554(16) \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$  is Newton’s constant of gravity (Quinn et al. 2014). A definition of a reference potential in line with Eq. (2) is  $U_0 = 0$  for  $r = \infty$ .

In an attempt to describe the physical process(es) that lead to the gravitational redshift, Wolf et al. (2010) and Müller et al. (2010) disagreed on whether the frequency of an atomic clock is sensitive to the gravitational potential  $U$  (according to Wolf et al.) or, as suggested by Müller et al., to the local gravity field  $\mathbf{g} = \nabla U$ . Support for the first alternative can be found in many publications (e.g., Einstein 1908; von Laue 1920; Schiff 1960; Will 1974; Okun et al. 2000; Sinha and Samuel 2011), but it is, indeed, not obvious how an atom can locally sense the gravitational potential  $U$ .

Many experiments on Earth (Pound and Rebka 1959; Cranshaw et al. 1960; Krause and Lüders 1961; Pound and Snider 1965), in space (Vessot et al. 1980; Bauch and Weyers 2002) and in the Sun-Earth system (St. John 1928; Blamont and Roddier 1961; Brault 1962, 1963; Snider 1972; LoPresto et al. 1991; Cacciani et al. 2006; Takeda and Ueno 2012) have, however, quantitatively confirmed in the static weak field approximation a relative frequency shift of

$$\frac{\nu - \nu_0}{\nu_0} = \frac{\Delta \nu}{\nu_0} \approx \frac{\Delta U}{c_0^2} = \frac{U - U_0}{c_0^2}, \tag{3}$$

where  $\nu_0 = c_0/\lambda_0$  is the frequency of the radiation emitted by a certain transition at  $U_0$  and  $\nu$  the observed frequency there, if the emission caused by the same transition had occurred at a potential  $U$ .

In addition to the redshift, the deflection of light near gravitational centres is of fundamental importance. For a close solar fly-by Soldner (1804) and Einstein (1911) obtained  $0.87''$  under the assumption that radiation would be affected in the same way as matter. *Twice* this value was then derived in the framework of the General Theory of Relativity (GTR, Einstein 1916),<sup>1</sup> and later by Schiff (1960) using the equivalence principle and the Special Theory of Relativity (STR, Einstein 1905a). The high value was confirmed during the total solar eclipse in 1919 for the first time (Dyson et al. 1920). This and later observations have been summarized by Mikhailov (1959) and combined to a mean value of approximately  $2''$ .

## 2 Graviton interactions

A model of gravitational interactions based on a modified impact concept has been proposed for massive bodies (Wilhelm et al. 2013, Paper 1), and the difficulties of the old theory proposed by Nicolas Fatio de Duillier (1949) (cf. Bopp 1929; Gagnebin 1949) have been considered in the

<sup>1</sup>It is of interest in the context of this paper that Einstein employed Huygens’ Principle in his calculation of the deflection.

light of the SRT and the non-local behaviour of virtual particles (cf. Nimtz and Stahlhofen 2008). The basic idea is that impacting *gravitons*—originally named *quadrupoles*—with no mass and a speed  $c_0$  are absorbed by massive particles and re-emitted with reduced energy  $T_G^-$  according to

$$T_G^- = T_G(1 - Y), \tag{4}$$

where  $T_G$  is the energy of a graviton in the background flux and  $0 < Y \ll 1$ . A spherically symmetric emission of a liberated graviton with a reduction parameter  $Y$  had been assumed in Paper 1. Further studies have, however, indicated that an anti-parallel emission with respect to the incoming graviton is more appropriate, because conflicts with the energy and momentum conversation principles in closed systems can be avoided by the second choice (cf. Wilhelm and Dwivedi 2015). Newton’s law of gravitation could be explained with this model, however, a secular mass increase of matter was a consequence of its application. This poses the question of how the interaction of gravity with photons can be understood, since the photon mass is in all likelihood zero.<sup>2</sup> If the mass of a photon is indeed zero, the interaction process must be different. An initial attempt at solving that problem has been made in Wilhelm and Dwivedi (2013, Paper 2) and is summarized here under the assumption of an anti-parallel re-emission of the gravitons, both for massive particles and photons.

A physical process will then be outlined that provides information on the gravitational potential  $U$  at the site of a photon emission. This aspect had not been covered in our earlier paper on the gravitational redshift (Wilhelm and Dwivedi 2014).

Interactions between massive bodies have been treated in Paper 1 with an absorption rate of *half* the intrinsic de Broglie frequency  $mc_0^2/h$  for a mass  $m$  (cf. de Broglie 1923), because *two* virtual gravitons have to be emitted for one interaction, whereas in Paper 2 it is assumed that a photon causes a reflection with an interaction rate of  $\nu = E_\nu/h$  with Planck’s constant  $h$ . The momentum transfer to a photon will thus be twice as high as to a massive body with a mass equivalent to  $E_\nu/c_0^2$ .

If we apply the momentum conservation principle to photon-graviton pairs in the same way as to photons (cf. Landau and Lifchitz 1972), we can write after a reflection of  $\mathbf{p}_G$

$$\mathbf{p}_\nu + 2\mathbf{p}_G = \mathbf{p}_\nu^* \tag{5}$$

with  $|\mathbf{p}_G| = p_G = T_G/c_0$ .

<sup>2</sup>A zero mass of photons follows from the STR and a speed of light in vacuum  $c_0$  constant for all frequencies. Einstein (1905b) used “Lichtquant” for a quantum of electromagnetic radiation; the term “photon” was introduced by Lewis (1926). With various methods the photon mass could be constrained to  $m_\nu < 10^{-49} \text{ kg}$  (Goldhaber and Nieto 1971; Amsler et al. 2008).

We assume, applying Eq. (5) with  $p_G \ll p_v = |\mathbf{p}_v|$ , that under the influence of a gravitational centre relevant interactions occur on opposite sides of a photon with  $p_G$  and  $p_G(1 - Y)$  transferring a net momentum of  $2Yp_G$ . Note, in this context, that the Doppler effect can only operate for interactions of photons with massive bodies (cf. Fermi 1932; Sommerfeld 1978). Consequently, there will be no energy change of the photon, because both gravitons are reflected with constant energies under these conditions, and we can write for a pair of interactions

$$E_v = |\mathbf{p}_v|c = |\mathbf{p}_v + 2Y \mathbf{p}_G|c' = |\mathbf{p}'_v|c' = E'_v, \tag{6}$$

where  $\mathbf{p}'_v$  is the photon momentum after the events. If  $\mathbf{p}_v$  and a component of  $2Y \mathbf{p}_G$  are pointing in the same direction, it is  $c' < c$ , the speed is reduced; an antiparallel direction leads to  $c' > c$ . Note that this could, however, not result in  $c' > c_0$ , because  $c = c_0$  can only be attained in a region with an isotropic distribution of gravitons with a momentum of  $p_G$ , i.e. with a gravitational potential  $U_0 = 0$ .

The momentum  $\mathbf{p}_v$  of a photon radially approaching a gravitational centre will be treated in line with Eq. (6) in Sect. 2 of Paper 2 for massive bodies, however, with twice the interaction rate (valid for photons as explained above). Since we know from observations that the deflection of light during a close fly-by at the Sun is very small—to simplify the calculations, we only treat this configuration—the momentum variation caused by the weak and static gravitational interaction is also very small. The momentum change rate of the photon can then be approximated by

$$\frac{\Delta \mathbf{p}_v}{\Delta t_M} \approx 2G_N M \frac{\hat{\mathbf{r}}}{r^2} \frac{p_v}{c_0}, \tag{7}$$

where  $M$  the mass of the gravitational centre,  $r = |\mathbf{r}|$  the distance of the photon from the centre, and the position vector of the photon is  $r\hat{\mathbf{r}}$  with a unit vector  $\hat{\mathbf{r}}$ . The small deflection angle also allows us to approximate the actual path by a straight line and use  $x \approx c_0 t_M$  along an  $x$  axis. The normalized momentum variation along the trajectory then is

$$-\frac{c_0}{p_v} \left( \frac{\Delta \mathbf{p}_v}{\Delta t_M} \right)_x = \frac{c_0}{p_v} \frac{\Delta p_v}{\Delta t_M} \cos \vartheta \approx 2G_N M \frac{x}{r^3}. \tag{8}$$

The corresponding component perpendicular to the trajectory is

$$-\frac{c_0}{p_v} \left( \frac{\Delta \mathbf{p}_v}{\Delta t_M} \right)_y = \frac{c_0}{p_v} \frac{\Delta p_v}{\Delta t_M} \sin \vartheta \approx 2G_N M \frac{R}{r^3}, \tag{9}$$

where  $R$  is the impact parameter of the trajectory. Integration of Eq. (8) over  $t_M$  from  $-\infty$  to  $x/c_0$  yields

$$\frac{1}{p_v} [d\mathbf{p}_v(r)]_x \approx \frac{2G_N M}{c_0^2 r} = \frac{2G_N M}{c_0^2 \sqrt{R^2 + x^2}}, \tag{10}$$

If we apply Eq. (6) to a photon approaching the mass  $M$  along the  $x$  axis starting from infinity with  $E_v = p_v c_0$ , and considering that the  $y$  component in Eq. (9) is much smaller than the  $x$  component in Eq. (8) for  $x \gg R$ , the photon speed  $c(r)$  as a function of  $r$  can be determined from

$$p_v c_0 \approx \{p_v + [d\mathbf{p}_v(r)]_x\} c(r). \tag{11}$$

Division by  $p_v c_0$  then gives with Eq. (10)

$$\frac{1}{[n_G(r)]_x} = \frac{c(r)}{c_0} \approx 1 - \frac{2G_N M}{c_0^2 r} = 1 + \frac{2U(r)}{c_0^2} \tag{12}$$

as a good approximation of the inverse gravitational index of refraction along the  $x$  axis. The same index has been obtained albeit with different arguments, e.g., by Boonserm et al. (2005) as well as by Ye and Lin (2008). The resulting speed of light is in agreement with evaluations by Schiff (1960), for a radial propagation<sup>3</sup> in a central gravitational field, and Okun (2000)—calculated on the basis of the standard Schwarzschild metric. A decrease of the speed of light near the Sun, consistent with Eq. (12), is not only supported by the predicted and subsequently observed Shapiro delay (Shapiro 1964; Reasenberg et al. 1979; Shapiro et al. 1971; Kramer et al. 2006; Ballmer et al. 2010; Kutschera and Zajiczek 2010), but also indirectly by the deflection of light (Dyson et al. 1920).

### 3 Gravitational redshift

Since Einstein discussed the gravitational redshift and published conflicting statements regarding this effect, the confusion could still not be cleared up consistently (cf., e.g., Mannheim 2006; Sotiriou et al. 2008). In most of his publications Einstein defined clocks as atomic clocks. Initially he assumed that the oscillation of an atom corresponding to a spectral line might be an intra-atomic process, the frequency of which would be determined by the atom alone (Einstein 1908, 1911). Scott (2015) also felt that the equivalence principle and the notion of an ideal clock running independently of acceleration suggest that such clocks are unaffected by gravity. Einstein (1916) later concluded that clocks would slow down near gravitational centres thus causing a redshift.

The question whether the gravitational redshift is caused by the emission process (Case a) or during the transmission phase (Case b) is nevertheless still a matter of recent debates. Proponents of (a) are, e.g., Møller (1957), Desloge (1990), Schiff (1960), Cranshaw et al. (1960), Ohanian (1976), Earman and Glymour (1980), Okun (2000), Okun et al. (2000) and of (b): Hay et al. (1960), Feynman et al. (1995), Straumann (2004), Fließbach (2006), Randall (2006), Will (2006).

<sup>3</sup>Einstein (1912) states explicitly that the speed at a certain location is not dependent on the direction of the propagation.

There is general agreement on the observational and experimental facts and most of the arguments are formally consistent with them, but different physical processes or mathematical concepts are considered. In particular, it is surprising that the same team of experimenters, albeit with different first authors (Cranshaw et al. and Hay et al.) published different views on the process of the Pound–Rebka–Experiment. Pound and Snider (1965) and Pound (2000) pointed out, however, that this experiment could not distinguish between the two options, because the invariance of the velocity of the radiation had not been demonstrated. Bondi (1986) and Dicke (1960) also left the question open. In many cases, the confusion results from the unclear definitions of clocks and times as detailed, for instance, by Ashby and Allan (1979) and Okun (2000).

Einstein (1917) emphasized that for an elementary emission process not only the energy exchange, but also the momentum transfer is of importance (cf., as well Poincaré 1900; Abraham 1903; Fermi 1932). Taking these considerations into account, Wilhelm and Dwivedi (2014) formulated a photon emission process at a gravitational potential  $U$  assuming that:

- (1) The atom cannot sense the potential  $U$ , in line with the original proposal by Einstein (1908, 1911), and initially emits the same energy  $\Delta E_0$  at  $U < 0$  and  $U_0 = 0$ .
- (2) It also cannot directly sense the speed of light at the location with a potential  $U$ . The initial momentum thus is  $p_0 = \Delta E_0/c_0$ .
- (3) As the local speed of light is, however,  $c(U) \neq c_0$ , a photon having an energy of  $\Delta E_0$  and a momentum  $p_0$  is not able to propagate. The necessary adjustments of the photon energy and momentum as well as the corresponding atomic quantities then lead in the *interaction region* to a redshift consistent with  $h\nu = \Delta E_0(1 + U/c_0^2)$  and observations.

As outlined in Sect. 2, there is general agreement in the literature that the local speed of light is

$$c(U) \approx c_0 \left( 1 + \frac{2U}{c_0^2} \right) \quad (13)$$

in line with Eq. (12). It has, however, to be noted that in Sect. 2 the speed  $c(U)$  was obtained for a photon propagating from  $U_0$  to  $U$ , and, therefore, the physical process which controls the speed of newly emitted photons is not established. An attempt to do that will be made in the next section.

#### 4 An aether model with photons as solitons

Before we suggest a specific aether model, a few statements on the aether concept in general should be mentioned. Fol-

lowing the famous experiment of Michelson and Morley (1887), Einstein (1905a, 1908) concluded that the concept of a light aether as carrier of the electric and magnetic forces is not consistent with the STR. In response to critical remarks by Wiechert (1911), cf. Schröder (1990) for Wiechert's support of the aether, von Laue (1912) wrote that the existence of an aether is not a physical, but a philosophical problem, but later differentiated between the physical world and its mathematical formulation. A four-dimensional 'world' is only a valuable mathematical trick; deeper insight, which some people want to see behind it, is not involved (von Laue 1959).

In contrast to his earlier statements, Einstein said at the end of a speech in Leiden that according to the GTR a space without aether cannot be conceived (Einstein 1920); and even more detailed: Thus one could instead of talking about 'aether' as well discuss the 'physical properties of space'. In theoretical physics we cannot do without aether, i.e., a continuum endowed with physical properties (Einstein 1924). Michelson et al. (1928) confessed at a meeting in Pasadena in the presence of H.A. Lorentz that he clings a little to the aether; and Dirac (1951) wrote in a letter to Nature that there are good reasons for postulating an aether.

Wilhelm et al. (2014) proposed an impact model for the electrostatic force based on massless *dipoles*. The vacuum is thought to be permeated by these dipoles that are, in the absence of electromagnetic or gravitational disturbances, oriented and directed randomly propagating along their dipole axis with a speed of  $c_0$ . There is little or no interaction among them. Note that such electric dipoles have no mean interaction energy, even in the classical theory (see, e.g., Jackson 2006). We suggest to identify the dipole distribution with an aether. This is very similar to the conclusion of Preston (1875):

“[...] first, that the normal state of the component particles of the ether is a state of motion; second, that this motion of the particles takes place in straight lines; and third, that this motion takes place towards every possible direction.”

Einstein's aether mentioned above may, however, be more related to the gravitational interactions (cf. Granek 2001). In this case, we have to consider the graviton distribution as another component of the aether.

If we assume that an individual dipole interacts with gravitons in the same way as photons, see Eq. (6), according to

$$E_D = |\mathbf{p}_D|c = |\mathbf{p}_D + 2Y \mathbf{p}_G|c' = |\mathbf{p}'_D|c' = E'_D, \quad (14)$$

where  $E_D$  and  $\mathbf{p}_D$  refer to the energy and momentum of a dipole. We can then modify Eqs. (7) to (11) by changing  $\nu$  to  $D$  and find that Eqs. (12) and (13) are also valid for dipoles with a speed of  $c_0$  for  $U_0 = 0$ . One exception from Preston's

“ether” is that dipoles can, according to a modified Eq. (9), be deflected by graviton interactions.

Considering that many suggestions have been made to describe photons as solitons (e.g., Dirac 1927; Vigier 1991; Kamenov and Slavov 1998; Meulenberg 2013; Bersons 2013; Bersons et al. 2014), we also propose that a photon is a soliton propagating in the dipole aether with a speed of  $c(U)$ , cf., Eq. (13), controlled by the dipoles moving in the direction of propagation of the photon. The dipole distribution thus determines the gravitational index of refraction, cf. Eq. (12), and consequently the speed of light  $c(U)$  at the potential  $U$ . This solves the problem formulated at the end of Sect. 3 and might be relevant for other phenomena, such as gravitational lensing and the cosmological redshift (cf., e.g., Ellis 2010; Chen and Kantowski 2008).

We will further assume that the dipoles constituting a photon will have turned the orientation of their axes to a direction perpendicular to the photon velocity vector. This avoids any electrostatic interactions during emission and absorption processes of photons, and will probably also be required by their polarization effects.

## 5 Discussion and conclusion

Our aim was to identify a physical process that leads to a speed  $c(U)$  of photons controlled by the gravitational potential  $U$ . This could be achieved by postulating an aether model with moving dipoles, in which a gravitational index of refraction  $n_G(U) = c_0/c(U)$  regulates the emission and propagation of photons as required by energy and momentum conservation principles. The emission process thus follows Steps (1) to (3) in Sect. 3, where the local speed of light is given by the gravitational index of refraction  $n$ . In this sense, the statement that an atom cannot detect the potential  $U$  by Müller et al. (2010) is correct; the local gravity field  $g$ , however, is not controlling the emission process.

A photon will be emitted by an atom with appropriate energy and momentum values, because the local speed of light requires an adjustment of the momentum. This occurs in the interaction region between the atom and its environment as outlined in Step (3) of Sect. 3. A receiver of the same type next to the emitter would also not be able to determine the potential either, because the energy and momentum restrictions apply for the absorption process as well.

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