

## Chapter 3

### **Crack propagation at the interface of composite media**

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#### **3.1 Introduction**

Over past many years investigations in fracture theories were restricted in mode-I crack type problems. But in most of the physical and engineering problems loadings are mixed-mode type, which is actually combination of Irwin's modes, arising due to effect of tension and shear. Several mathematical models were proposed to characterize mixed mode cracking for finding the direction of crack extension and critical load etc.

Sih (1974) first proposed the energy based approach, where strain energy density function measures the strength of the elastic energy field near the crack tip. It is shown that the crack will likely to propagate at the crack tip in the radial direction where strain energy density is minimum and then start to propagate when it attains its critical value depending upon the material. After that Erdogan and Sih (1963) proposed a model based on maximum tangential stress to predict the crack propagation and its direction. According to this criteria crack will propagate in the direction along which the tangential stress is maximum after reaching its critical value at function to get approximate solution of energy release rate.

It is shown that the crack will propagate in the radial direction where energy release rate is maximum and attains its critical the crack tip. Later, Palaniswamy (1972) handled the mixed mode fracture problem using stress value. Mixed mode fracture problems had been studied by several investigators like Nuismer (1975), Erdogan and Sih (1963), and

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etc. Recently interfacial Griffith crack propagation problems had been studied by Beom et al. (2001, 2014) and Mukherjee and Das (2007, 2005) etc.

In composite materials, interface plays an important role as it governs the strength of structural components of various components during bonding, which may be weakened due to the presence of cracks or flaws along the bonding (Hutchinson and Suo (1992), Sih (1991)). When a crack is embedded in a body and body is under loading condition, then cracked surfaces may open and slide relative to each other simultaneously. Then crack under these condition are called mixed mode condition. In brittle isotropic homogeneous materials, crack advances in the direction that maintains Mode-I (Opening mode) at the crack tip, whereas in composite materials crack at the interfaces may grow under mixed mode condition. Under mixed mode fracture in ductile material, fracture toughness can be lower under mixed mode or Mode-II loading compared to Mode-I.

It is experimentally verified that under the mixed mode loading, there is a sharp transition of crack growth behavior from Mode-I to Mode-II type. In Mode-II crack grows if the crack opening displacement (COD) dominated by the shear components parallel to the surface of the crack. The correlated crack propagation direction to where the energy release rate is maximum and assumed that crack begins to propagate when the energy release rate reaches some critical value. Lee (2000) showed the crack at the interface of two isotropic materials will propagate when strain energy release rate touches the crack energy. He showed that there will be circular path with the critical stress intensity factor from where the crack will propagate. Mukherjee and Das (2007) showed that path will be elliptic during the propagation of the crack lying at the interface of two orthotropic half planes. But to the best of my knowledge the required

path has not yet been shown by any researcher when the crack starts to propagate at the interface of the two orthotropic strips.

In this chapter the author has made an endeavour to determine the path for the propagation of the crack under mixed mode loading situated at the interface of two orthotropic strips and. The locus of fracture surface are shown for different particular cases through finding the critical crack energy required while plotting the numerical results of strain energy release rate and crack energy.

### 3.2 Formulation of the Problem

Consider an elastodynamic plane problem on an orthotropic elastic strip between two dissimilar orthotropic strips and a crack is moving at the interface of the two materials. It is considered that crack is moving with constant velocity  $c$  along positive X-axis direction without changing its length, as predicted in the model of Yoffe (1951).

In orthotropic media the displacement equations of motion, under the assumption of plane strain are

$$C_{11}^{(i)} \frac{\partial^2 U^{(i)}}{\partial X^2} + C_{66}^{(i)} \frac{\partial^2 U^{(i)}}{\partial Y^2} + (C_{12}^{(i)} + C_{66}^{(i)}) \frac{\partial^2 V^{(i)}}{\partial X \partial Y} = \rho^{(i)} \frac{\partial^2 U^{(i)}}{\partial t^2}, \quad (3.1)$$

$$C_{22}^{(i)} \frac{\partial^2 V^{(i)}}{\partial Y^2} + C_{66}^{(i)} \frac{\partial^2 V^{(i)}}{\partial X^2} + (C_{12}^{(i)} + C_{66}^{(i)}) \frac{\partial^2 U^{(i)}}{\partial X \partial Y} = \rho^{(i)} \frac{\partial^2 V^{(i)}}{\partial t^2}, \quad (3.2)$$

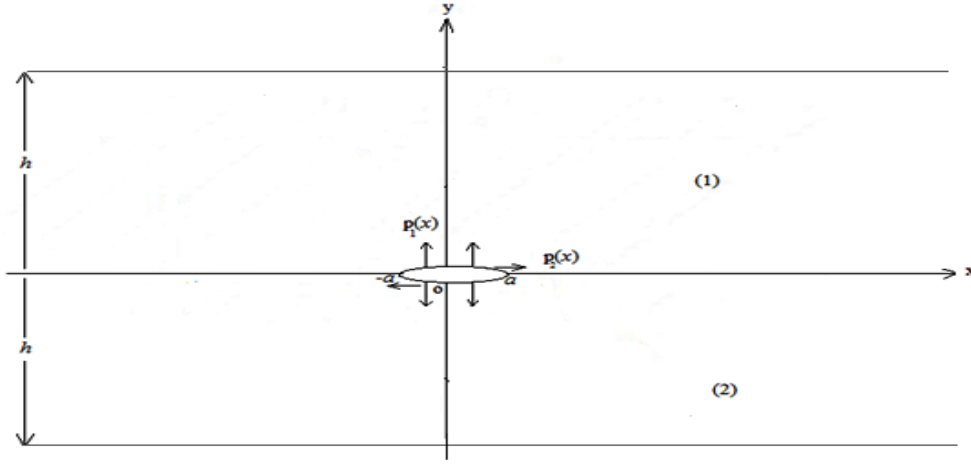
where  $t$ ,  $\rho^{(i)}$ ,  $C_{jk}^{(i)}$  represents time densities and elastic constants of the respective materials and  $i = 1, 2$  indicates for  $0 \leq Y \leq h$  and  $-h \leq Y \leq 0$ , respectively. Using

Galilean transformation,  $x = X - ct$ ,  $y = Y$  and  $t = t$  equations (1) and (2) become

$$(C_{11}^{(i)} - c^2 \rho^{(i)}) \frac{\partial^2 u^{(i)}}{\partial x^2} + C_{66}^{(i)} \frac{\partial^2 u^{(i)}}{\partial y^2} + (C_{12}^{(i)} + C_{66}^{(i)}) \frac{\partial^2 v^{(i)}}{\partial x \partial y} = 0, \quad (3.3)$$

$$(C_{22}^{(i)} - c^2 \rho^{(i)}) \frac{\partial^2 v^{(i)}}{\partial y^2} + C_{66}^{(i)} \frac{\partial^2 v^{(i)}}{\partial x^2} + (C_{12}^{(i)} + C_{66}^{(i)}) \frac{\partial^2 u^{(i)}}{\partial x \partial y} = 0, \quad (3.4)$$

where  $u^{(i)}(x, y) = U(X, Y, t)$ ,  $v^{(i)}(x, y) = V^{(i)}(X, Y, t)$ .



**Fig. 3.1** Geometry of the problem

Now it is considered that the crack defined by  $|x| \leq 1$ ,  $y = \pm 0$  is opened by internal tractions  $p_1(x)$  and  $p_2(x)$  such that it fulfills the symmetry requirements. Then boundary conditions at the interface  $y = 0$  are

$$\sigma_{yy}^{(1)}(x, 0) = -p_1(x), \quad |x| \leq 1, \quad (3.5)$$

$$\sigma_{xy}^{(1)}(x, 0) = -p_2(x), \quad |x| \leq 1, \quad (3.6)$$

$$u^{(1)}(x, 0) = u^{(2)}(x, 0), \quad |x| > 1, \quad (3.7)$$

$$v^{(1)}(x, 0) = v^{(2)}(x, 0), \quad |x| > 1, \quad (3.8)$$

$$\sigma_{yy}^{(1)}(x, 0) = \sigma_{yy}^{(2)}(x, 0), \quad -\infty < x < \infty, \quad (3.9)$$

$$\sigma_{xy}^{(1)}(x, 0) = \sigma_{xy}^{(2)}(x, 0), \quad -\infty < x < \infty. \quad (3.10)$$

Boundary conditions on  $y = \pm h$  are

$$u^{(1)}(x, h) = 0, \quad v^{(1)}(x, h) = 0, \quad -\infty < x < \infty, \quad (3.11)$$

$$u^{(2)}(x, -h) = 0, \quad v^{(2)}(x, -h) = 0, \quad -\infty < x < \infty. \quad (3.12)$$

It is considered that Mack numbers  $M_j^{(i)} = c/v_j^{(i)}$  ( $j=1, 2$ ),

where  $v_j^{(i)} = \sqrt{\frac{C_{11}^{(i)}}{\rho^{(i)}}}$ , ( $i=1,2, j=1,2$ ) are less than one for subsonic propagation.

### 3.3 Solution of the problem

The appropriate integral solutions of equations (3.3) and (3.4) are taken as

$$u^{(i)}(x, y) = \int_0^{\infty} [A_1^{(i)}(s)ch(\gamma_1^{(i)}sy) + A_2^{(i)}(s)ch(\gamma_2^{(i)}sy) + C_1^{(i)}(s)sh(\gamma_1^{(i)}sy) + C_2^{(i)}(s)sh(\gamma_2^{(i)}sy)] \sin sx ds, \quad (3.13)$$

$$v^{(i)}(x, y) = - \int_0^{\infty} \left[ \frac{\alpha_1^{(i)}}{\gamma_1^{(i)}} A_1^{(i)}(s)sh(\gamma_1^{(i)}sy) + \frac{\alpha_2^{(i)}}{\gamma_2^{(i)}} A_2^{(i)}(s)sh(\gamma_2^{(i)}sy) + \frac{\alpha_1^{(i)}}{\gamma_1^{(i)}} C_1^{(i)}(s)ch(\gamma_1^{(i)}sy) + \frac{\alpha_2^{(i)}}{\gamma_2^{(i)}} C_2^{(i)}(s)ch(\gamma_2^{(i)}sy) \right] \cos sx ds, \quad (3.14)$$

where  $\gamma_1^{(i)}$  and  $\gamma_2^{(i)} (< \gamma_1^{(i)})$ ,  $i=1,2$  are the positive roots of the equation

$$C_{66}^{(i)}C_{22}^{(i)}\gamma^4 + [(C_{12}^{(i)} + C_{66}^{(i)})^2 - C_{22}^{(i)}(C_{11}^{(i)} - c^2\rho^{(i)}) - C_{66}^{(i)}(C_{66}^{(i)} - c^2\rho^{(i)})]\gamma^2 + (C_{11}^{(i)} - c^2\rho^{(i)})(C_{66}^{(i)} - c^2\rho^{(i)}) = 0, \quad (3.15)$$

$$\text{with } \alpha_j^{(i)} = (C_{11}^{(i)} - c^2\rho^{(i)} - (\gamma_j^{(i)})^2 C_{66}^{(i)}) / (C_{12}^{(i)} + C_{66}^{(i)}), \quad i=1,2, \quad j=1,2. \quad (3.16)$$

Equations (3.5) and (3.6), with the aid of boundary conditions (3.7) - (3.12) give rise to following integral equations

$$\phi_l(x) + \frac{1}{\pi i e r_l} \int_{-1}^1 \frac{\phi_l(t)}{t-x} dt + \int_{-1}^1 [K_{l1}^*(x,t)\phi_1(t)dt + K_{l2}^*(x,t)\phi_2(t)]dt = -g_l(x), \quad -1 \leq x \leq 1, \quad l=1,2, \quad (3.17)$$

where the unknown functions  $f_i(x)$  are satisfying the conditions

$$\int_{-1}^1 f_i(t)dt = 0, \quad i=1, 2. \quad (3.18)$$

$$\text{with } \phi_l(x) = \sqrt{a_1 b_1} f_1(x) + i r_l \sqrt{c_1 d_1} f_2(x), \quad l=1, 2.$$

$$\varepsilon_l = \sqrt{a_1 b_1 c_1 d_1}, \text{ and } r_l = (-1)^l, \quad l = 1, 2.$$

$$g_l(x) = \frac{2}{\pi} \left( \sqrt{b_1/a_1} p_1(x) + i r_l \sqrt{d_1/c_1} p_2(x) \right),$$

$$2\pi K_{lm}^* = \left( \frac{1}{a_1} K_{11}(x,t) + r_l r_m \frac{1}{c_1} K_{22}(x,t) \right) + i \varepsilon_l^{-1} (r_l d_1 K_{21}(x,t) - r_m b_1 K_{12}(x,t)), \quad l, m = 1, 2,$$

where

$$a_1 = -[\eta_1^{(1)} W_{22}(\infty) - \eta_2^{(1)} W_{21}(\infty)], \quad c_1 = -[\mu_1^{(1)} W_{12}(\infty) - \mu_2^{(1)} W_{11}(\infty)],$$

$$\frac{1}{b_1} = [\eta_1^{(1)} W_{12}(\infty) - \eta_2^{(1)} W_{11}(\infty)], \quad \frac{1}{d_1} = [\mu_1^{(1)} W_{22}(\infty) - \mu_2^{(1)} W_{21}(\infty)],$$

$$K_{ij}(x,t) = \int_0^\infty d_{ij}(s) \cos s(t-x) ds, \quad \text{if } i = j$$

$$= \int_0^\infty d_{ij}(s) \sin s(t-x) ds, \quad \text{if } i \neq j, \quad i, j = 1, 2.$$

$$d_{11}(s) = -\eta_1^{(1)} W_{22}^{(1)}(s) - \eta_1^{(1)} \delta_1^{(1)}(s) W_{22}(s) + \eta_2^{(1)} \delta_3^{(1)}(s) W_{22}(s) + \eta_2^{(1)} W_{21}^{(1)}(s)$$

$$+ \eta_1^{(1)} \delta_2^{(1)}(s) W_{21}(s) - \eta_2^{(1)} \delta_4^{(1)}(s) W_{21}(s),$$

$$d_{12}(s) = \eta_1^{(1)} W_{12}^{(1)}(s) + \eta_1^{(1)} \delta_1^{(1)}(s) W_{12}(s) - \eta_2^{(1)} \delta_3^{(1)}(s) W_{12}(s) - \eta_2^{(1)} W_{11}^{(1)}(s)$$

$$- \eta_1^{(1)} \delta_2^{(1)}(s) W_{11}(s) + \eta_2^{(1)} \delta_4^{(1)}(s) W_{11}(s),$$

$$d_{21}(s) = -\mu_1^{(1)} W_{22}^{(1)}(s) + \mu_2^{(1)} W_{21}^{(1)}(s), \quad d_{22}(s) = -\mu_1^{(1)} W_{12}^{(1)}(s) + \mu_2^{(1)} W_{11}^{(1)}(s),$$

where

$$W_{ij}(s) = \omega_{ij}(s) / [\omega_{11}(s)\omega_{22}(s) - \omega_{12}(s)\omega_{21}(s)], \quad i, j = 1, 2.$$

$$W_{ij}(\infty) = \lim_{s \rightarrow \infty} W_{ij}(s).$$

$$\omega_{11}(s) = -[1 + \delta_1^{(1)}(s) - \delta_3^{(1)}(s)] + \{[1 + \delta_1^{(2)}(s)]\{\mu_2^{(2)}\omega_1^{(1)}(s) - \mu_1^{(1)}\omega_2^{(2)}(s)\}$$

$$- [1 + \delta_2^{(2)}(s) - \delta_4^{(2)}(s)] + \{\mu_1^{(2)}\omega_1^{(1)}(s) - \mu_1^{(1)}\omega_1^{(2)}(s)\} / [\mu_1^{(2)}\omega_2^{(2)}(s) - \mu_2^{(2)}\omega_1^{(2)}(s)],$$

$$\omega_{12}(s) = -[1 + \delta_2^{(1)}(s) - \delta_4^{(1)}(s)] + \{[1 + \delta_1^{(2)}(s) - \delta_3^{(2)}(s)]\{\mu_2^{(2)}\omega_2^{(1)}(s) + \mu_2^{(1)}\omega_2^{(2)}(s)\}$$

$$\begin{aligned}
 & -\{1+\delta_2^{(2)}(s)-\delta_4^{(2)}(s)\}+\{\mu_1^{(2)}\omega_2^{(1)}(s)-\mu_2^{(1)}\omega_1^{(2)}(s)\}/[\mu_1^{(2)}\omega_2^{(2)}(s)-\mu_2^{(2)}\omega_1^{(2)}(s)], \\
 \omega_{21}(s) &= \frac{\alpha_1^{(1)}}{\gamma_1^{(1)}} + \frac{\alpha_1^{(2)}}{\gamma_1^{(2)}} \frac{\mu_2^{(2)}\omega_1^{(1)}(s) - \mu_1^{(1)}\omega_2^{(2)}(s)}{\mu_1^{(2)}\omega_2^{(2)}(s) - \mu_2^{(2)}\omega_1^{(2)}(s)} - \frac{\alpha_2^{(2)}}{\gamma_2^{(2)}} \frac{\mu_1^{(2)}\omega_1^{(1)}(s) - \mu_1^{(1)}\omega_1^{(2)}(s)}{\mu_1^{(2)}\omega_2^{(2)}(s) - \mu_2^{(2)}\omega_1^{(2)}(s)}, \\
 \omega_{22}(s) &= \frac{\alpha_2^{(1)}}{\gamma_2^{(1)}} + \frac{\alpha_1^{(2)}}{\gamma_1^{(2)}} \frac{\mu_2^{(2)}\omega_2^{(1)}(s) + \mu_2^{(1)}\omega_2^{(2)}(s)}{\mu_1^{(2)}\omega_2^{(2)}(s) - \mu_2^{(2)}\omega_1^{(2)}(s)} + \frac{\alpha_2^{(2)}}{\gamma_2^{(2)}} \frac{\mu_1^{(2)}\omega_2^{(1)}(s) + \mu_2^{(1)}\omega_1^{(2)}(s)}{\mu_1^{(2)}\omega_2^{(2)}(s) - \mu_2^{(2)}\omega_1^{(2)}(s)}, \\
 \delta_1^{(i)} &= 2\left\{\left(\frac{\alpha_1^{(i)}}{\gamma_1^{(i)}} + \frac{\alpha_2^{(i)}}{\gamma_2^{(i)}}\right)e^{-2\gamma_1^{(i)}sh} + \left(\frac{\alpha_1^{(i)}}{\gamma_1^{(i)}} - \frac{\alpha_2^{(i)}}{\gamma_2^{(i)}}\right)e^{-2(\gamma_1^{(i)}+\gamma_2^{(i)})sh}\right\}/D^{(i)}, \\
 \delta_2^{(i)} &= 4\frac{\alpha_2^{(i)}}{\gamma_2^{(i)}}e^{-(\gamma_1^{(i)}+\gamma_2^{(i)})sh}/D^{(i)}, \quad \delta_3^{(i)} = \frac{4\alpha_1^{(i)}}{\gamma_1^{(i)}}e^{-(\gamma_1^{(i)}+\gamma_2^{(i)})sh}/D^{(i)}, \\
 \delta_4^{(i)} &= 2\left\{\left(\frac{\alpha_1^{(i)}}{\gamma_1^{(i)}} + \frac{\alpha_2^{(i)}}{\gamma_2^{(i)}}\right)e^{-2\gamma_2^{(i)}sh} - \left(\frac{\alpha_1^{(i)}}{\gamma_1^{(i)}} - \frac{\alpha_2^{(i)}}{\gamma_2^{(i)}}\right)e^{-2(\gamma_1^{(i)}+\gamma_2^{(i)})sh}\right\}/D^{(i)}, \\
 D^{(i)} &= \left(\frac{\alpha_1^{(i)}}{\gamma_1^{(i)}} - \frac{\alpha_2^{(i)}}{\gamma_2^{(i)}}\right)\{1 - e^{-2(\gamma_1^{(i)}+\gamma_2^{(i)})sh} - \left(\frac{\alpha_1^{(i)}}{\gamma_1^{(i)}} + \frac{\alpha_2^{(i)}}{\gamma_2^{(i)}}\right)\{e^{-2\gamma_1^{(i)}sh} - e^{-2\gamma_2^{(i)}sh}\}, \quad i=1, 2. \tag{3.19}
 \end{aligned}$$

$$\eta_j^{(i)} = C_{12}^{(i)} - C_{22}^{(i)}\alpha_j^{(i)}, \quad \mu_j^{(i)} = C_{66}^{(i)}\frac{\beta_j^{(i)}}{\gamma_j^{(i)}}, \quad i, j = 1, 2. \tag{3.20}$$

Taking the solution of the integral equation (3.17) as

$$\phi_l(x) = \omega_l(x) \sum_{n=0}^{\infty} C_{ln} P_n^{(\alpha_l, \beta_l)}(x), \quad l = 1, 2, \tag{3.21}$$

where  $\omega_l(x) = (1-x)^{\alpha_l}(1+x)^{\beta_l}$ ,  $\alpha_l = -\frac{1}{2} + i\omega_l$ ,  $\beta_l = -\frac{1}{2} - i\omega_l$ ,  $\omega_l = r_l\omega$ ,

$$\omega = \frac{1}{2\pi} \ln \left| \frac{1+\varepsilon}{1-\varepsilon} \right|,$$

the integral equation (17) reduces to

$$\frac{\sqrt{1-\varepsilon^2}}{2i\varepsilon r_l} C_{lj+1} \theta_j^{(-\alpha_k-\beta_k)} + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} C_{mn} L_{lmj}^* = -F_{lj}, \tag{3.22}$$

$$\text{where } \theta_j^{(\alpha, \beta)} = \frac{2^{\alpha+\beta+1} \Gamma(j+\alpha+1) \Gamma(j+\beta+1)}{j! (2j+\alpha+\beta+1) \Gamma(j+\alpha+\beta+1)},$$

$$L_{lmj}^* = \int_{-1}^1 L_{lmn}(x) \omega_l^{-1}(x) P_j^{(-\alpha_k, -\beta_k)}(x) dx,$$

$$L_{lmn}(x) = \int_{-1}^1 K_{lm}^*(x, t) \omega_m(t) P_n^{(\alpha_m, \beta_m)}(t) dt,$$

$$F_{lj}(x) = \int_{-1}^1 g_l(x) \omega_l^{-1}(x) P_j^{(-\alpha_l, -\beta_l)}(x) d(x), \quad l = 1, 2, \quad j = 0, 1, 2.$$

The stress intensity factors near the crack tip at  $x=1$  are calculated as

$$\sqrt{b_1/a_1} K_I + i r_l \sqrt{d_1/c_1} K_{II} = \frac{-i\pi\sqrt{1-\varepsilon^2}}{2\varepsilon r_l} \sum_{n=1}^{\infty} C_{ln} P_n^{(\alpha_l, \beta_l)}(1), \quad l = 1, 2. \quad (3.23)$$

The strain energy release rate is given as

$$\frac{G}{p^2} = \frac{\varepsilon}{2} \left[ \frac{b_1}{a_1} K_I^2 + \frac{d_1}{c_1} K_{II}^2 \right]. \quad (3.24)$$

Crack energy is calculated as

$$\begin{aligned} W &= - \int_{-1}^1 p_1(x) [v^{(1)}(x, 0) - v^{(2)}(x, 0)] dx \\ &= \frac{\pi p}{4i\sqrt{c_1 d_1}} \left( \frac{1}{4} + \omega^2 \right) \frac{(C_{11} - C_{21})}{\cosh \pi \omega}. \end{aligned} \quad (3.25)$$

### 3.4 Results and discussion

In this section the orthotropic materials 1 and 2 are considered as  $\alpha$ -Uranium and Beryllium respectively. The stress intensity factors, strain energy release rate at the crack tip  $x=1$  and the crack energy are calculated for subsonic propagation.

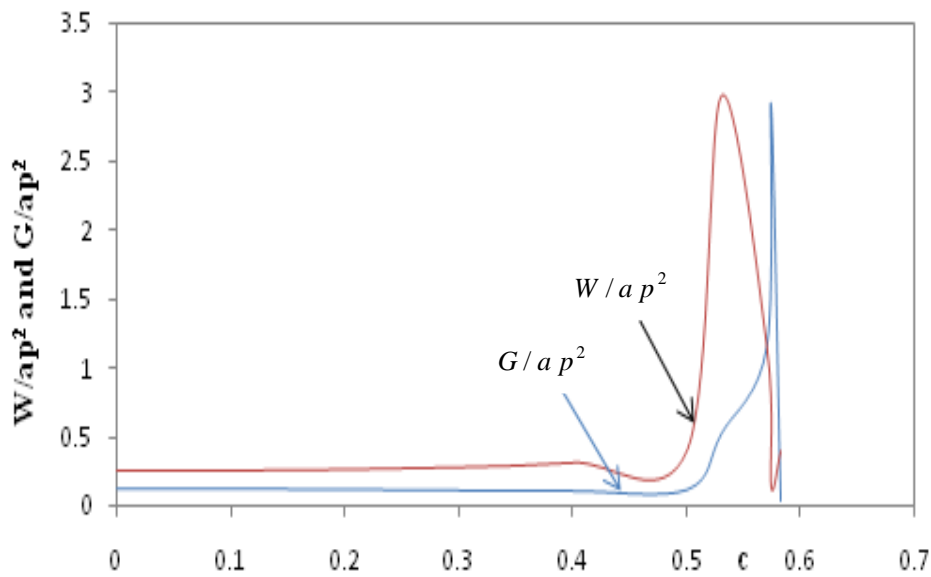
It is seen from Fig. 3.2 that at  $h=5$ , initially the amplitude of the strain energy release rate  $G/a p^2$  is less than that of crack energy  $W/a p^2$  and both the curves intersect at

$c = 0.57351$  (the Mach numbers at this crack velocity are

$M_1^{(1)} = 0.540511, M_2^{(1)} = 0.918812, M_1^{(2)} = 0.1371, M_2^{(2)} = 0.229508$ ) and after that it



oscillates. Similar oscillation phenomenon was observed by England (1965), William (1959), Sneddon and Lowengrub (1969), Das and Patra (1998,1999). As we know that the fracture occurs when  $G \geq W$ . Thus the fracture condition in mixed mode loading problem is given by  $\left(\frac{K_I}{K_{IC}}\right)^2 + \left(\frac{K_{II}}{K_{IIC}}\right)^2 = 1$ , which implies that fractured surface, i.e., the locus of fracture is elliptic (see Fig. 3.6), where  $K_{IC} = \sqrt{W_C a_1 / b_1}$ ,  $K_{IIC} = \sqrt{W_C c_1 / d_1}$  are critical stress intensity factors,  $W_C$  is critical crack energy. Similar natures are found for  $h=10, 15$  and  $20$  and at the values of  $c = 0.57351$  where the curves of  $G$  and  $W$  intersects (Figs.3.3 - 3.5). The elliptic paths corresponding to various depths of the strips ( $h$ ) are depicted through Fig. 3.6 The interesting observation from Fig. 3.6 is as  $h$  increases the semi major and semi minor axes of elliptic paths increase which means that the values of critical stress intensity factors increases when the depths of the strips increases. Thus the propagation tendency of an interface crack will be less as the depth of the composite medium becomes higher.



**Fig. 3.2** Plots of  $W / a p^2$  and  $G / a p^2$  versus  $c$  for  $h=5$

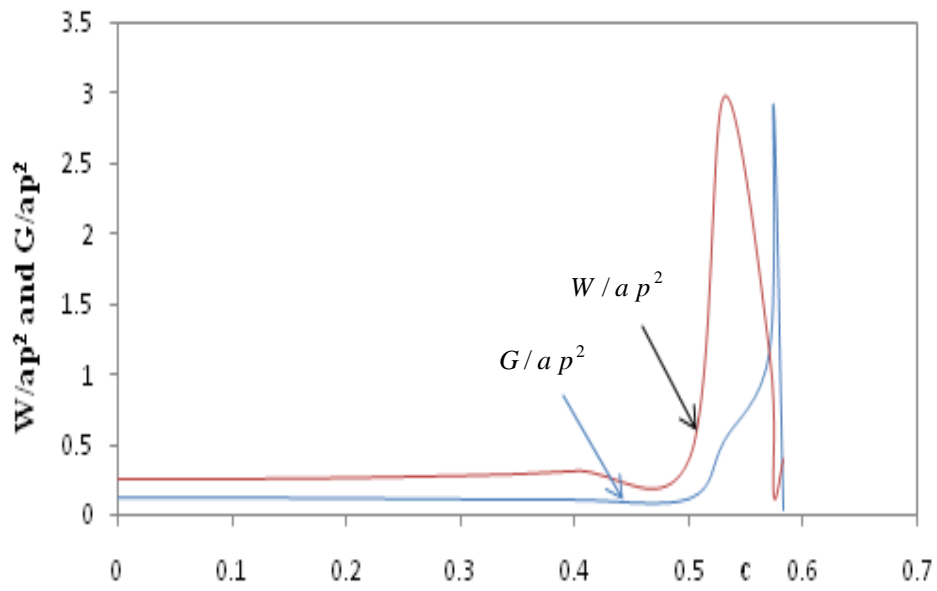


Fig. 3.3 Plots of  $W/a p^2$  and  $G/a p^2$  versus  $c$  for  $h=10$

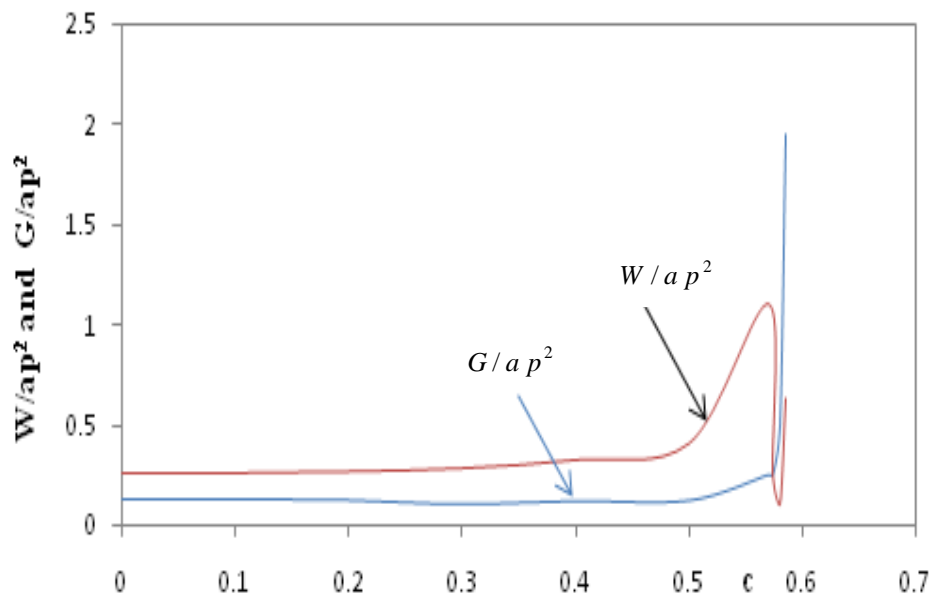


Fig. 3.4 Plots of  $W/a p^2$  and  $G/a p^2$  versus  $c$  for  $h=15$

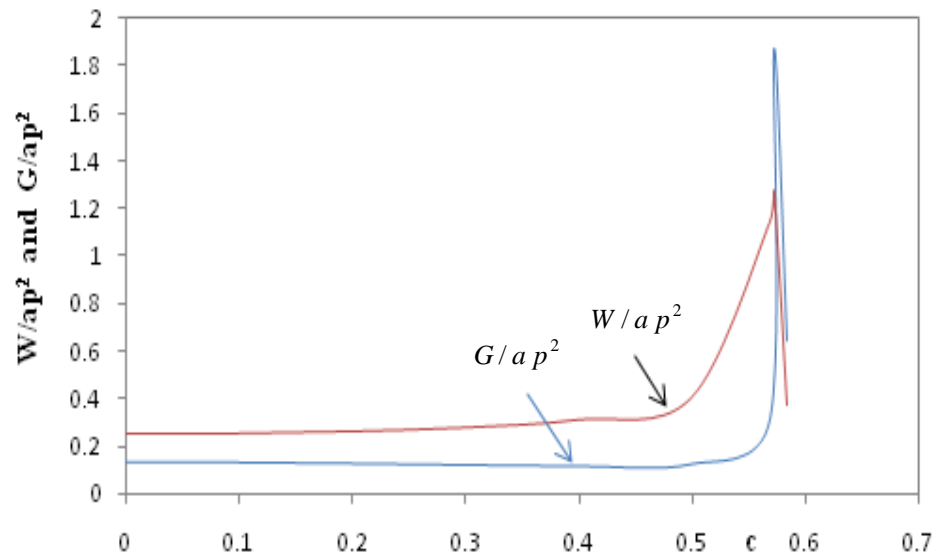


Fig. 3.5 Plots of  $W/a p^2$  and  $G/a p^2$  versus  $c$  for  $h=20$

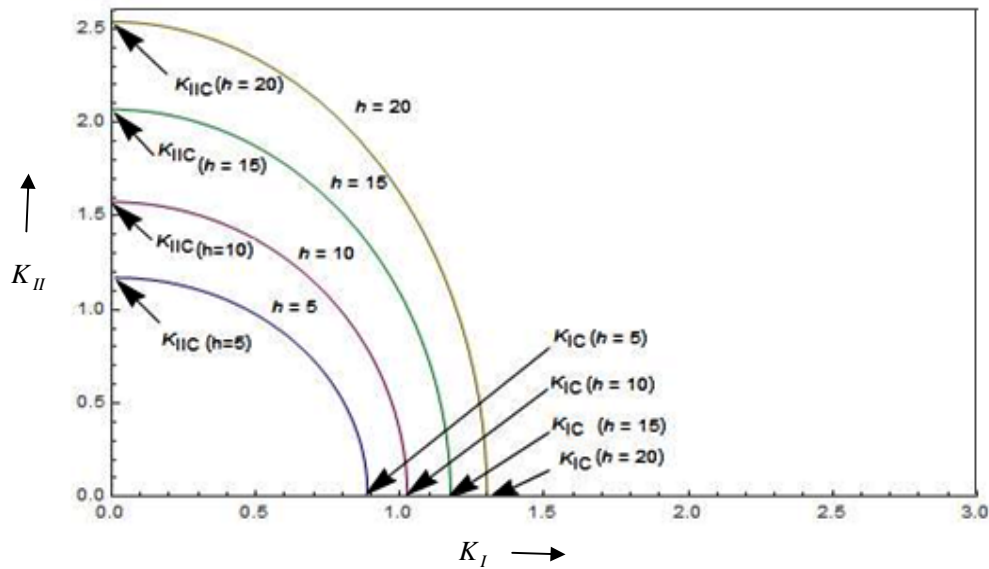


Fig. 3.6 Plots for combined mode fracture surfaces for various values of  $h$

### 3.5 Conclusion

Main aim of this chapter is to study the locus of the fractured surface of a moving crack under mixed mode loading situated at the interface of two orthotropic strips of equal height and limiting values of parameters. First achievement of this study is finding the expression of critical value of SIFs, crack energy and energy release rate for the

interfacial crack. Second one is the graphical presentation of crack energy and energy release rate for different particular cases. Third one is the path of the fractured surface for different particular cases. The given mathematical model is very much useful to predict the critical path for fractured surface and also helps to resist damage of the cracked orthotropic composite medium through increasing the height of the strips. The author is optimist that the proposed mathematical model will be beneficial for the engineers working in the field of composite medium.