

Contents		Page No.
Acknowledgements		xi
Contents		xiii
List of Figures		xix
Preface		xxv
Chapter 1	Introduction	01
§1.1	Fractional calculus	01
	▪ 1.1.1 Brief history	01
	▪ 1.1.2 Applications	03
	▪ 1.1.3 Fractional derivative	07
	▪ 1.1.4 Leibniz rule	11
	▪ 1.1.5 Fractional differential equation	12
§1.2	Dynamical system	13
	▪ 1.2.1 Classification of dynamical systems	14
§1.3	Chaos	15
	▪ 1.3.1 Definition of chaos	16
	▪ 1.3.2 Attractor and strange attractor	17
	▪ 1.3.3 Chaos in differential equations	17
	▪ 1.3.4 Chaos in fractional order systems	21
	▪ 1.3.5 Asymptotic stability of commensurate fractional order system	23
§1.4	Synchronization	24
	▪ 1.4.1 Types of synchronization	25
§1.5	Methods for synchronization	33

	<ul style="list-style-type: none"> ▪ 1.5.1 Active control method ▪ 1.5.2 Nonlinear control method ▪ 1.5.3 Backstepping method 	<p>34</p> <p>35</p> <p>37</p>
§1.6	Delay differential equations	37
	<ul style="list-style-type: none"> ▪ 1.6.1 Existence and uniqueness of a solution: Method of steps ▪ 1.6.2 Numerical solution of a DDE ▪ 1.6.3 Fractional delay differential equations 	<p>39</p> <p>40</p> <p>41</p>
Chapter 2	Synchronization between fractional order complex chaotic systems	45
§2.1	Introduction	45
§2.2	Problem formulation and synchronization	47
§2.3	Systems' descriptions	49
	<ul style="list-style-type: none"> ▪ 2.3.1 The fractional order complex Lorenz system ▪ 2.3.2 The fractional order complex Lu system ▪ 2.3.3 The fractional order complex T system 	<p>49</p> <p>53</p> <p>57</p>
§2.4	Synchronization between fractional order complex Lorenz and Lu systems	61
	<ul style="list-style-type: none"> ▪ 2.4.1 Numerical simulation and results 	67
§2.5	Synchronization between fractional order complex Lu and T systems	68
	<ul style="list-style-type: none"> ▪ 2.5.1 Numerical simulation and results 	74
§2.6	Synchronization between fractional order complex Lorenz and T systems	74
	<ul style="list-style-type: none"> ▪ 2.6.1 Numerical simulation and results 	79

§2.7	Conclusion	79
Chapter 3	Synchronization between fractional order complex chaotic systems with uncertainty	81
§3.1	Introduction	81
§3.2	Problem formulation	84
	▪ 3.2.1 Active control method	85
§3.3	Systems' descriptions	86
	▪ 3.3.1 The fractional order complex Lorenz system with uncertainty	86
	▪ 3.3.2 The fractional order complex T system with uncertainty	90
§3.4	Synchronization of Lorenz and T systems via active control method	94
§3.5	Numerical simulation and results	99
§3.6	Conclusion	99
Chapter 4	Comparative study of synchronization methods of fractional order chaotic systems	101
§4.1	Introduction	101
§4.2	Systems' descriptions	103
	▪ 4.2.1 Fractional order Chen system	103
	▪ 4.2.2 Fractional order Qi system	104
§4.3	Synchronization of fractional order Chen and Qi systems using active control method	105

§4.4	Synchronization of fractional order Chen and Qi systems using backstepping approach	110
§4.5	Numerical simulation and results	117
§4.6	Conclusion	117
Chapter 5	Synchronization of time-delay chaotic systems with uncertainties and external disturbances	119
§5.1	Introduction	119
§5.2	Problem statement	121
§5.3	Systems' descriptions	121
	▪ 5.3.1 Time-delay advanced Lorenz system	121
	▪ 5.3.2 Double time-delay Rossler system	124
§5.4	Synchronization using active control method	125
§5.5	Simulation results	130
§5.6	Concluding remarks	130
Chapter 6	Nonlinear control technique for dual combination synchronization of complex chaotic systems	131
§6.1	Introduction	131
§6.2	The scheme for dual combination synchronization	133
§6.3	Stability analysis	135
§6.4	Systems' descriptions	137
	▪ 6.4.1 Complex Lorenz system	137
	▪ 6.4.2 Complex Lu systems	139

	<ul style="list-style-type: none"> ▪ 6.4.3 Complex T system ▪ 6.4.4 Complex Chen system ▪ 6.4.5 Complex two coupled dynamos system ▪ 6.4.6 Nonlinear complex chaotic system 	<p>141</p> <p>143</p> <p>145</p> <p>147</p>
§6.5	Illustration of the scheme	149
§6.6	Simulation results and discussion	156
§6.7	Conclusion	157
Chapter 7	Dual combination synchronization of the fractional order complex chaotic systems	159
§7.1	Introduction	159
§7.2	The scheme for dual combination synchronization	162
§7.3	Stability analysis	166
§7.4	Systems' descriptions	167
	<ul style="list-style-type: none"> ▪ 7.4.1 The fractional order complex Lorenz system ▪ 7.4.2 The fractional order complex T system 	<p>167</p> <p>171</p>
§7.5	Illustration of the scheme	175
§7.6	Simulation results and discussion	185
§7.6	Conclusion	186
Bibliography		187
List of Publications		201