

Chapter 5 : A PDE-BASED GENERAL FRAMEWORK ADAPTED TO RAYLEIGH'S, RICIAN'S AND GAUSSIAN'S DISTRIBUTED NOISE FOR RESTORATION AND ENHANCEMENT OF MAGNETIC RESONANCE IMAGES

In this chapter, a partial differential equation-based general framework adapted to Rayleigh's, Rician's and Gaussian's distributed noise for restoration and enhancement of magnetic resonance image is presented. The proposed framework is obtained by casting the noise removal problem into a variational framework. This framework automatically identifies various types of noise present in the MRI and filters them by choosing an appropriate filter. This filter consists of two terms wherein the first term is a data likelihood term and the second term is a prior function. The first term is obtained by minimizing the negative log likelihood of the corresponding pdfs: Gaussian or Rayleigh or Rician. Further, due to the ill-posedness of the likelihood term, a prior function is needed. This chapter examines three PDE based priors which include total variation (TV) based prior, anisotropic diffusion (AD) based prior and a complex diffusion (CD) based prior. A regularization parameter is used to balance the trade off between data fidelity term and prior. The finite difference scheme is used for discretization of the proposed method. The performance analysis and comparative study of the proposed method with other standard methods is presented for Brain Web dataset at varying noise levels in terms of PSNR, MSE, SSIM, and CP. From the simulation results, it is observed that the proposed framework with CD based prior is performing better in comparison to other methods and priors in consideration.

5.1. Introduction

Objective of this chapter is to present the design and development of a general framework for restoration and enhancement of MR Image. In MR image Rician noise is

one of the prominent noise, however Gaussian and Rayleigh noise are also present. These type of noises in the MRI can be identified by measuring SNR value of image data. In the literature, variety of methods have been developed for MRI de-noising, but first time Henkelman [32] presented a method to estimate the noiseless magnitude MR image from its noisy version. The key importance is the estimation of noise variance from magnetic resonance images (MRI) often as an input parameter for image post processing tasks. The quality measurement of the MR data is done by estimated noise variance. Like noise reduction, segmentation and parameter estimation or clustering [113], the noise variance is a crucial parameter in image processing algorithms.

In the literature, for estimation of the noise level from MRI several methods have been proposed like filtering approach, Transform domain approach and Statistical approach. Examples of filtering approach linear filtering such as spatial filter [88] and temporal filter [88]. Non-linear filtering such as anisotropic diffusion filter (ADF) [37], adaptive ADF filter [89], noise driven ADF filter [90], noise adaptive ADF filter, fourth order PDE filter [91], adaptive 4th order PDE filter [38], 4th order complex PDE filters [92], non local means filter (NLM) [39], fast NLM filter [93], Block wise optimized NLM filter [52], Unbiased NLM filter [19], dynamic NLM filter [94], enhanced NLM filter [54], adaptive NLM filter [40]. Combination of domain and range filters [41], bilateral domain and range filters [42], trilateral domain and range filters [42].

Examples of Transform domain [66] approaches are curvelet [95], contourlet [69] and wavelet [96], adaptive multiscale product thresholding [97], multiwavelet [98], undecimated wavelet [99]. Examples of Statistical approach are Maximum likelihood estimation [74, 18], Linear minimum mean square error estimation [18], Phase error estimation [72], Nonparametric estimation [80], Singularity function analysis [53, 54] were presented. Some other Rician noise removal of MRI approaches were proposed in

literature such as machine learning-based approaches [100,101,102,103,104,105], DCT-based filter [106], PCA-based technique [107], and conventional approaches [71].

5.2. Related work

From the background area of the magnitude MR image many methods named Rayleigh distributed [80,76] have been proposed earlier for the estimation of noise level. Unfortunately, these methods proved to be useless for images because of unavailability of background information. Other than the brain for MR images background data may not be available in Cardiac or lung images. Noise assumption based on Rayleigh distribution is failed in the case which is small like the field of view (FAV) [76] Most of the noisy background is also eliminated by new scanning techniques and software. These techniques may also affects the methods based on Rayleigh model which require certain amount of background pixels for proper estimation of noise level [21]. Zero mean Gaussian probability density function (PDF) illustrates the raw complex MR data acquired in the Fourier domain. The linearity and the orthogonality of the Fourier transform may cause Gaussian for the noise distribution in the real and imaginary components, after the inverse Fourier transforms. However, due to the subsequent transform to a magnitude image, the noise distribution will be no longer Gaussian but Rician distributed. If I is the original signal amplitude, then the PDF of the reconstructed magnitude image M will be:

$$p(I/M) = \frac{M}{\sigma^2} \exp\left(-\frac{M^2 + I^2}{2\sigma^2}\right) J_0\left(\frac{IM}{\sigma^2}\right) \in(M) \quad (5.1)$$

where I denotes amplitude of a noise-free image, σ^2 denotes the Gaussian noise variance, $J_0(\cdot)$ show that modified zero order Bessel function. $\in(\cdot)$ is the unit step Heaviside function, and M is the magnitude MR image. The Rician PDF is only valid

for nonnegative values of M [18]. In the image background, where the SNR is low (SNR ≈ 0), the Rician PDF reduces to a Rayleigh distribution [21] with PDF:

$$p(I / M) = \frac{M}{\sigma^2} \exp\left(-\frac{I^2}{2\sigma^2}\right) \in (M) \quad (5.2)$$

When SNR is high (greater than 3dB), then the Rician distribution becomes Gaussian distribution [21] with mean $\sqrt{I^2 + \sigma^2}$ and variance σ^2 given as follows:

$$p(I / M) = \frac{1}{\sqrt{2\pi}\sigma^2} \exp\left(-\frac{M^2 - \sqrt{I^2 + \sigma^2}}{2\sigma^2}\right) \in (M) \quad (5.3)$$

For the estimation of noise variance a method based on the local computation of the skewness of the magnitude data distribution is proposed by Rajan *et al.*, [76]. It is to be concluded that Rician distribution are always in between the moments of Rayleigh and Gaussian distributions. The relationship between σ^2 and the variance of a Rician distribution σ_R^2 at low and high SNR can be written as

$$\sigma^2 = \sigma_R^2 \left(2 - \frac{\pi}{2}\right)^{-1} \quad (5.4)$$

and $\sigma^2 = \sigma_R^2$, (5.5)

respectively. In general, σ^2 in terms of σ_R^2 can be written as

$$\sigma^2 = \sigma_R^2 \times \psi, \quad (5.6)$$

where ψ is a correction factor in the range $[1; \left(2 - \frac{\pi}{2}\right)^{-1}]$, i.e. when the Rician distribution approaches a Rayleigh distribution (at low SNR), the correction factor tends to $\left(2 - \frac{\pi}{2}\right)^{-1}$ and when the Rician distribution approaches a Gaussian (at high SNR), the correction factor tends to 1.

In view of above discussion and limitations of the existing method such as non capability of removal of this type of noise and lower restoration accuracy, in this paper

we propose a PDE based general framework for restoration and enhancement of MR data. The propose method is capable of removing all possible type of noise that may be present in MR data.

5.3. Method and Models

The Rician, Rayleigh, and Gaussian noise removal and regularization of MR image data is obtained by minimizing the following nonlinear energy functional of the image I within a continuous domain Ω , using the variational framework [112]:

$$E(I) = \arg \min_{\Omega} \left\{ \int_{\Omega} [L(p(I/M)) + \lambda \cdot \phi(\|\nabla I\|)] d\Omega \right\} \quad (5.7)$$

Where $L(p(I/M))$ shows the negative likelihood term of Rician or Rayleigh or Gaussian distributed noise in MRI, given by equation (5.1-5.2-5.3). During the filtering process log likelihood term measures the dissimilarities at a pixel between M and its estimated value I . $L(p(I/M))$ acts as the data attachment term or the likelihood term in equation (5.7).

Maximization of log likelihood or minimization of the negative log likelihood leads to de-noising of image data, but is an ill-posed problem and hence regularization is needed. That's why the second term $\phi(\|\nabla I\|)$ in equation (5.7) is needed and it acts as a regularization or penalty function or prior term. In the equation (5.7), λ is a regularization parameter, which has a constant value and makes a balance between the data attachment term and regularization function. The value of λ has been determined experimentally and is set to a value for which peak signal to noise ratio is maximum during the iteration process of filtration. The nonlinear complex diffusion based, anisotropic diffusion based and total variation based prior is suitable choice for the energy term $\phi(\|\nabla I\|)$ based on the concept of energy function.

$$\phi(\|\nabla I\|) = f(I) \quad (5.8)$$

$f(I)$ is the diffusion PDE based prior obtained by minimization of $E(I)$, From equation (8) substituting the value of $\phi(\|\nabla I\|)$ in equation (5.7) reads:

$$E(I) = \arg \min_{\Omega} \left\{ \int_{\Omega} [L(p(I/M)) + \lambda \cdot f(I)] d\Omega \right\} \quad (5.9)$$

In case of only Rician noise, after solving modified zero order Bessel function [109], when we taking log and differentiating equation (5.1) w.r.t. I we get the loglikelihood term of Rician's pdf as:

$$L\{p(I/M)\} = -\frac{I}{\sigma^2} + \frac{2k_1}{I} \quad (5.10)$$

where k_1 represents positive integer.

In case of only Gaussian noise, we put value of unit step Heaviside function is one in equation (5.3), after taking logarithmic of equation (5.3) becomes:

$$\log\{p(I/M)\} = \log\left\{\frac{1}{\sqrt{2\pi}\sigma^2} \exp\left(-\frac{M^2 - \sqrt{I^2 + \sigma^2}}{2\sigma^2}\right)\right\} \quad (5.11)$$

Differentiating equation (5.11) w.r.t I we get the loglikelihood term of Gaussian's pdf as:

$$\frac{\partial}{\partial I} \{\log p(I/M)\} = \frac{\partial}{\partial I} \left\{ -2\log(\sigma) - \frac{1}{2}\log 2\pi - \frac{M^2}{2\sigma^2} - \frac{(I^2 + \sigma^2)^{1/2}}{2\sigma^2} \right\} \quad (5.12)$$

$$\text{or} \quad L\{p(I/M)\} = -\frac{I}{2\sigma^2(I^2 + \sigma^2)^{1/2}} \quad (5.13)$$

In case of only Rayleigh noise, the loglikelihood term of Rayleigh's pdf proposed by Srivastava *et al.*, [117] as follows:

$$L\{p(I/M)\} = -\frac{I}{\sigma^2} \quad (5.14)$$

Hence when we combine equation (5.10), (5.13) and (5.14) then we get combined loglikelihood term given as follows:

$$L\{p(I/M)\} = -\left\{ \frac{I}{\sigma^2} - \frac{2k_1}{I} + \frac{I}{2\sigma^2(I^2 + \sigma^2)^{1/2}} \right\} \quad (5.15)$$

Where $L\{p(I/M)\}$ shows that the negative likelihood term of combined Rayleigh's, Rician's and Gaussian's distributed noise in MRI. When we put the value of likelihood term from equation (5.15) in to the equation (5.9) then we get the proposed general framework using Euler-Lagrange minimization technique combined with gradient descent approach.

Therefore, the proposed general framework based model adapted to Rayleigh's, Rician's and Gaussian's distributed noise reads:

$$\frac{\partial I}{\partial t} = -\left[\lambda_1 \left(\frac{I}{\sigma^2} \right) - \lambda_2 \left(\frac{2k_1}{I} \right) + \lambda_3 \left(\frac{I}{2\sigma^2(I^2 + \sigma^2)^{1/2}} \right) \right] + \lambda \cdot f(I) \quad (5.16a)$$

with initial condition

$$I_{t=0} = I_0 \quad (5.16b)$$

where λ_1 , λ_2 and λ_3 are the constants to be set according to noise pattern, and λ is the regularization parameter, I_0 is the noisy image data.

Restoration of MRI for different noise distribution

Case-1: Gaussian noise distribution ($SNR = \frac{M}{\sigma} > 3$ dB)

When $\lambda_1 = \lambda_2 = 0$ and $\lambda_3 = 1$ then equation (5.16) become adapted to Gaussian distribution.

$$\frac{\partial I}{\partial t} = -\left(\frac{I}{2\sigma^2(I^2 + \sigma^2)^{1/2}} \right) + \lambda \cdot f(I) \quad (5.17a)$$

with initial condition

$$I_{t=0} = I_0 \quad (5.17b)$$

Case-2: Rician noise distribution ($0 < SNR = \frac{M}{\sigma} < 3$ dB)

When $\lambda_1 = \lambda_2 = 1$ and $\lambda_3 = 0$ then equation (5.16) become adapted to Rician distribution.

$$\frac{\partial I}{\partial t} = -\left(\frac{I}{\sigma^2} - \frac{2k_1}{I}\right) + \lambda \cdot f(I) \quad (5.18a)$$

with initial condition

$$I_{t=0} = I_0 \quad (5.18b)$$

Case-3: Rayleigh noise distribution ($SNR = \frac{M}{\sigma} \approx 0$ dB)

When $\lambda_2 = \lambda_3 = 0$ and $\lambda_1 = 1$ then equation (5.16) become adapted to Rayleigh distribution.

$$\frac{\partial I}{\partial t} = -\left(\frac{I}{\sigma^2}\right) + \lambda \cdot f(I) \quad (5.19a)$$

with initial condition

$$I_{t=0} = I_0 \quad (5.19b)$$

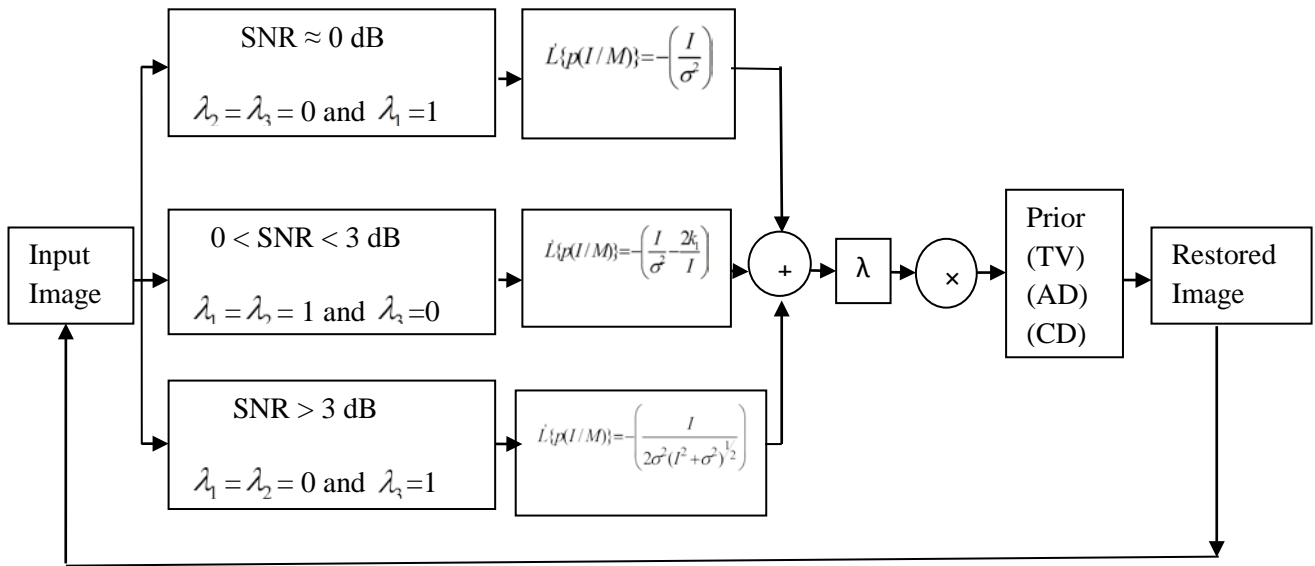


Fig. 5.1: Restoration of MRI data for different noise distribution with different priors

Fig. 5.1 illustrates the operation of the proposed general framework for restoration and enhancement of MRI data.

Selection of prior terms

The following three types of diffusion based prior terms are used and examined for their efficacy in the proposed methods.

1. Total variation (TV) based method
2. Anisotropic diffusion (AD) based method
3. The complex diffusion (CD) based method

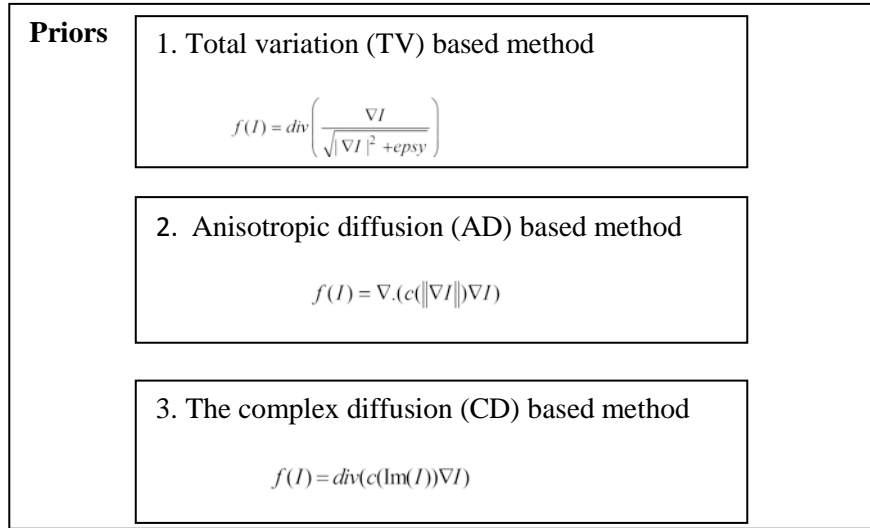


Fig. 5.2: Selection of prior terms

1. Total variation (TV) based method

The TV regularization approach was first proposed by Rudin *et al.*, [118] presented in to de- noise an image corrupted with additive white Gaussian noise. In Total variation (TV) based framework for Rayleigh's, Rician's and Gaussian's noise the regularization function is defined as [118].

$$f(I) = |\nabla I| = \sqrt{I_x^2 + I_y^2} \tag{5.20}$$

In discrete case, TV is defined as:

$$TV(I) = |\nabla I| = \sum_{j,k=1}^n \sqrt{|(\nabla I)_{j,k}^x|^2 + |(\nabla I)_{j,k}^y|^2} \quad (5.21)$$

Where

$$(\nabla I)_{j,k}^x = I_{j+1,k} - I_{j,k} \text{ for } j < n = 0 \text{ for } j = n \quad (5.22)$$

and

$$(\nabla I)_{j,k}^y = I_{j,k+1} - I_{j,k} \text{ for } k < n = 0 \text{ for } k = n \quad (5.23)$$

$$f(I) = \text{div} \left(\frac{\nabla I}{|\nabla I|} \right) \quad (5.24)$$

For numerical implementations, the derivatives can be discretized using standard centered difference approximations and the quantity $|\nabla I|$ is replaced with $\sqrt{|\nabla I|^2 + \text{eps}}$ for some small positive value of eps such as 0.00000000001. The value of eps can be assigned to lowest machine number to avoid divide by zero conditions during implementations.

$$f(I) = \text{div} \left(\frac{\nabla I}{\sqrt{|\nabla I|^2 + \text{eps}}} \right) \quad (5.25)$$

2. Anisotropic diffusion based method

In Anisotropic diffusion based framework for Rayleigh's, Rician's and Gaussian's noise the regularization function is defined as [37]:

$$f(I) = \nabla \cdot (c(\|\nabla I\|) \nabla I) \quad (5.26)$$

where the diffusion coefficient $c(\|\nabla I\|)$ is defined as [37],

$$c(\|\nabla I\|) = \frac{1}{1 + \left(\frac{\|\nabla I\|}{\gamma}\right)^2} \quad (5.27)$$

Where γ is the threshold parameter.

3. The complex diffusion based method

In nonlinear complex diffusion based framework for Rayleigh's, Rician's and Gaussian's noise the regularization function is defined as [119]:

$$f(I) = \text{div}(c(\text{Im}(I))\nabla I) \quad (5.28)$$

The diffusion coefficient $c(\text{Im}(I))$ is defined as follows [119]:

$$c(\text{Im}(I)) = \frac{e^{i\theta}}{1 + \left(\frac{\text{Im}(I)}{k\theta}\right)^2} \quad (5.29)$$

Here, k is known as threshold parameter and for digital images [119] the value of k is ranges from 1 to 1.5. Eqs.(5.28) describe the nonlinear complex diffusion process , where linear forward diffusion control the evolution of real part of the images, and both the real and imaginary equations control the evolution of imaginary part of the image.

A qualitative property of edge detection i.e. second smoothed derivative is described by the imaginary part of the image for small value of θ , where as real values depict the properties of ordinary Gaussian scale -space. For large values of θ , the imaginary part feeds back into the real part creating the wave like ringing effect which is an undesirable property. Here, for experimentation purposes the value of θ is chosen to be $\frac{\pi}{30}$. The adaptive value of edge threshold parameter is used in Eq.(5.29). It is defined as

negative exponential distribution:

$$kt \approx k_0 \exp(-\alpha t) \quad (5.30)$$

where α and k_0 are constants, usually 1.

Discretization of the proposed model:

For digital implementations, the Eqs. (5.16a), (5.16b) can be discretized using finite differences schemes [109]. For example, the discretized form of TV based proposed model reads:

$$I^{n+1} = I^n + \Delta t. \left[- \left\{ \lambda_1 \left(\frac{I^n}{\sigma^2} \right) - \lambda_2 \left(\frac{2k_1}{I^n} \right) + \lambda_3 \left(\frac{I^n}{2\sigma^2 (I^{2n} + \sigma^2)^{1/2}} \right) \right\} + \lambda \operatorname{div} \left(\frac{\nabla I^n}{\sqrt{|\nabla I^n|^2 + \epsilon \text{psy}}} \right) \right] \quad (5.31)$$

Similarly, for anisotropic diffusion based model can be discretized using finite difference scheme reads:

$$I^{n+1} = I^n + \Delta t. \left[- \left\{ \lambda_1 \left(\frac{I^n}{\sigma^2} \right) - \lambda_2 \left(\frac{2k_1}{I^n} \right) + \lambda_3 \left(\frac{I^n}{2\sigma^2 (I^{2n} + \sigma^2)^{1/2}} \right) \right\} + \lambda \nabla \cdot (c(\|\nabla I^n\|) \nabla I^n) \right] \quad (5.32)$$

Similarly, for nonlinear complex diffusion model can be discretized using finite difference scheme reads:

$$I^{n+1} = I^n + \Delta t. \left[- \left\{ \lambda_1 \left(\frac{I^n}{\sigma^2} \right) - \lambda_2 \left(\frac{2k_1}{I^n} \right) + \lambda_3 \left(\frac{I^n}{2\sigma^2 (I^{2n} + \sigma^2)^{1/2}} \right) \right\} + \lambda \operatorname{div}(c(\operatorname{Im}(I^n)) \nabla I^n) \right] \quad (5.33)$$

$$I_{t=0} = I_0 \quad (5.34)$$

The von-Neumann analysis [109], shows that condition require $\frac{\Delta t}{(\Delta x)^2} < \frac{1}{4}$, for the numerical scheme, given by equation (5.31-5.34) to become stable. If the size of the grid is set to be $\Delta x=1$, after that $\Delta t < \frac{1}{4}$ i.e. $\Delta t < 0.25$. Hence, for the stability of equation (5.31-5.34), the value of Δt is set to be 0.24.

5.4. Results and discussions

Brain Web database [1] is used for simulated (synthetic) and real (clinical) data sets of normal brain MR images, to compare the effectiveness of the proposed technique. There are three modalities (pulse sequences) dataset present in the Brain Web databases [1] which are T1, T2 and PD weighted. The proposed method and other standard methods used for comparison purposes were implemented using MATLAB R2014.

The performance of restoration results are analyzed for images artificially degraded by mainly Rician's noise and partially Gaussian's noise and Rayleigh's noise if image background is present. LMMSE [78], RLMMSE [49], and RSNLMMSE [50], are familiar existing techniques used for comparing the proposed method in the case of Rician noise. For Rician noise the best setups as proposed by the authors and the free parameters of these methods are used during experimentation. To obtain the best results the relevant values of the parameters are given below:

- LMMSE [78]: window of size 5×5 , Linear Minimum Mean Square Error Estimator.
- RLMMSE [49]: Recursive version of Linear Minimum Mean Square Error Estimator, window of size 5×5 .
- RSNLMMSE [50]: window using a 5×5 , recursive version of SNR-based Nonlocal LMMSE.

The parameters are adjusted empirically for de-noising MR images and the setup of all the parameters using the proposed scheme is shown in the Table 5.1. The ground truth MR data are artificially contaminated with a noise variance having the range 5–30 % to evaluate the quantitative metrics. Based on SSIM and MSE average restoration results for Rician noise and based on PSNR, MSE, SSIM and CP average restoration results for Gaussian and Rayleigh noise over 4-50 iterations or till the convergence of all these de-noising methods are computed.

The performance analysis and comparative study of the proposed method with other standard methods are represented on the basis of quantitative results SSIM (MSE) for different levels of Rician noise in Table 5.2. The value of PSNR, MSE, SSIM and CP represented for different levels of Gaussian and Rayleigh noise in Table 5.3 and Table 5.4 respectively. In the case of Rician noise SSIM and MSE values show that at

low as well as at high rates of Rician noise, the proposed method has much better restoration results than existing methods.

Fig. 5.3 and Fig. 5.4 illustrate detailed results, obtained with the close up view of the restored images for better inspection, in order to compare the visual performance, existing and proposed approaches, incorporates real image, noisy image and the restored image. The visual results for simulated MR slice is corrupted with 10 % level of Rician noise is presented in Fig. 5.3, Gaussian and Rayleigh noise in Fig. 5.4. On the basis of quantitative and visual results it is apparent that the proposed approach has produced more accurate results such as more noise removing ability, and preservation of edges and structural information, at all levels of Rician noise.

Retaining the important structural information, such as texture and edges, is considered as an important task in image restoration during noise smoothing process. The detailed information, present in the image do not quantify MSE. A well-known quantitative measure SSIM is used for measure the detail preservation performance of the proposed filter shown in the Table 5.2. The proposed technique is superior in terms of retaining structural information at all noise levels clearly shown in Fig. 5.5(a), 5.5(b), 5.5(c).

The solution can be computed in one single step (or a few steps for the RLMMSE filter), making it computationally efficient for large data sets; this is the main advantage of the LMMSE [78] filter (and to some extent for the RLMMSE filter). The proposed technique is an deterioration in terms of MSE comparison with the best performing RSNLMMSE [50] and RLMMSE [49], at a low noise rate (5 %), 5.08 for T1, 4.58 for T2, and 5.18 for PD. Result, shows that the efficiency of proposed filter also increases as the noise rate increases.

At high noise rates, the proposed technique accurately differentiates the low and high noise regions, hence the better result obtained. Similarly, in the case of SSIM Table 5.2 shows that the proposed scheme outperforms the existing techniques. Comparison of the proposed filter using MSE values are shown in Fig. 5.5(d), 5.5(e), 5.5(f) respectively using simulated data sets. The above figure clearly indicates that the proposed technique is superior at all noise levels.

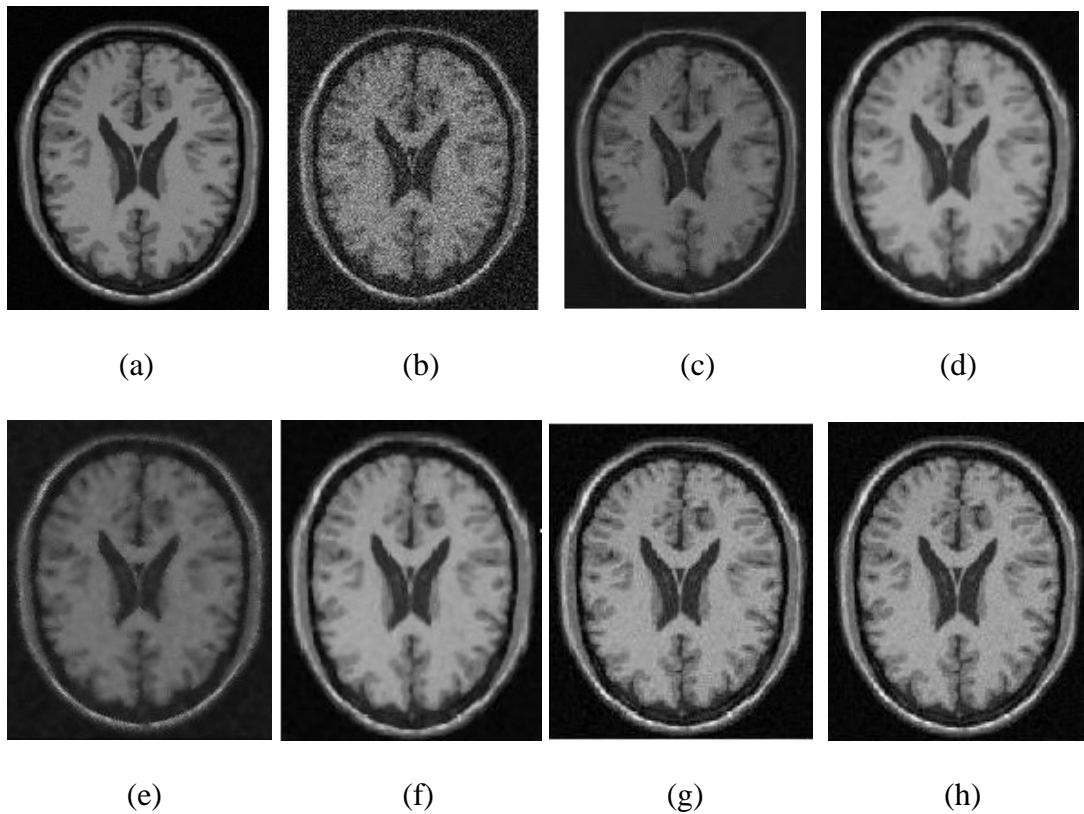


Fig. 5.3: Simulated T1 weighted MR image with Rician noise (a) Original image (b) 10% noisy image (c) RLMMSE (d) RSNLMMSE (e) LMMSE (f) Proposed with TV (g) Proposed with AD (h) Proposed with CD.

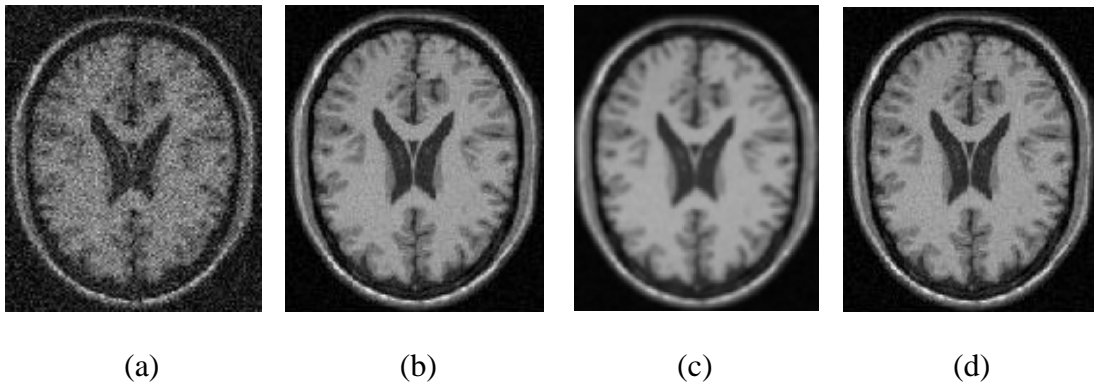
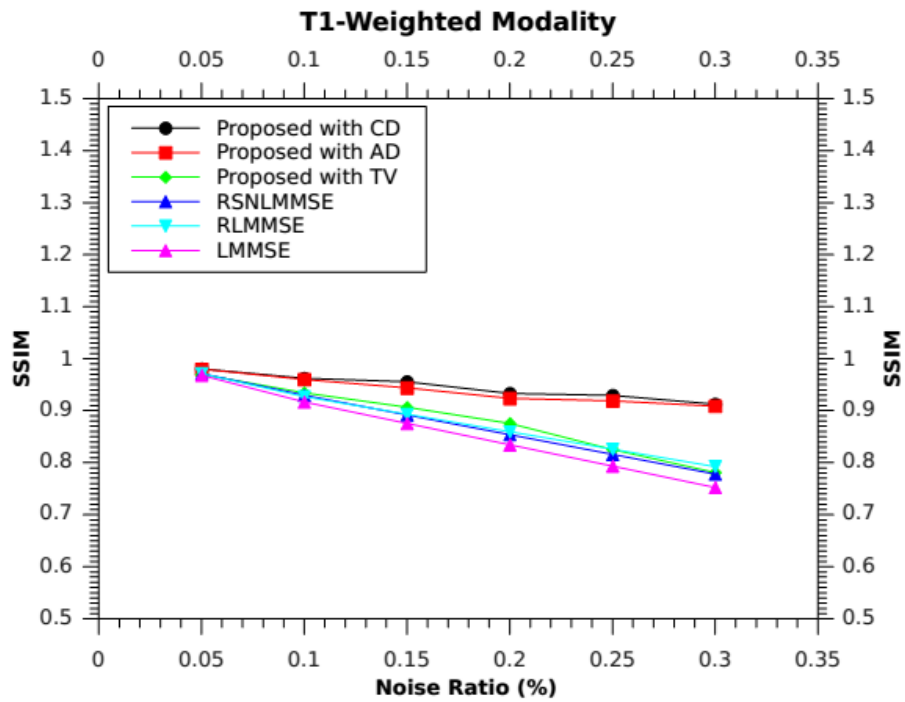
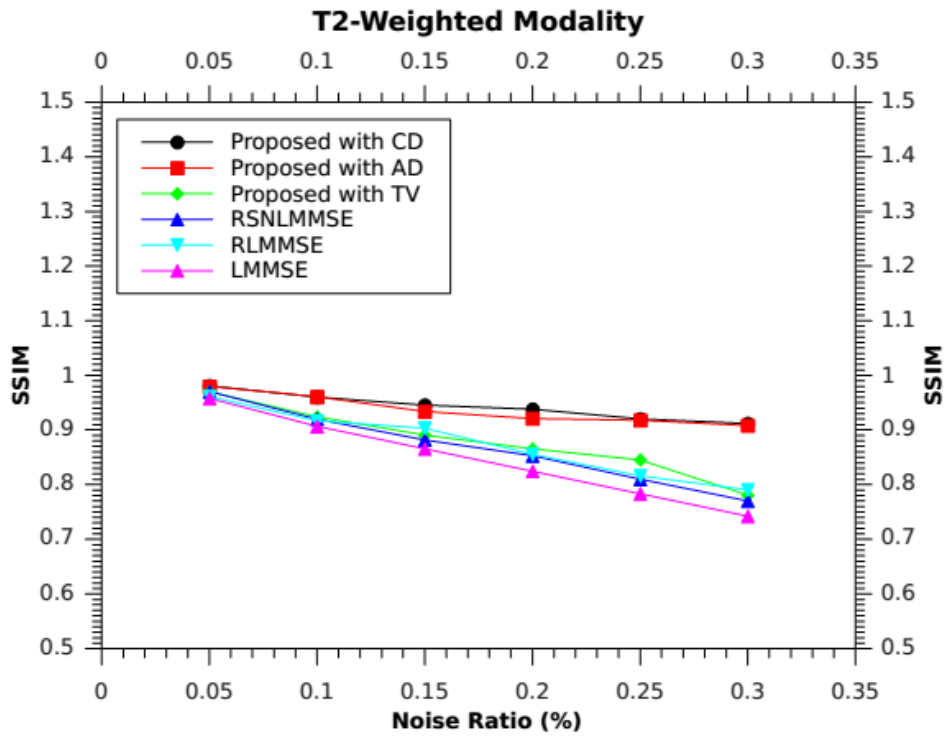


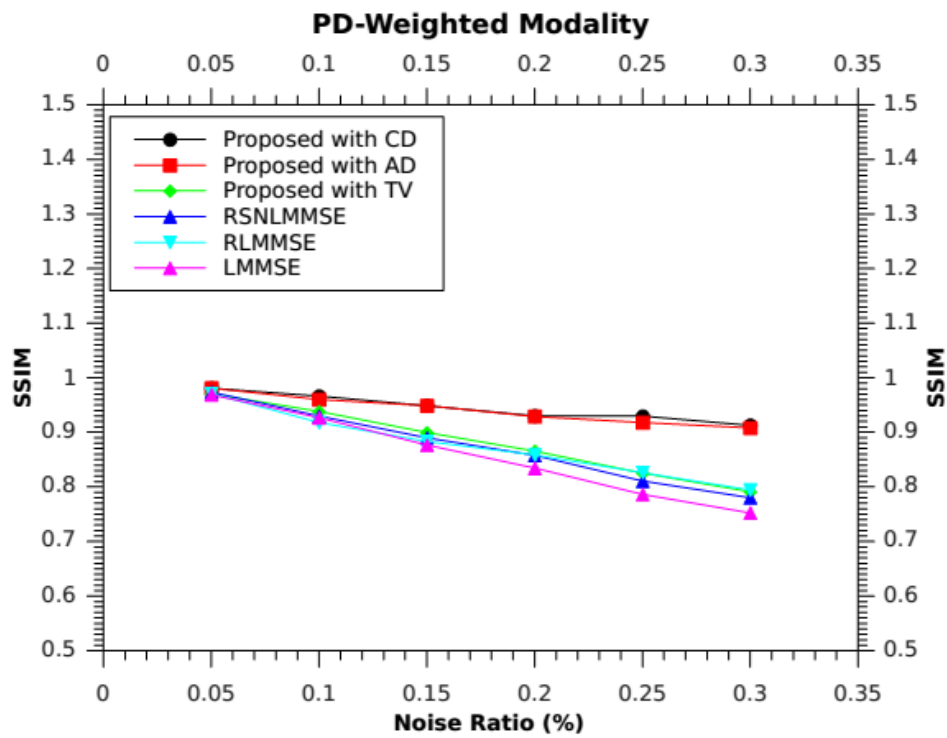
Fig. 5.4: Simulated T1 weighted MR image with Gaussian and Rayleigh noise (a) Gaussian noise corrupted MR image (b) Restored image with proposed method from Gaussian noise corrupted MR image (c) Rayleigh noise corrupted MR image (d) Restored image with proposed method from Rayleigh noise corrupted MR image.



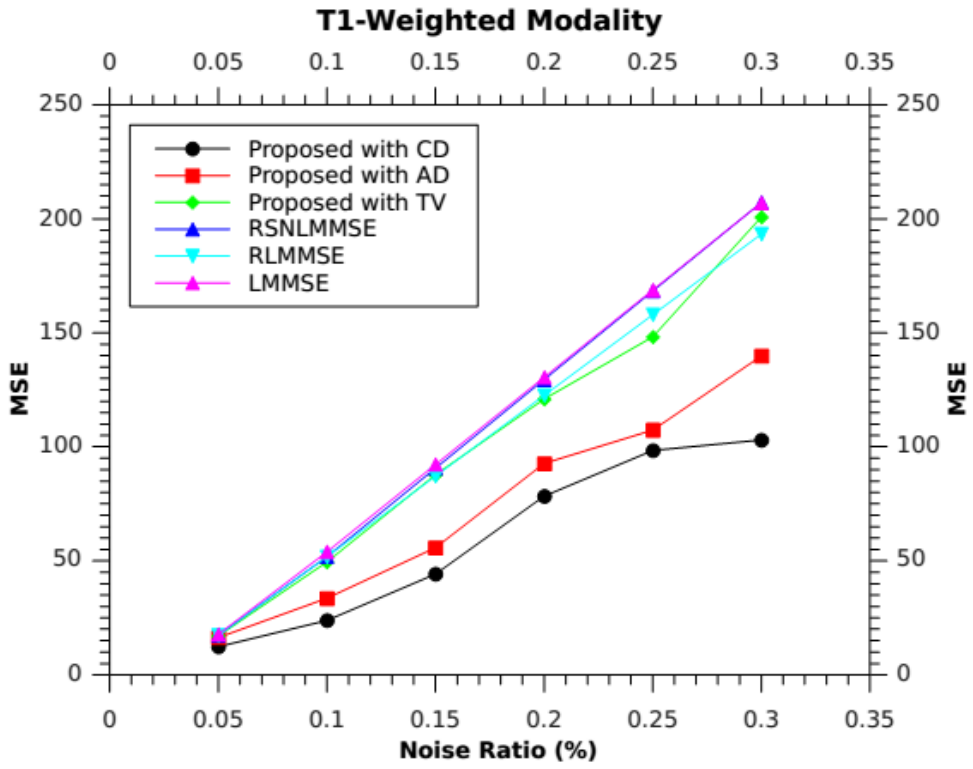
(a)



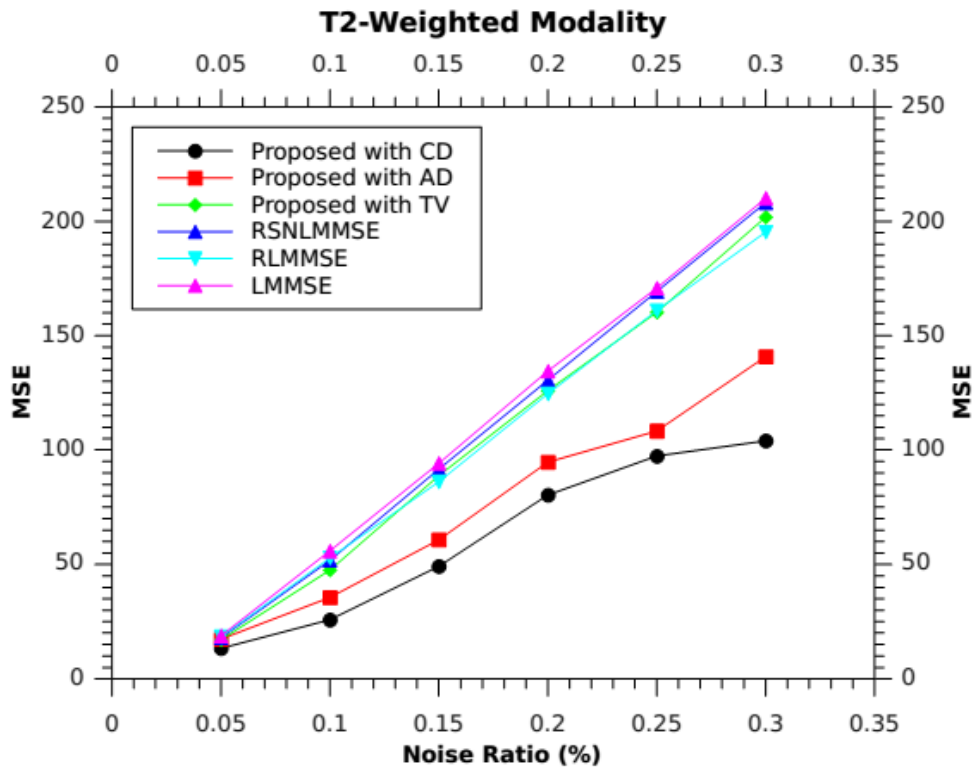
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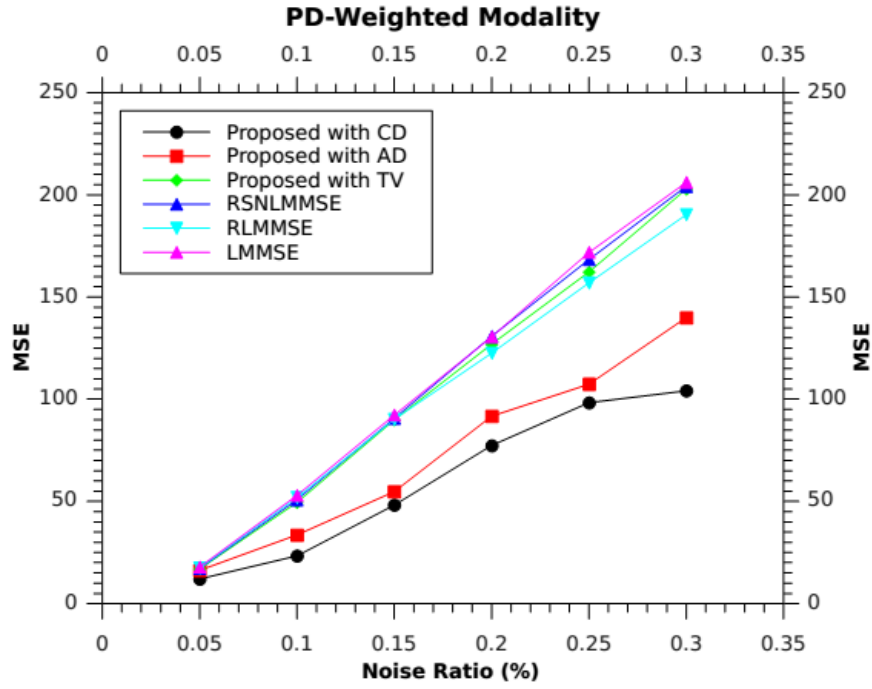
(c)



(d)



(e)



(f)

Figure 5.5: SSIM based comparison of various methods: (a) T1-weighted modality (b) T2-weighted modality (c) PD-weighted modality and MSE based comparison of various methods: (d) T1-weighted modality (e) T2-weighted modality (f) PD-weighted modality.

Table 5.1: Parameters setup of the proposed method for de-noising MR images

Parameter	Description	Value
Num_Iter	Number of iterations used as a parameter to getting desired output at four in the proposed method.	4.0
Δt	Integration constant which is used as a parameter to calculate the desired output at zero point one in the proposed method.	0.10
k	Edge threshold parameter used to controls the diffusion, getting desired output at one point four in the proposed method.	1.4

θ	Used as a parameter in the diffusion coefficient, getting desired output at $\pi/30$ in the proposed method.	$\frac{\pi}{30}$
λ	Regularization parameter used for making balance between likelihood term and regularization function, getting desired output at zero point nine in the proposed method.	0.9
k_1	Positive number used to calculate the Rician noise, getting desired output at one in the proposed method.	1
epsy	The value of eps can be assigned to lowest machine number to avoid divide by zero conditions during implementations	0.0000000001
γ	Gradient modulus threshold used as a parameter that controls the conduction, getting desired output at four in the proposed method.	4.0

Table 5.2: Quantitative comparison on Simulated MR data (Brain Web) for Rician noise using SSIM (MSE)

Modality (slice)	Noise ratio	LMMSE [78]	RLMMSE [49]	RSNLMMS E [50]	Proposed with TV	Proposed with AD	Proposed with CD
T1-weighted (slice 70)	0.05	0.96(17.79)	0.97(17.40)	0.97(17.45)	0.97(17.11)	0.98(16.38)	0.98(12.37)
	0.10	0.91(53.97)	0.92(51.81)	0.92(51.84)	0.93(49.55)	0.95(33.62)	0.96(23.88)
	0.15	0.87(92.25)	0.89(87.19)	0.89(90.68)	0.90(87.80)	0.94(55.80)	0.95(44.22)
	0.20	0.83(130.53)	0.85(122.5)	0.85(129.51)	0.87(120.94)	0.92(92.71)	0.93(78.37)
	0.25	0.79(168.81)	0.82(157.9)	0.81(168.34)	0.82(160.26)	0.91(107.37)	0.92(98.43)
	0.30	0.75(207.09)	0.79(193.3)	0.77(207.17)	0.78(200.83)	0.90(139.85)	0.91(103.03)
	Mean	0.85(111.74)	0.87(105.0)	0.87(110.83)	0.88(104.08)	0.93(74.29)	0.94(60.05)
T2-weighted (slice 70)	0.05	0.95(18.79)	0.96(18.40)	0.97(17.95)	0.96(17.01)	0.98(17.38)	0.98(13.37)
	0.10	0.90(55.97)	0.91(52.81)	0.91(51.84)	0.92(47.55)	0.96(35.62)	0.96(25.88)
	0.15	0.86(94.25)	0.90(86.19)	0.88(91.68)	0.89(88.80)	0.93(60.80)	0.94(49.22)
	0.20	0.82(134.53)	0.85(124.5)	0.85(130.51)	0.86(125.94)	0.92(94.71)	0.93(80.37)
	0.25	0.78(170.81)	0.81(160.9)	0.81(169.34)	0.84(160.26)	0.91(108.37)	0.92(97.43)
	0.30	0.74(210.09)	0.79(195.3)	0.77(208.17)	0.78(201.83)	0.90(140.85)	0.91(104.03)
	Mean	0.84(111.92)	0.87(104.9)	0.86(111.58)	0.87(106.91)	0.93(76.29)	0.94(61.72)

PD-weighted (slice 50)	0.05	0.96(17.89)	0.97(17.50)	0.97(17.25)	0.97(17.14)	0.98(16.30)	0.98(12.07)
	0.10	0.92(52.97)	0.91(51.88)	0.92(50.84)	0.93(49.55)	0.96(33.62)	0.96(23.38)
	0.15	0.87(92.25)	0.88(90.19)	0.88(90.68)	0.89(90.90)	0.94(54.80)	0.94(48.22)
	0.20	0.83(130.53)	0.85(122.5)	0.85(130.81)	0.86(126.94)	0.92(91.71)	0.93(77.37)
	0.25	0.78(171.81)	0.82(156.9)	0.81(168.34)	0.82(162.26)	0.91(107.37)	0.92(98.33)
	0.30	0.75(206.09)	0.79(190.3)	0.78(204.17)	0.79(202.83)	0.90(139.95)	0.91(104.13)
	Mean	0.85(111.92)	0.87(104.9)	0.87(110.35)	0.88(108.08)	0.94(73.96)	0.94(60.58)
Overall Mean		0.84(111.86)	0.87(104.9)	0.86(110.92)	0.85(106.35)	0.93(74.84)	0.94(60.78)

Table 5.3: Quantitative comparison of proposed method on Simulated MR data (Brain Web) for Gaussian noise using PSNR, MSE, SSIM and CP

Modality (slice)	Noise ratio	PSNR-TV	PSNR-AD	PSNR-CD	MSE-TV	MSE-AD	MSE-CD
T1-weighted (slice 70)	0.05	30.6981	35.9323	48.0611	49.0349	16.5701	12.0162
	0.10	29.1353	34.1822	45.9927	64.7059	20.9823	18.6103
	0.15	26.7679	33.9802	43.6477	81.3862	31.0412	23.9809
	0.20	22.5606	33.0856	40.1873	100.9685	43.9964	30.0242
	0.25	20.2709	32.9734	39.5916	121.5862	64.0451	41.8231
	0.30	19.0696	32.0946	38.8859	144.5862	85.1161	53.0327
	Mean	24.7504	33.7080	42.7277	93.7113	43.6252	29.9145
	Noise ratio	SSIM-TV	SSIM-AD	SSIM-CD	CP-TV	CP-AD	CP-CD
	0.05	0.9051	0.9846	0.9879	0.9519	0.9857	0.9885
	0.10	0.8431	0.9628	0.9702	0.9082	0.9659	0.9708
	0.15	0.7876	0.9595	0.9655	0.8099	0.9377	0.9389
	0.20	0.7025	0.9465	0.9575	0.7158	0.9089	0.9196
	0.25	0.6362	0.9075	0.9253	0.6531	0.9011	0.9094
	0.30	0.5194	0.8973	0.9054	0.6015	0.8815	0.8998
	Mean	0.7323	0.9430	0.9519	0.7734	0.9301	0.9378

Table 5.4: Quantitative comparison of proposed method on Simulated MR data (Brain Web) for Rayleigh noise using PSNR, MSE, SSIM and CP

Modality (slice)	Noise ratio	PSNR-TV	PSNR-AD	PSNR-CD	MSE-TV	MSE-AD	MSE-CD
T1-weighted (slice 70)	0.05	34.0182	36.6902	53.4115	35.1719	13.1882	10.5691
	0.10	31.5019	35.8007	52.8658	56.1409	33.1353	18.6167
	0.15	29.7152	34.8623	50.4752	77.5652	44.8061	25.7233
	0.20	26.3026	34.0087	47.2959	89.9408	51.0374	31.2191
	0.25	23.3632	33.9042	44.2978	98.5446	70.4395	41.8444
	0.30	21.8798	33.6601	43.2258	118.5376	80.8573	48.0198
	Mean	27.7968	34.8210	48.5953	79.3168	48.9106	29.3320
	Noise ratio	SSIM-TV	SSIM-AD	SSIM-CD	CP-TV	CP-AD	CP-CD
	0.05	0.9646	0.9802	0.9804	0.9797	0.9801	0.9808

	0.10	0.9071	0.9485	0.9795	0.9578	0.9689	0.9783
	0.15	0.8858	0.9091	0.9391	0.9051	0.9397	0.9417
	0.20	0.8445	0.8988	0.9095	0.8922	0.9012	0.9094
	0.25	0.8037	0.8844	0.8991	0.8711	0.8822	0.8924
	0.30	0.7955	0.8631	0.8794	0.8521	0.8601	0.8644
	Mean	0.8668	0.9140	0.9311	0.9096	0.9220	0.9278

5.5. Conclusions

In this chapter, we have proposed a partial differential equation-based general framework filter adapted to Rician noise, Gaussian noise and Rayleigh noise for restoration and enhancement of magnetic resonance images. The proposed filter consists of two terms namely data fidelity and prior. The data fidelity term i.e. likelihood term is derived from Rician pdf, Gaussian pdf and Rayleigh pdf and total variation (TV) based prior, anisotropic diffusion (AD) based prior and a nonlinear complex diffusion (CD) based prior are used. Further, mathematical simplifications have been introduced for likelihood term for efficient implementation of the algorithm. The proposed method was tested on Brain Web data set for varying noise levels and performance was evaluated in terms of MSE and SSIM for Rician noise. Similarly the proposed method also removes Gaussian noise as well as Rayleigh noise. From obtained results and comparative analysis with other standard methods, it is observed that the proposed method is performing better. Further, visual results clearly indicate that the proposed technique has the capability of better noise removal.