Chapter 4 : ORIENTATION DEPENDENT ANISOTROPIC ADAPTIVE FUZZY DIFFUSION BASED FILTER FOR RESTORATION AND ENHANCEMENT OF MRI

In this chapter, orientation dependent anisotropic adaptive fuzzy diffusion based method for restoration and enhancement of MRI have been proposed and implemented. This chapter is divided into the following sections. Section 4.1 presents introduction of the proposed method. Section 4.2 presents the background of the proposed method. 4.3 present the method and models of the proposed method. 4.4 present their result analysis and discussion with their qualitative and quantitative analysis of the proposed model. 4.5 finally the conclusion of this chapter is presented.

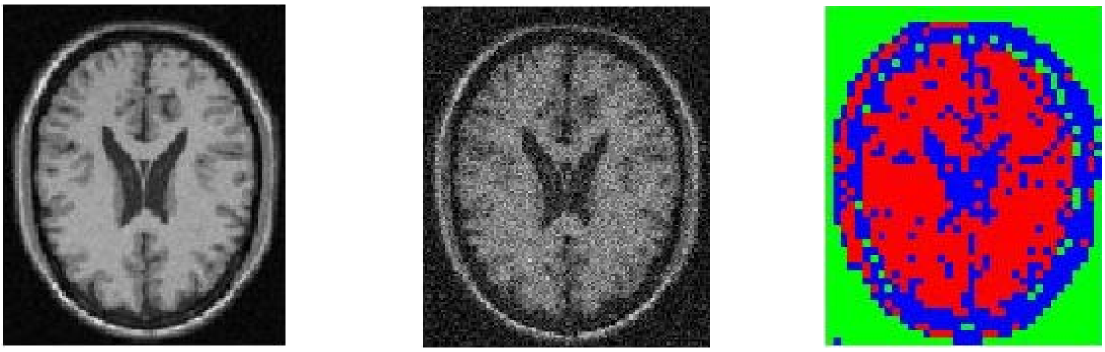
4.1. Introduction

In this section a compressive literature review related to proposed filter on the magnetic resonance images is presented. The various approaches for MRI noise removal can be broadly categorized as filtering approach, transform domain approach and statistical approach. Examples of filtering approach include linear filtering such as spatial filter [88] and temporal filter [88]. Non-linear filtering methods such as anisotropic diffusion filter (ADF) [37], adaptive ADF filter [89], noise driven ADF filter [90], noise adaptive ADF filter. Fourth order PDE filters [91], adaptive fourth order PDE filter [38], fourth order complex PDE filters [92]. Non local means filter (NLM) [39], fast NLM filter [93], Block wise optimised NLM filter [52], Unbiased NLM filter [19], dynamic NLM filter [94], enhanced NLM filter [54], adaptive NLM filter [40]. Combination of domain and range filters [41], bilateral domain and range filters [42], trilateral domain and range filters [42].

Example of transform domain [66] approaches include curvelet [95], contourlet [69] and wavelet [96], adaptive multiscale product thresholding [97], multiwavelet [98], undecimated wavelet [99] .Example of Statistical approach include maximum likelihood estimation [74,18], linear minimum mean square error estimation [18], phase error estimation [72], nonparametric estimation [80], singularity function analysis [53,54] were presented. Some other Rician noise removal of MRI approaches were proposed in literature such as machine learning-based approaches [100,101,102,103,104,105], DCT-based filter [106], PCA-based technique [107], and conventional approaches [71].

In MR images, three categories of noises mainly Rician, Gaussian and Rayleigh noise are present, but Rician noise is one of the prominent noise. However sometimes other noise Gaussian and Rayleigh are also dominant which affect the quality of the reconstructed image. By measuring the SNR value of the image data in MRI these types of noises can be identified. In this chapter identification of Rician, Gaussian and Rayleigh noise on the basis of signal-to-noise ratio (SNR) is presented which is shown in the Figure 4.1. Rician noise can be removed by many available methods in the literature starting from Henkelman [32] who presented a method from noisy version of MR image to estimate the noiseless magnitude MR image.

An input parameter for image post processing tasks is the estimation of noise variance from the magnetic resonance images (MRI) which is often of key importance. The estimated noise variance is used to measure the quality of MR data. In image processing algorithms, the noise variance is a crucial parameter just like segmentation, noise reduction, parameter estimation or clustering [113].



(a) (b) (c)

Fig. 4.1: Simulated T1 weighted MR image (a) Noise free MR image (b) 15% noisy MR image (c) Rayleigh (green), Rician (Blue) and Gaussian (Red) noise at different SNR level in MRI

In this chapter, orientation dependent anisotropic adaptive fuzzy diffusion based methods are proposed for restoration and enhancement of magnetic resonance images. The proposed method is capable to remove Gaussian as well as Rician noise from MRI using orientation dependent anisotropic adaptive fuzzy diffusion based prior.

The proposed methods are casted into a variational framework as an energy functional. Euler-Lagrange minimization technique is used to derive the filters. The derived filters are adapted to the specific type of noises such as Gaussian or Rician depending on the type of noise present in the image. To deal with the ill-posedness problem of the derived filters an orientation dependent anisotropic adaptive fuzzy diffusion based prior is used because it performs better in comparison to other methods. The basic advantage of using this prior is that in addition to producing smooth noise free images it also preserves the edges and fine structures in the image data. The proposed filter automatically filters the type of noise present in the image data depending on the SNR values of the image data.

4.2. Background

For proper estimation of noise level [86] methods based on Rayleigh model require a certain amount of background pixels which may be affected by these techniques. The raw complex MR data acquired in the Fourier domain are illustrated by Zero mean Gaussian probability density function (PDF).Gaussian for the noise distribution in the real and imaginary components may be caused by the linearity and orthogonality of the Fourier transform, after the inverse Fourier transforms. The noise distribution will no longer be Gaussian but Rician distributed due to the subsequent to the subsequent transform to a magnitude image. The PDF of the reconstructed magnitude image M, if I is the original signal amplitude will be:

$$p(I/M) = \frac{M}{\sigma^2}\exp\left(-\frac{M^2+I^2}{2\sigma^2}\right)J_0\left(\frac{IM}{\sigma^2}\right)\in(M) \tag{4.1}$$

where I denotes amplitude of a noise-free image, σ^2 denotes the Gaussian noise variance, $J_0(.)$ show that modified zero order Bessel function. $\in(.)$ is the unit step Heaviside function, and M is the magnitude MR image.

The Rician PDF is only valid for nonnegative values of M [18]. In the image background, where the SNR is low (SNR ≈ 0), the Rician PDF reduces to a Rayleigh distribution [76] with PDF:

$$p(I/M) = \frac{M}{\sigma^2}\exp(-\frac{I^2}{2\sigma^2})\in(M) \tag{4.2}$$

when SNR is high (greater than 3dB), then the Rician distribution becomes Gaussian distribution [21] with mean $\sqrt{I^2+\sigma^2}$ and variance σ^2 given as follows:

$$p(I/M)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp(-\frac{M^2-\sqrt{I^2+\sigma^2}}{2\sigma^2})\in(M) \quad (4.3)$$

based on the local computation of the skewness of the magnitude data distribution is proposed by Rajan *et al.,* [76] for the estimation of noise variance. In between the moments of Rayleigh and Gaussian distributions it can be concluded that Rician distribution is always present. The relationship between σ^2 and the variance of a Rician distribution σ_R^2 at high and low SNR can be written as:

$$\sigma^2=\sigma_R^2\left(2-\frac{\pi}{2}\right)^{-1} \quad (4.4)$$

and $\sigma^2=\sigma_R^2$, (4.5)

respectively. In general, σ^2 in terms of σ_R^2 can be written as

$$\sigma^2=\sigma_R^2\times\psi, \quad (4.6)$$

where ψ is a correction factor in the range [1; $\left(2-\pi/2\right)^{-1}$], i.e. when the Rician distribution approaches a Rayleigh distribution (at low SNR), the correction factor tends to $\left(2-\pi/2\right)^{-1}$ and when the Rician distribution approaches a Gaussian (at high SNR), the correction factor tends to 1.

The proposed method, automatically identify various types of noise present into the MRI and filters them by choosing an appropriate filter. The different categories of noises during the acquisition of MR image may be corrupted generally due to the external or internal causes. The external causes lead to an additive noise pattern which follows a Gaussian distribution (pdf). Causes of internal noise in MR image are basically the intrinsic noise that is generated during the acquisition process. Normally intrinsic noise in MR image follows the Rician distribution (pdf). The proposed filter gets adapted for the removal of specific types of noise based on SNR values of image data. The performance analysis and comparative study of the proposed method with other standard methods is presented for Brain Web dataset at varying noise levels in terms of PSNR and RMSE. From the simulation results, it is observed that the proposed framework with ODAAFD based prior is performing better in comparison to other priors.

4.3. Method and Models

In this section, the proposed method working on the orientation dependent anisotropic adaptive fuzzy diffusion based framework of a PDE filter for removal of Gaussian as well as Rician noise from MRI. The regularization of MR image data and the removal of the Gaussian and Rician noise are obtained by minimizing the following nonlinear energy functional of the image I within a continuous domain Ω, using the variational framework [112].

$$E(I) = \arg_{\min}\left\{\int_{\Omega}\left[L(p(I/M)) + \lambda.\phi(\|\nabla I\|)\right]d\Omega\right\} \quad (4.7)$$

where $L(p(I/M)$ shows the negative likelihood term of Rician or Gaussian distributed noise in MRI, given by equation (4.2) and (4.3). During the filtering process log likelihood term measures the dissimilarities at a pixel between M and its estimated value I. $L(p(I/M)$ acts as the data attachment term or the likelihood term in equation (4.7).

De-noising of image data is led by maximization of log likelihood or minimization of the negative log likelihood, but is an ill-posed problem and hence regularization is needed. That’s why the second term $\phi(\|\nabla I\|)$ in equation (4.7) is needed and it acts as a prior term. In the equation (4.7), λ is a regularization parameter, which has a constant value and makes a balance between regularization function and the data attachment term. The value of λ has been determined experimentally and is set to a value for which peak signal to noise ratio is maximum during the iteration process of filtration. The total variation based prior is suitable choice for the energy term $\phi(\|\nabla I\|)$ based on the concept of the energy function.

$$\phi(\|\nabla I\|) = f(I) \tag{4.8}$$

$f(I)$ is the diffusion PDE based prior obtained by minimization of $E(I)$, From equation (4.8) substituting the value of $\phi(\|\nabla I\|)$ in equation (4.7) reads:

$$E(I) = \arg_{\min}\left\{\int_{\Omega}\left[L(p(I/M)) + \lambda. f(I)\right]d\Omega\right\} \tag{4.9}$$

In case of Gaussian noise, we put value of unit step Heaviside function is one in equation (4.3), after taking logarithmic of equation (4.3) becomes:

$$\log\{p(I/M)\} = \log\{\frac{1}{\sqrt{2\pi\sigma^2}}\exp(-\frac{M^2 - \sqrt{I^2+\sigma^2}}{2\sigma^2})\} \tag{4.10}$$

Differentiating equation (4.10) w.r.t I we get the loglikelihood term of Gaussian's pdf as:

$$L'\{p(I/M)\} = -\frac{I}{2\sigma^2(I^2+\sigma^2)^{1/2}} \tag{4.11}$$

In case of Rician noise, the loglikelihood term of Rician's pdf proposed by Yadav *et. al.*, [114] as follows:

$$L'\{p(I/M)\} = -\frac{I}{\sigma^2} + \frac{2k_1}{I} \tag{4.12}$$

Restoration of MRI for different noise distribution

Gaussian and Rician noise removal:

Gaussian noise distribution ($SNR = \frac{M}{\sigma}$ > 3 dB)

The restoration model for Gaussian noise distribution can be written as:

$$\frac{\partial I}{\partial t} = -\left(\frac{I}{2\sigma^2 (I^2 + \sigma^2)^{1/2}} \right) + \lambda . f(I) \qquad (4.13a)$$

with initial condition

$$I_{t=0} = I_0 \qquad (4.13b)$$

Rician noise distribution (0 < $SNR = \frac{M}{\sigma}$ < 3 dB)

The restoration model for Rician noise distribution can be written as:

$$\frac{\partial I}{\partial t} = -\left(\frac{I}{\sigma^2} - \frac{2k_1}{I} \right) + \lambda . f(I) \qquad (4.14a)$$

with initial condition

$$I_{t=0} = I_0 \qquad (4.14b)$$

Following Fig.4.2 illustrates the operation of the Gaussian and Rician noise based proposed method for restoration and enhancement of MRI data.

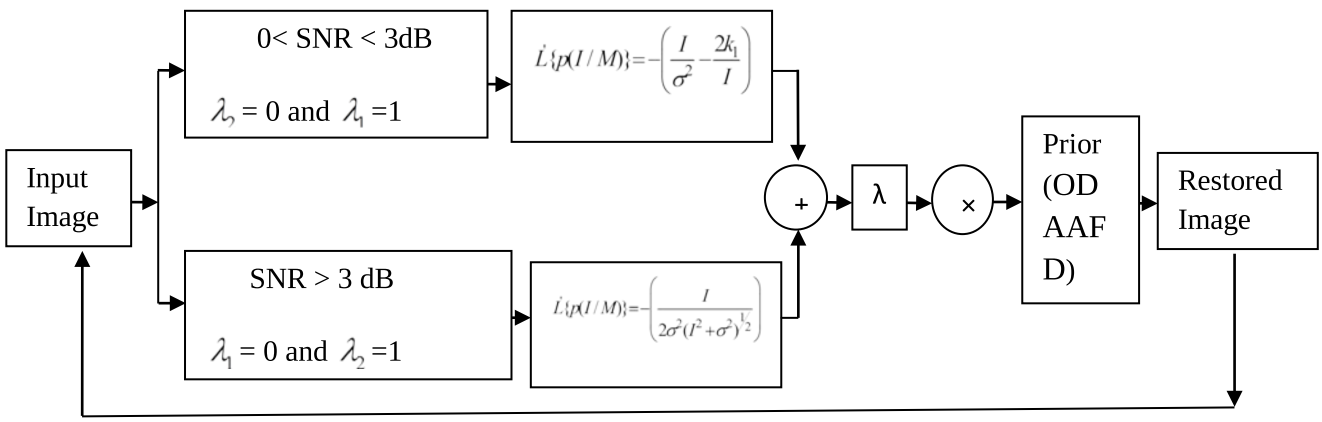


Fig.4.2: Restoration of MRI for Gaussian and Rician noise distribution with ODAAFD prior.

Hence when we combine the likelihood terms of the equation (4.13) and (4.14) then we get combined log-likelihood term given as follows:

$$L^{'}\{p(I/M)\} = -\left\{\frac{I}{\sigma^2} - \frac{2k_1}{I} + \frac{I}{2\sigma^2(I^2+\sigma^2)^{1/2}}\right\} \tag{4.15}$$

Therefore, the proposed general framework based model adapted to Rician's and Gaussian's distributed noise reads:

$$\frac{\partial I}{\partial t} = -\left[\lambda_1\left(\frac{I}{\sigma^2} - \frac{2k_1}{I}\right) + \lambda_2\left(\frac{I}{2\sigma^2(I^2+\sigma^2)^{1/2}}\right)\right] + \lambda . f(I) \tag{4.16a}$$

with initial condition

$$I_{t=0} = I_0 \tag{4.16b}$$

where λ_1 and λ_2 are the constants to be set according to noise pattern, and λ is the regularization parameter, I_0 is the noisy image data.

Orientation dependent anisotropic adaptive fuzzy diffusion (ODAAFD) based method

In this chapter we introduced orientation dependent anisotropic adaptive fuzzy diffusion (ODAAFD) regularization framework based prior terms which is examined for their efficacy in the proposed methods. The orientation dependent anisotropic adaptive fuzzy diffusion based prior [115]:

Orientation dependent anisotropic adaptive fuzzy diffusion based method
$f(I) = \nabla.\left\{w_{s,s-1,t,t}(\lVert\nabla I_x\rVert)I_x + w_{s,s,t,t-1}(\lVert\nabla I_y\rVert)I_y\right\}$

Fig.4.3: Selection of prior term

Here, equation given below is the anisotropic diffusion equation defined as [37]:

$$f\ (I_t) = \nabla.(c(\lVert\nabla I\rVert)\nabla I) \tag{4.17}$$

$$I_{t=0}(x,y) = I_0(x,y) \tag{4.18}$$

where $c(\|\nabla I\|)$ is the diffusion coefficient.

$$c(\|\nabla I\|) = \frac{1}{1 + (\frac{\nabla I}{\gamma})^2} \qquad (4.19)$$

Where γ is the threshold parameter, $\nabla.$ is the divergence operator and ∇ is the gradient operator, $I_{t=0} = I_0$ is the initial condition for noisy image.

$$f\ (I_t) = \nabla.\left\{c(\|\nabla I\|)\nabla I\right\} \qquad (4.20)$$

$$= \nabla.\left\{c(\|\nabla I\|)(I_x + I_y)\right\} \qquad (4.21)$$

$$= \nabla.\left\{c_x(\|\nabla I_x\|)I_x + c_y(\|\nabla I_y\|)I_y\right\} \qquad (4.22)$$

Now we replace $c_x = w_{s,s-1,t,t}$ and $c_y = w_{s,s,t,t-1}$ in equation (4.22)

$$= \nabla.\left\{w_{s,s-1,t,t}(\|\nabla I_x\|)I_x + w_{s,s,t,t-1}(\|\nabla I_y\|)I_y\right\} \qquad (4.23)$$

where

$$w_{s,s-1,t,t} = \omega \exp\left\{ 1 \Big/ 1 + \left((I_{s,t} - I_{s-1,t})/k_2\right)^2 \right\} \qquad (4.24)$$

$$w_{s,s,t,t-1} = \omega \exp\left\{ 1 \Big/ 1 + \left((I_{s,t} - I_{s,t-1})/k_2\right)^2 \right\} \qquad (4.25)$$

where s and t are the indices of the location of the attenuation coefficient along in plane domain (slice). k_2 is a scale factor which controls the strength of the diffusion during each iteration.

In the above Eq. (4.24) and (4.25), the diffusion process can be considered as a fuzzy classification in the view of fuzzy mathematics. That is, the more pixels belonging to the flat region, the stronger the diffusion effects. Therefore, the diffusion coefficient can be controlled by a membership function which gives the degree of belongingness in the region to be smoothed. Therefore, we adopt a membership function described as follow [116]:

$$\omega_{bj} = \frac{\exp\left(-\left\|I(n_j) - I(n_m)\right\|_E^2 \big/ h^2\right)}{d_{jm}} \tag{4.26}$$

$$\begin{aligned} I(n_j) &= \{I_l : l \in n_j\} \\ I(n_m) &= \{I_l : l \in n_m\} \end{aligned} \tag{4.27}$$

$$\left\|I(n_j) - I(n_m)\right\|_E^2 = \left\{I_{l \in n_j} - I_{l \in n_m}\right\}^2 \tag{4.28}$$

$$d_{jm} = \frac{1}{\sqrt{(j_x - m_x)^2 + (j_y - m_y)^2}} \tag{4.29}$$

where $I(n_j)$ and $I(n_m)$ represents the pixel value in two comparing neighbourhood n_j centred on the pixel *j* and n_m centred on the pixel *m* respectively. The parameter *h* is a factor which controls the decay of the pixel *m* and *j* respectively. The weight reflects the degree of similarity or connectivity between the pixel *j* and pixel *m* in the neighbourhood.

The value of k_2 is set to which is minimum absolute deviation (MAD) of the gradient of an image. The adaptive value of k_2 is estimated as:

$$k_2 = \sigma_e = 1.4826 \times median_I \left[\left\|\nabla I - median_I \left(\left\|\nabla I\right\|\right)\right\|\right] \tag{4.30}$$

Discretization of the proposed model:

For digital implementations, the Eqs. (4.16a), (4.16b) can be discretized using finite differences schemes [109]. For Orientation dependent anisotropic adaptive fuzzy diffusion (ODAAFD) regularization framework based model can be discretized using finite difference scheme reads

$$I^{n+1} = I^n + \Delta t.\left[-\left\{ \lambda_1 \left(\frac{I^n}{\sigma^2} \right) - \lambda_2 \left(\frac{2k_1}{I^n} \right) + \lambda_3 \left(\frac{I^n}{2\sigma^2 (I^{2n} + \sigma^2)^{1/2}} \right) \right\} + \lambda f^n(I) \right] \quad (4.31a)$$

where $f^n(I) = \nabla.\left\{ w_{s,s-1,t,t}(\left\|\nabla I_x^n\right\|) I_x^n + w_{s,s,t,t-1}(\left\|\nabla I_y^n\right\|) I_y^n \right\}$ (4.31b)

$$I_{t=0} = I_0 \quad (4.31c)$$

The von-Neumann analysis [109], shows that condition require $\frac{\Delta t}{(\Delta x)^2} < 1/4$, for the numerical scheme, given by equation (4.31a) to become stable. If the size of the grid is set to be Δx=1, after that Δt < ¼ i.e. Δt < 0.25. Hence, for the stability of equation (4.31a), the value of Δt is set to be 0.24.

4.4. Results and Discussions

To compare the effectiveness of the proposed technique, Brain Web database [1] is used for the simulated (synthetic) and real (clinical) data sets of normal brain MR images. T1, T2 and PD weighted are the three modalities (pulse sequences) dataset present in the Brain Web databases [1]. MATLAB R2014 was used for the implementation of the proposed method with different methods used for comparison purposes.

For de-noising MR images the parameters are adjusted empirically (Gaussian membership function, using MATLAB fuzzy logic toolbox) and the setup of all the parameters using the proposed scheme is shown in the Table 4.1. To evaluate the quantitative metrics, a noise variance having the range 5–30 % is mixed with the ground truth MR data artificially contaminated. The computation over 4-50 iterations or till the convergence of all these de-noising methods, required for Gaussian and Rician noise removal based on peak signal to noise ratio (PSNR) and root mean square error (RMSE).

The low and high noise regions are accurately differentiated by the proposed technique at high noise rates; hence the obtained result is better. Fig. 4.5(d), 4.5(e) and 4.5(f) show the comparison of the proposed filter using RMSE values for simulated data sets. The above figure clearly indicates that the proposed technique is superior at all noise levels. Comparison of the proposed filter using PSNR values is shown in Fig. 4.5(a), 4.5(b) and 4.5(c) for simulated data sets. The above figure clearly indicates that the proposed technique is superior at all noise levels.

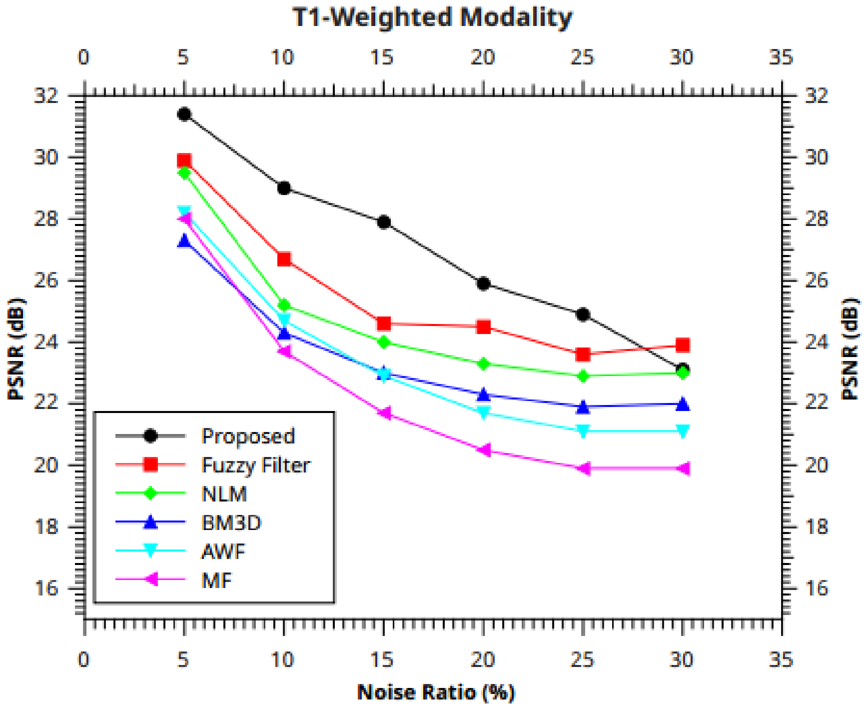
Fig.4.1 illustrate detailed results, obtained with the close up view of the images for better inspection, in order to compare the visual performance at different SNR value, Gaussian and Rician noise clearly identified. The proposed approaches, incorporates real image, noisy image and the restored image presented in Fig.4.4. The visual results for the simulated MR slice is corrupted with 15 % level of Rician and Gaussian noise are presented in Fig. 4.1. On the basis of quantitative and visual results it is apparent in Fig.4.4, that the proposed approach has produced more accurate results such as more noise removing ability and preservation of edges and structural information.

(a) (b) (c)

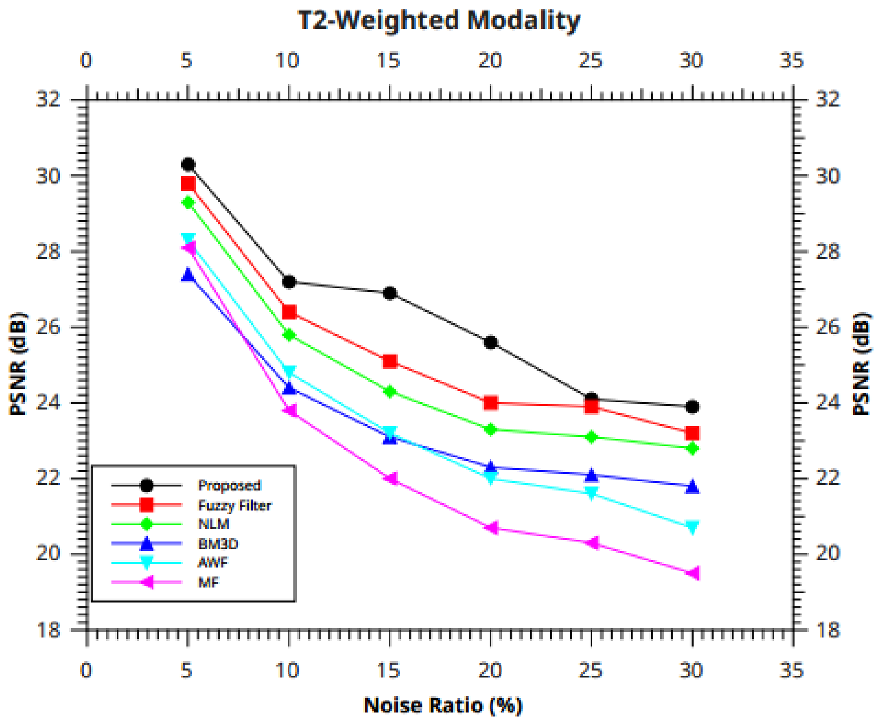
Fig.4.4: Simulated T1 weighted MR image with noise (a) Gaussian and Rician noise corrupted MR image (b) Restored image with proposed method from Gaussian noise (c) Restored image with proposed method from Rician noise.

Table 4.1: Parameters setup of the proposed method for de-noising MR images

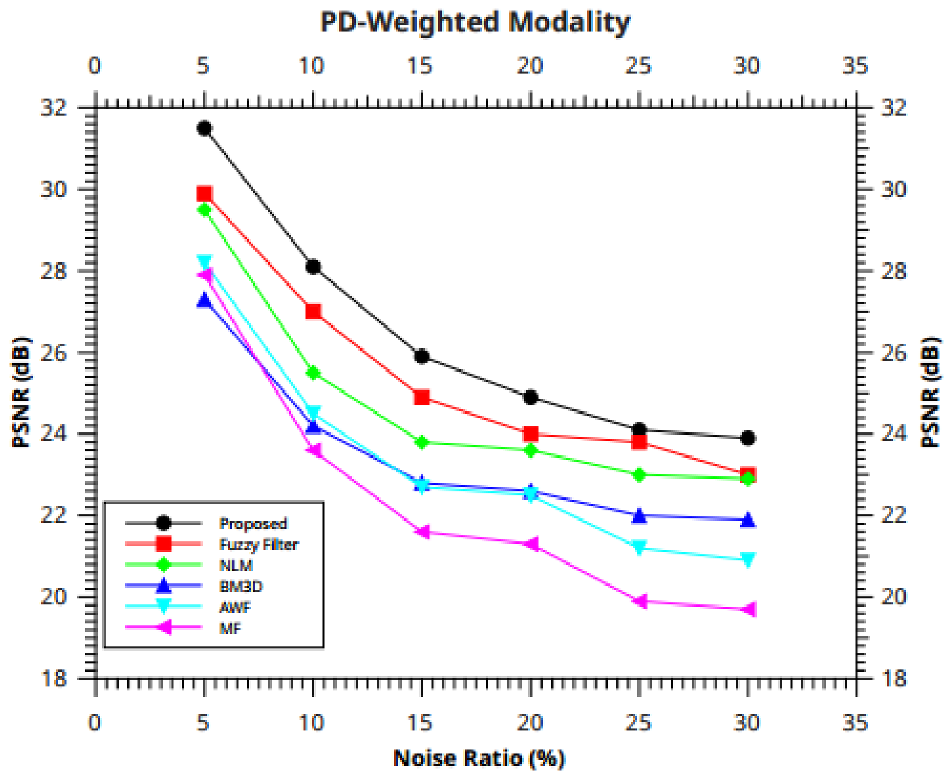
Parameter	Description	Value
Num_Iter	Number of iterations used as a parameter to getting desired output at five in the proposed method.	5.0
Δt	Integration constant which is used as a parameter to calculate the desired output at zero point one in the proposed method.	0.10
γ	Gradient modulus threshold used as a parameter that controls the conduction, getting desired output at four in the proposed method.	4.0
λ	Regularization parameter used for making balance between likelihood term and regularization function, getting desired output at zero point nine in the proposed method.	0.9


T1-Weighted Modality
PSNR (dB)
Noise Ratio (%)
Proposed
Fuzzy Filter
NLM
BM3D
AWF
MF

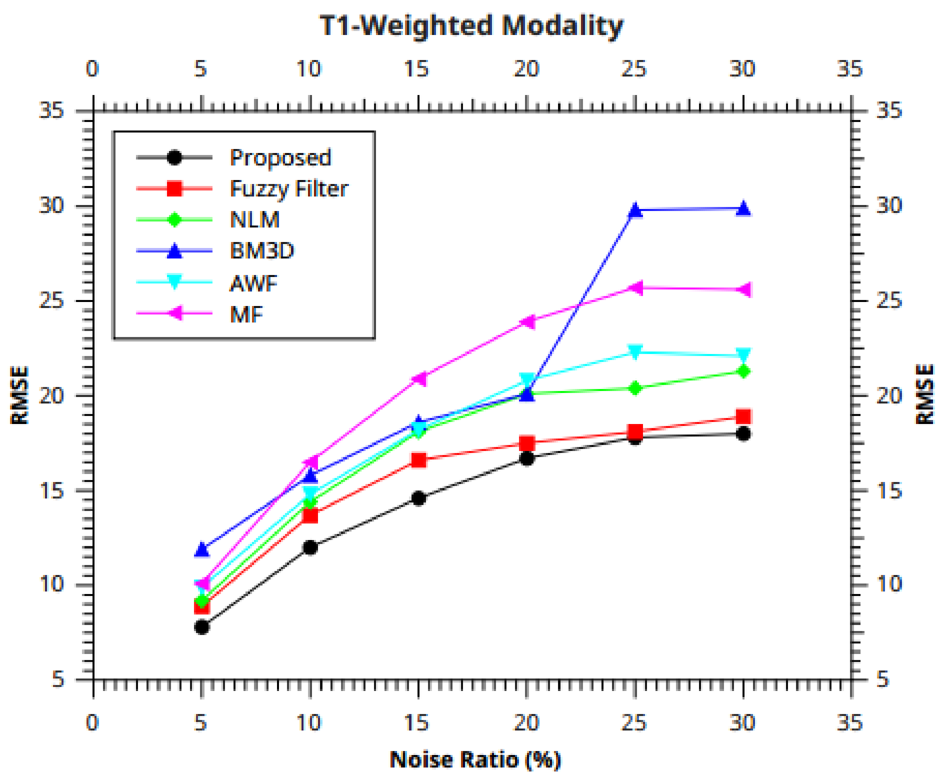
(a)


T2-Weighted Modality
PSNR (dB)
Noise Ratio (%)
Proposed
Fuzzy Filter
NLM
BM3D
AWF
MF

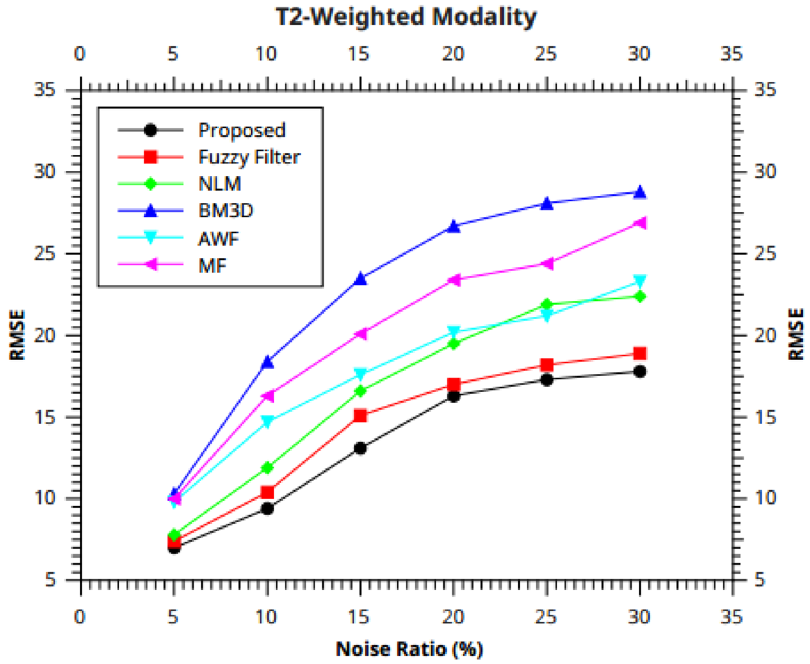
(b)


PD-Weighted Modality
PSNR (dB)
Noise Ratio (%)
Proposed
Fuzzy Filter
NLM
BM3D
AWF
MF

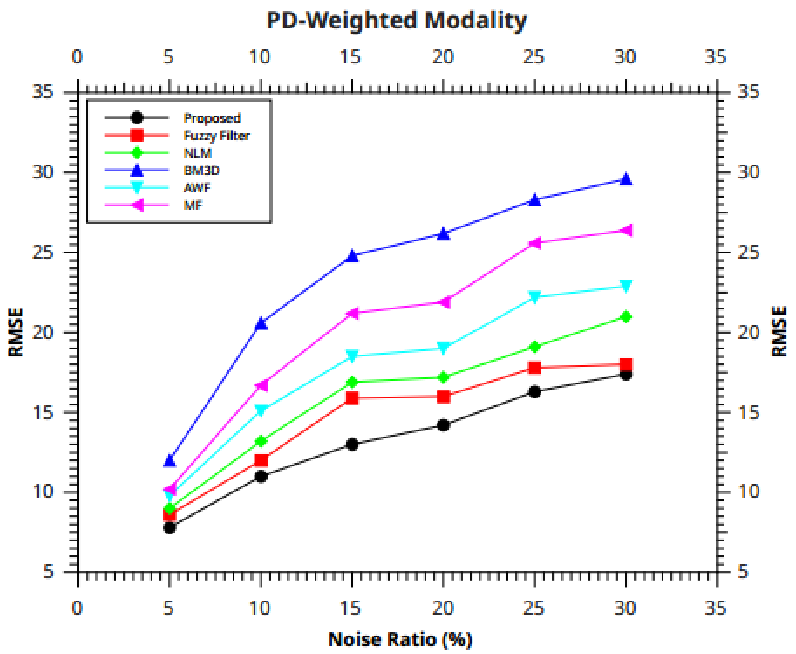
(c)


T1-Weighted Modality
RMSE
Noise Ratio (%)
Proposed
Fuzzy Filter
NLM
BM3D
AWF
MF

(d)



(e)



(f)

Figure 4.5: PSNR based comparison of various methods: (a) T1-weighted modality (b) T2-weighted modality (c) PD-weighted modality and RMSE based comparison of various methods: (d) T1-weighted modality (e) T2-weighted modality (f) PD-weighted modality.

4.5. Conclusions

This chapter presented orientation dependent anisotropic adaptive fuzzy diffusion based filter for restoration and enhancement of magnetic resonance images using experimental study with various methods. The proposed filter consists of two terms, namely data fidelity and prior. The data fidelity term, i.e. likelihood term is derived from Gaussian pdf and Rician pdf and an orientation dependent anisotropic adaptive fuzzy diffusion based prior is used. The Brain Web data set was used for testing of varying noise levels with different existing methods from proposed method. PSNR and RMSE were evaluated as a performance metrics. The better performance of the proposed method with ODAAFD prior is observed for comparative analysis and obtained results with other methods. Further, capability of better noise removal in the proposed technique with ODAAFD prior is clearly indicated by the visual results.