Chapter 3 : DESIGN AND DEVELOPMENT OF NONLINEAR PDE BASED FILTERS FOR RESTORATION AND ENHANCEMENT OF MR IMAGES

In this chapter, two proposed method for restoration and enhancement of magnetic resonance images have been studied and focuses on improving statistical iterative restoration algorithms by incorporating a suitable filters. This chapter is divided into the following sections. Section 3.3 formulates the first proposed model an efficient PDE-Based nonlinear filter adapted to Rician noise for restoration and enhancement of magnetic resonance images, their result analysis and discussion with their qualitative and quantitative analysis. Section 3.4 presents the second proposed model based on the modified complex diffusion based nonlinear filter for restoration and enhancement of magnetic resonance images, their result analysis and discussion with their qualitative and quantitative analysis. In section 3.5, final conclusion of this chapter is presented.

3.1. Introduction

Magnetic resonance imaging (MRI) is one of the most powerful and effective tool in the field of medical imaging. Storage or reproduction phases of processing, acquisition, preprocessing, compression, and transmission [84] are responsible for degradation of MRI. After the acquisition, removing the noise and increasing the accuracy of the clinical diagnostic system, post-processing, de-noising, and enhancement techniques are suitable alternatives. The Gaussian noise distribution is transformed into a Rician noise distribution in order to transform the MR image from complex to magnitude [85].

The performance of post-processing techniques is affected by the signal-dependent noise model for example, segmentation, registration, parametric image synthesis, or tensor estimation. In diffusion tensor performance depends on the categorization of Rician noise in magnitude MR images [86]. Noise removal in MR images is a critical and challenging task because MR signal has low SNR when containing more structural features. In literature, several Rician noise removal techniques have been reported.

First time, Henkelman [32] introduced the effect of the noise on MRI which is scrutinized for the estimation of magnitude MRI, from a noisy data degraded by Rician noise. A lot of filtering methods based on the signal averaging principal is used for the natural spatial pattern redundancy in the images. The filter is a common and simple approach which is used in some de-noising applications [87] because Gaussian filter blurs the edges and affects the high frequency region of the images. The edge preserving filters such as complex diffusion filters (CDFs) [85] mitigated this problem.

In this section a brief literature review on the MRI noise removal is presented. The various approaches for MRI noise removal can be broadly categorised as filtering approach, transform domain approach and statistical approach. Examples of filtering approach include linear filtering such as spatial filter [88] and temporal filter [88]. Non-linear filtering methods such as anisotropic diffusion filter (ADF) [37], adaptive ADF filter [89], noise driven ADF filter [90], noise adaptive ADF filter, fourth order PDE filters [91], adaptive fourth order PDE filter (Samsonov et al., 2004), fourth order complex PDE filters [92], non-local means filter (NLM) [39], fast NLM filter [93], Block wise optimised NLM filter [52], Unbiased NLM filter [19], dynamic NLM filter [94], enhanced NLM filter [54], adaptive NLM filter [40]. Combination of domain and range filters [41], bilateral domain and range filters [42] ,trilateral domain and range filters [42].

Examples of transform domain [66] approaches include curvelet [95], contourlet [69] and wavelet [96], adaptive multiscale product thresholding [97], multiwavelet [98], undecimated wavelet [99]. Examples of Statistical approach include maximum likelihood estimation [74, 18], linear minimum mean square error estimation [18], phase error estimation [72], nonparametric estimation [80], singularity function analysis [53,54] were presented. Some other Rician noise removal of MRI approaches was proposed in literature such as machine learning-based approaches [100,101,102,103,104,105], DCT-based filter [106], PCA-based technique [107], and conventional approaches [71].

In case of Rician noise removal, the above methods remove high-frequency signal components results in an introducing some extra bias in the quantification process and blurring the boundaries. Therefore to modify this drawback, advanced image restoration methods are required. In this chapter to address the problems mentioned above, the proposed technique used for enhancement, restoration and a comparative analysis of the same with other standard methods.

3.2. Background

The noise free MR image can be estimated directly from a real and imaginary component of complex valued MR image by transforming the complex value into magnitude image. The magnitude image shows the anatomical and physiological quantities in the MRI [50, 72]. For automated computer analysis [96], the magnitude MR images are real valued and can be visualized easily. The non-linear operation performed during transformation [57] converts the MRI distribution from Gaussian to Rician.

The Rician noise has amplitude given as $f(x,y) = f_R(x,y) + f_I(x,y)$ where $f_R(x,y)$ and $f_I(x,y)$ are zero mean, independent Gaussian random variables for some variance. The intensity field of Rician noise is given as $n(x,y) = \left|f(x,y)\right|^2 = f_R{}^2 + f_I{}^2$.

The image observation model for Rician noise reads:

$$I_0(x,y) = I(x,y) \times n(x,y) + \eta(x,y) \tag{3.1}$$

Where $I_0(x,y)$ is the observed Rician noised image; $I(x,y)$ is the original noise free image and $n(x,y)$ is the multiplicative noise with zero mean and known variance σ_n^2 and $\eta(x,y)$ is the detector noise which is additive in nature. Assuming the detector noise to be zero, the general observation model reads:

$$I_0(x,y) = I(x,y) \times n(x,y) \tag{3.2}$$

The Rician noise present in the acquired MRI image may lead to misinterpretation of medical image during diagnosis, therefore it must be reduced. The Rician noise is normally multiplicative in nature and distributed according to Rician's probability density function (pdf) in 2D MR image given as follows:

$$p(I/M) = \frac{M}{\sigma^2}\exp\left(-\frac{M^2+I^2}{2\sigma^2}\right)J_0\left(\frac{IM}{\sigma^2}\right)u(M) \tag{3.3}$$

Where I denotes amplitude of a noise-free image, σ^2 denotes the Gaussian noise variance, $J_0(.)$ show that modified zero order Bessel function. $u(.)$ is the unit step Heaviside function, and M is the magnitude MR image. The Rician PDF is only valid for nonnegative values of M.

3.3. An efficient PDE-Based nonlinear filter adapted to Rician noise for restoration and enhancement of magnetic resonance images

In the present chapter, a PDE based nonlinear filter adapted to Rician noise is proposed for removal of Rician noise from MR images. The proposed method is casted into a variational framework. The introduced filter consists of two terms wherein the first term is a data fidelity term and the second term is a prior function. The first term is obtained by minimizing the negative log likelihood of Rician pdf. Since the solution of the first term is ill-posed in nature and hence a prior function is introduced which is a nonlinear anisotropic diffusion based filter. To balance the trade off between data fidelity term and prior a regularization parameter has been introduced. The performance analysis and comparative study of the proposed method with other standard methods is presented for Brain Web dataset at varying noise levels in terms of PSNR and SSIM. From the simulation results, it is observed that the proposed method is performing better noise removal in comparison to other methods.

3.3.1. Methods and Models

The Rician noise removal and regularization of MR image data is obtained by minimizing the following nonlinear energy functional of the image I within a continuous domain Ω:

$$E(I) = \arg_{m}\left\{\int_{\Omega}\left[L(P(I/M)) + \lambda.\phi(\|\nabla I\|)\right]\right\} \tag{3.4}$$

where $L(P(I/M))$ shows the negative likelihood term of Rician distributed noise in MRI. During the filtering process log likelihood term measures the dissimilarities at a pixel between M and its estimated value I. For positive integer the value of unit step Heaviside function $u(.)$ in equation (3.3) is equal to one. To calculate the log likelihood of above mentioned Rician probability distribution function, at first we take the logarithm of both sides and to make the computation easier and faster, we try to simplify the third term in equation (3.3), i.e., zero order modified Bessel's function $J_0(\frac{IM}{\sigma^2})$. According to Abramowitz and Stregun handbook of mathematical functions [108], the solution of $J_0(.)$, zero-order Bessel functions for positive integer is given as:

$$J_0(z) = \sum_{k_1=0}^{\infty} \frac{(-\frac{z^2}{4})^{k_1}}{k_1!^2} \tag{3.5}$$

Now taking the logarithm of the above function and we get the simplified solution with logarithmic domain. Substituting the value of $z = \frac{IM}{\sigma^2}$ in the above equation then we differentiating equation (3.5) get the third term of equation (3.3) as follows:

$$\frac{\partial}{\partial I}\left\{\log j_0(\frac{IM}{\sigma^2})\right\} = \frac{2k_1}{I} \tag{3.6}$$

To obtain maximum likelihood of I, the first order derivative of (3.3) Rician's pdf w.r.t. I reads:

$$L^{'}\{P(I/M)\} = -\frac{I}{\sigma^2} + \frac{2k_1}{I} \tag{3.7}$$

where $L^{'}\{P(I/M)\}$ acts as the data attachment term or the likelihood term in equation (3.4). Maximization of log likelihood or minimization of the negative log likelihood leads to de-noising of image data, but it is an ill-posed problem and hence regularization is needed. That's why the second term $\phi(\|\nabla I\|$ in equation (3.4) is needed and it acts as a regularization or penalty function. In the equation (3.4), λ is a regularization parameter, which has a constant value and makes a balance between the data attachment term and regularization function. For deriving anisotropic diffusion based filter defined by Perona and Malik [37] is the suitable choice for the energy term $\phi(\|\nabla I\|$ based on energy function. $\phi(\|\nabla I\|) = \|\nabla I\|^2$ which is the gradient norms of the image. Substituting the value of gradient norms in equation (3.4) we get:

$$E(I) = \arg_m\left\{\int_\Omega \left[L(P(I/M)) + \lambda.\phi\|\nabla I\|^2\right]\right\} \tag{3.8}$$

The anisotropic diffusion based filter originally proposed by Perona and Malik [37] and the final model for the restoration of MR image, corrupted from Rician Noise given as:

$$\frac{\partial I}{\partial t} = L^{'}(P(I/M)) + \lambda.div(c(\|\nabla I\|)\nabla I) \tag{3.9a}$$

with initial condition

$$I_{t=0} = I_0 \tag{3.9b}$$

The initial condition as the noisy image I_0 of the PDE equation (3.9b) creates after certain iterations till its convergence the filtered de-noised Rician image. Where term $c(\|\nabla I\|)$ in equation (3.9a) is known as conductivity coefficient and given as:

$$c(\|\nabla I\|) = \frac{1}{1 + \frac{\|\nabla I\|^2}{\gamma^2}} \tag{3.10}$$

where γ is known as gradient threshold which differentiates the homogeneous regions and area of edges and contours, the value of gradient threshold must be greater than zero.

Therefore, the proposed anisotropic diffusion based model adapted to Rician's distributed noise reads:

$$\frac{\partial I}{\partial t} = (\frac{2k_1}{I} - \frac{I}{\sigma^2}) + \lambda\Delta.(c(\|\nabla I\|)\nabla I) \tag{3.11a}$$

with initial condition

$$I_{t=0} = I_0 \tag{3.11b}$$

Discretization of the proposed model:

The equation (3.11a) and (3.11b) can be discretized using finite difference schemes [109], is the proposed model. The proposed PDE based model is represented in the discretized form given as:

$$I^{n+1}(x, y) = I^n(x, y) + \Delta t.\left[L'(p(I/M)) + \lambda.div(c(\|\nabla I^n(x, y)\|)\nabla I^n(x, y))\right] \tag{3.12}$$

$$I^{n+1}(x, y) = I^n(x, y) + \Delta t.\left[(\frac{2k_1}{I^n(x, y)} - \frac{I^n(x, y)}{\sigma^2}) + \lambda.div(c(\|\nabla I^n(x, y)\|)\nabla I^n(x, y))\right] \tag{3.13}$$

The von-Neumann analysis [109], shows that condition require $\Delta t / \Delta x \leq 1/4$ for the numerical scheme, given by above equation to become stable. If the size of the grid is set to be $\Delta x=1$, after that $\Delta t < 1/4$ i.e. $\Delta t < 0.25$. Hence, for the stability of above equation, the value of Δt is set to be 0.24.

3.3.2. Results and discussions

The Brain Web database [1] is used for simulated (synthetic) and real (clinical) data sets of normal brain MR images, to compare the effectiveness of the proposed technique. There are three modalities (pulse sequences) dataset present in the Brain Web data base [1] which are T1, T2 and PD weighted. The proposed method and other standard methods used for comparison purposes were implemented using MATLAB R2014.The performance of restoration results are analyzed for images artificially degraded by Rician noise. Non-local means (NLM) filter [19], adaptive Wiener filter (AWF) [110], Fuzzy-based hybrid filter [105], MF [110], and BM3D de-noising filter [111] are familiar existing techniques used for comparing the proposed method. The best setups as proposed by the authors and the free parameters of these methods are used during experimentation.

To obtain the best results the relevant values of the parameters are given below:

- BM3D [111]: for computing the noise level, actual noise variance is taken and other parameter adjusted according to the authors in the article.
- MF [110]: window of size 3×3.
- AWF [110]: window using a 5×5 neighborhood. The noise variance is manually locate to the real value to achieve the best performance.
- Fuzzy based hybrid filter [105]: the parameters are adjusted according to the authors in the article.

- NLM filter [19]: the noise level is computed by using the actual noise variance, radius of the searching area=5, radius of the local area=3, the correction constant=0.15.

The parameters are adjusted empirically for de-noising MR images and the setup of all the parameters using the proposed scheme is shown in the Table 3.1 Peak signal-to-noise ratio (PSNR) and structure similarity index map (SSIM) are used for quantitative comparison. Performance of the proposed approach is compared with few standard de-noising methods on simulated and real MR data sets. The ground truth MR data are artificially contaminated with a noise variance having the range 5–30 % to evaluate the quantitative metrics. Based on PSNR and SSIM average restoration results over 5-50 iterations or till the convergence of all these de-noising methods are computed.

The performance analysis and comparative study of the proposed method with other standard methods are represented in Table 3.2 on the basis of quantitative results (PSNR, SSIM) for different levels of Rician noise. The PSNR and SSIM values show that at low as well as at high rates of Rician noise, the proposed PDE based technique has much better restoration results than existing methods. Retaining the important structural information, such as texture and edges, is considered as an important task in image restoration during noise smoothing process. The detailed information, present in the image do not quantify PSNR. A well-known quantitative measure SSIM (Table 3.2) is used for measure the detail preservation performance of the proposed filter. The proposed technique is superior in terms of retaining structural information at all noise levels clearly shown in Fig. 3.2(d), 3.2(e), 3.2(f).

The non-local filter [19] gives good results at low-noise-corrupted detailed regions and the local filter [110] performs better at smooth regions degraded with high noise, whereas, the proposed technique performs better in both situations i.e. at low as well as high noise variance in image data. The hybrid filter [105] is based on noise contamination and its region characteristic, which adaptively assign appropriate fuzzy weights to local and non-local filters and the restoration results are better than NLM [19] based method. The proposed technique is an improvement in comparison with the best performing NLM filter and fuzzy based hybrid filter [105], at a low noise rate (5 %), 4.8 dB for T1, 5.1 dB for T2, and 4.8 dB for PD. Table I and Table II shows that the efficiency of proposed filter also increases as the noise rate increases. At high noise rates, the proposed technique accurately differentiates the low and high noise regions, hence the better result is obtained. Similarly, in the case of PSNR, Table 3.2 shown in Fig. 3.2(a), 3.2(b), 3.2(c) that the proposed scheme outperforms than the existing techniques.

Fig.3.1 illustrate detailed results, obtained with the close up view of the restored images for better inspection, in order to compare the visual performance, existing and proposed approaches, incorporates real image, noisy image and the restored image. The visual results for simulated MR slice is corrupted with 10 % level of Rician noise, in Figure 3.1. The BM3D and AWF are unable to smooth out noise completely and provide a blurred output as it can be observed from the Fig.3.1. Median filter is good at preserving some details at the cost of some noisy spots, NLM filter removes the noise completely, but taking much more time and most of the image structural information has been lost. Similarly, fuzzy based hybrid filter provides better results with noisy spots than NLM filter. To overcome these limitations, the proposed approach produces better results. On the basis of quantitative and visual results it is apparent that the proposed approach has produced more accurate results such as more noise removing ability, and preservation of edges and structural information, at all levels of Rician noise.

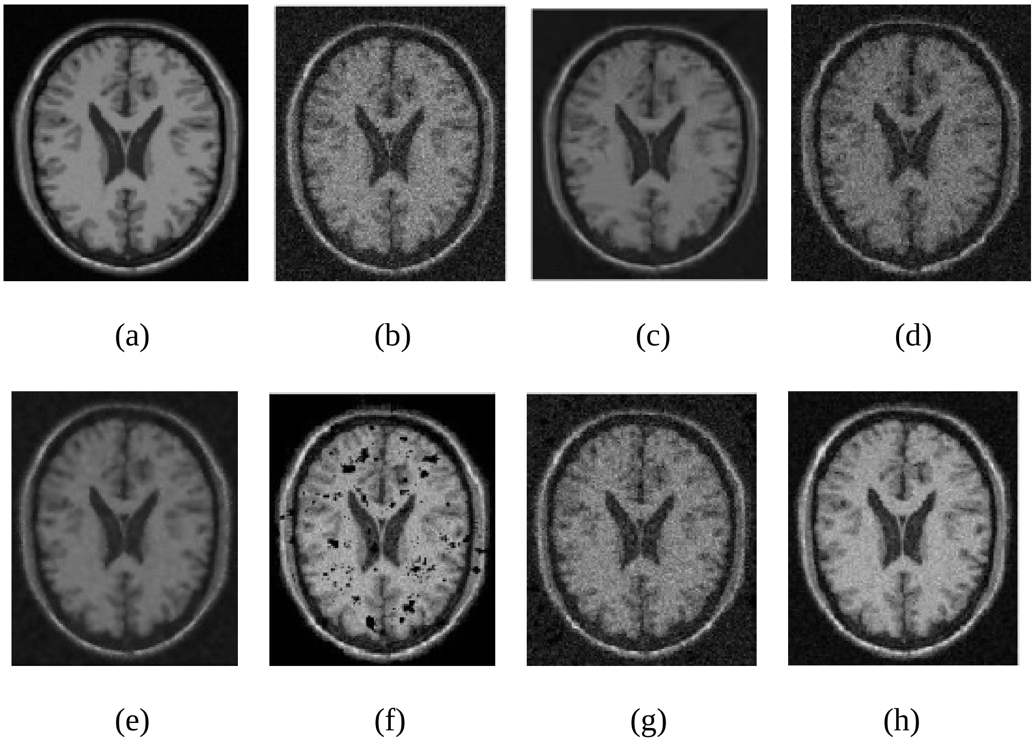


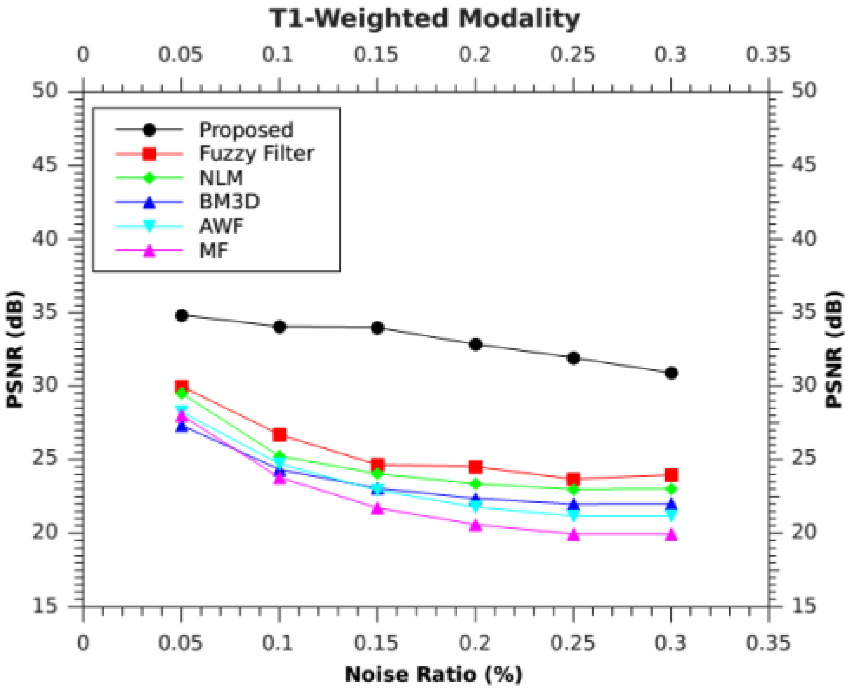
Fig.3.1: Simulated T1 weighted MR image with Rician noise (a) Original image (b) 10% noisy image (c) BM3D (d) MF (e) AWF (f) NLM (g) Fuzzy filter (h) Proposed.

Table 3.1: Parameters setup of the proposed method for de-noising MR images

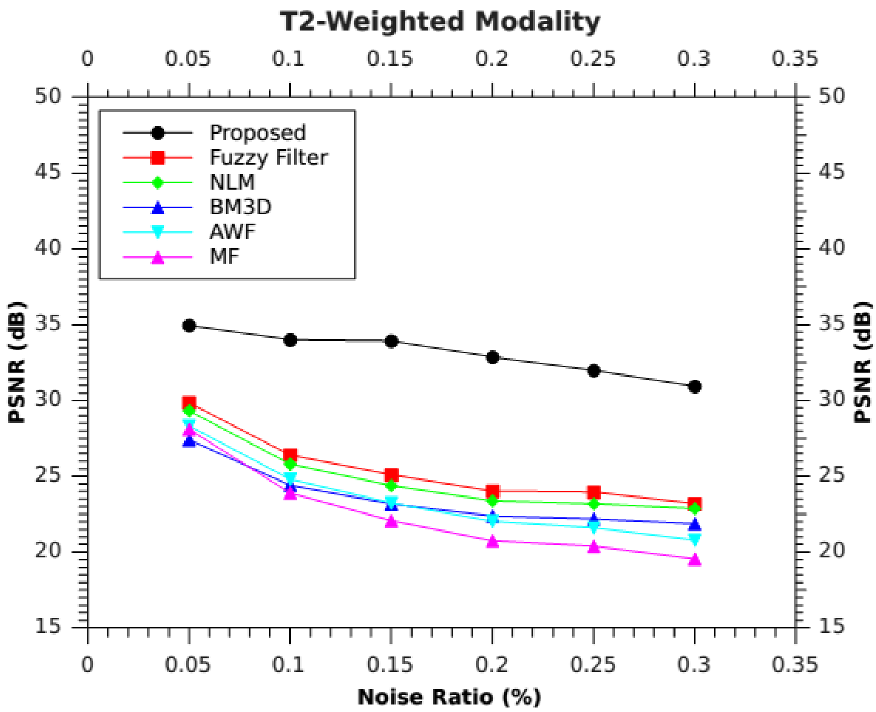
Parameter	**Description**	**Value**
Num_Iter	Number of iterations used as a parameter to getting desired output at five in the proposed method.	5.0
Δt	Integration constant which is used as a parameter to calculate the desired output at zero point one in the proposed method.	0.10
γ	Gradient modulus threshold used as a parameter that controls the conduction, getting desired output at four in the proposed method.	4.0
Option	Conduction coefficients [30] used option two for getting desired output in the proposed method. 1. $c = \exp(-(\|\nabla I\|/\gamma)^2)$ 2. $c = 1/(1+(\|\nabla I\|/\gamma)^2)$	2
λ	Regularization parameter used for making balance between likelihood term and regularization function, getting desired output at zero point nine in the proposed method.	0.9

Table 3.2: Quantitative comparison on Simulated MR data (Brain Web) using PSNR (SSIM)

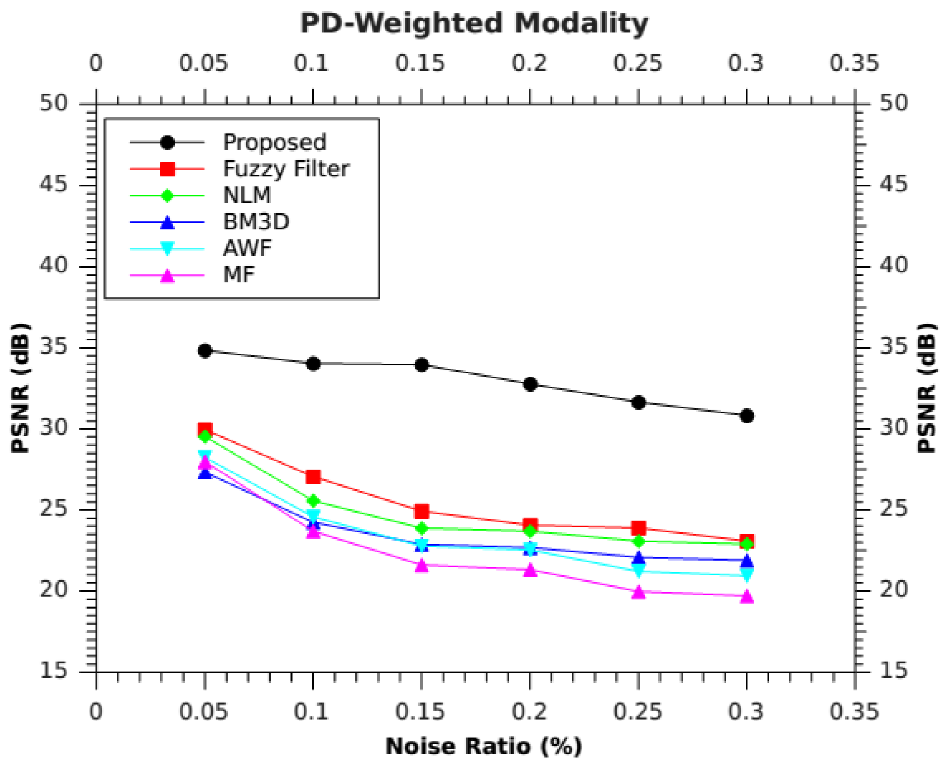
Modality (slice)	**Noise ratio (%)**	**MF [110]**	**AWF [110]**	**BM3D [111]**	**NLM method [19]**	**Fuzzy-filter [105]**	**Proposed Method**
T1-weighted (slice 70)	5	28.0(0.69)	28.2(0.66)	27.3(0.64)	29.5(0.97)	29.9(0.98)	34.8(0.98)
	10	23.7(0.47)	24.7(0.49)	24.3(0.46)	25.2(0.96)	26.7(0.96)	34.0(0.97)
	15	21.7(0.35)	22.9(0.42)	23.0(0.34)	24.0(0.90)	24.6(0.92)	33.9(0.96)
	20	20.5(0.30)	21.7(0.37)	22.3(0.29)	23.3(0.84)	24.5(0.85)	32.8(0.92)
	25	19.9(0.26)	21.1(0.34)	21.9(0.25)	22.9(0.76)	23.6(0.78)	31.9(0.90)
	30	19.9(0.24)	21.1(0.33)	22.0(0.23)	23.0(0.70)	23.9(0.72)	30.9(0.88)
	Mean	22.3(0.38)	23.3(0.43)	23.5(0.37)	24.7(0.86)	25.5(0.87)	33.0(0.93)
T2-weighted (slice 70)	5	28.1(0.69)	28.3(0.66)	27.4(0.61)	29.3(0.98)	29.8(0.98)	34.9(0.98)
	10	23.8(0.47)	24.8(0.49)	24.4(0.46)	25.8(0.95)	26.4(0.96)	34.0(0.97)
	15	22.0(0.36)	23.2(0.45)	23.1(0.35)	24.3(0.93)	25.1(0.94)	33.9(0.94)
	20	20.7(0.30)	22.0(0.38)	22.3(0.29)	23.3(0.92)	24.0(0.93)	32.8(0.94)
	25	20.3(0.26)	21.6(0.35)	22.1(0.25)	23.1(0.86)	23.9(0.87)	31.9(0.90)
	30	19.5(0.23)	20.7(0.32)	21.8(0.23)	22.8(0.70)	23.2(0.72)	30.9(0.89)
	Mean	22.4(0.39)	23.4(0.44)	23.5(0.37)	24.8(0.89)	25.4(0.90)	33.1(0.93)
PD-weighted (slice 50)	5	27.9(0.69)	28.2(0.66)	27.3(0.60)	29.5(0.98)	29.9(0.98)	34.8(0.98)
	10	23.6(0.47)	24.5(0.48)	24.2(0.46)	25.5(0.96)	27.0(0.97)	34.0(0.97)
	15	21.6(0.36)	22.7(0.42)	22.8(0.35)	23.8(0.95)	24.9(0.96)	33.9(0.96)
	20	21.3(0.30)	22.5(0.38)	22.6(0.29)	23.6(0.89)	24.0(0.90)	32.7(0.92)
	25	19.9(0.26)	21.2(0.35)	22.0(0.25)	23.0(0.77)	23.8(0.78)	31.6(0.90)
	30	19.7(0.23)	20.9(0.32)	21.9(0.22)	22.9(0.70)	23.0(0.71)	30.8(0.88)
	Mean	22.3(0.39)	23.3(0.44)	23.5(0.36)	24.7(0.88)	25.5(0.88)	33.0(0.93)
Overall Mean		22.3(0.39)	23.4(0.44)	23.5(0.37)	24.7(0.87)	25.5(0.88)	33.0(0.93)


T1-Weighted Modality
Proposed
Fuzzy Filter
NLM
BM3D
AWF
MF
PSNR (dB)
Noise Ratio (%)
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35
15 20 25 30 35 40 45 50

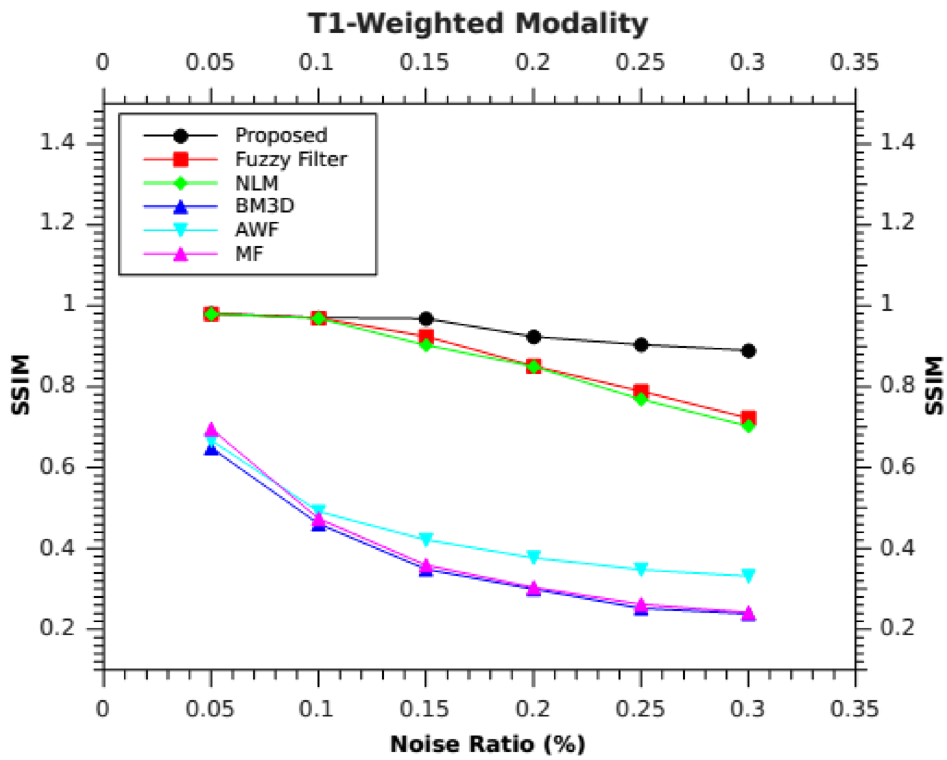
(a)


T2-Weighted Modality
Proposed
Fuzzy Filter
NLM
BM3D
AWF
MF
PSNR (dB)
Noise Ratio (%)
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35
15 20 25 30 35 40 45 50

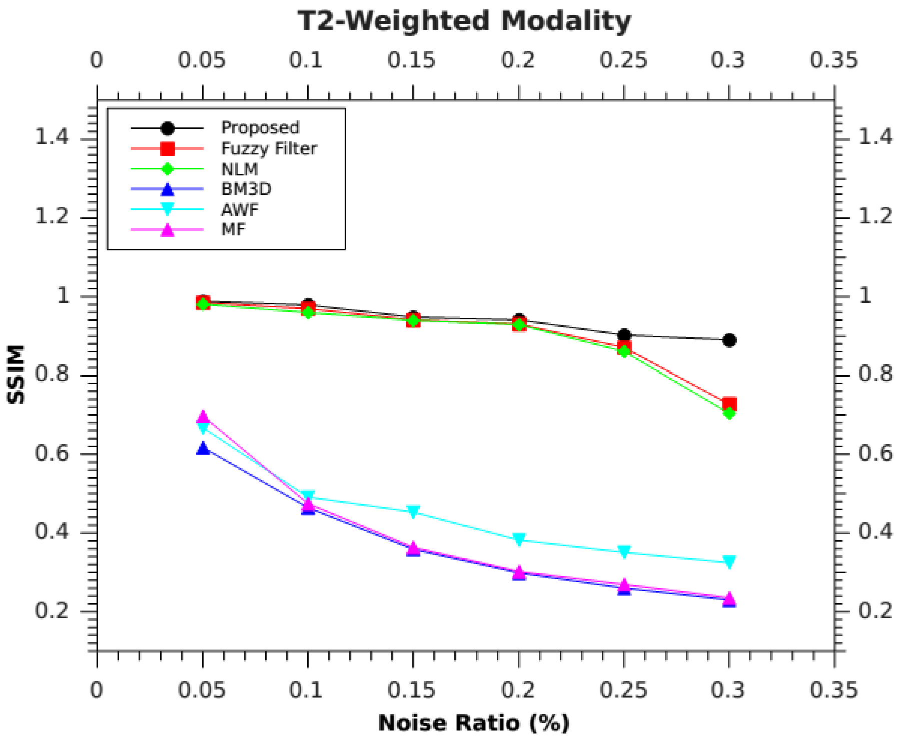
(b)


PD-Weighted Modality
PSNR (dB)
Noise Ratio (%)
Proposed
Fuzzy Filter
NLM
BM3D
AWF
MF

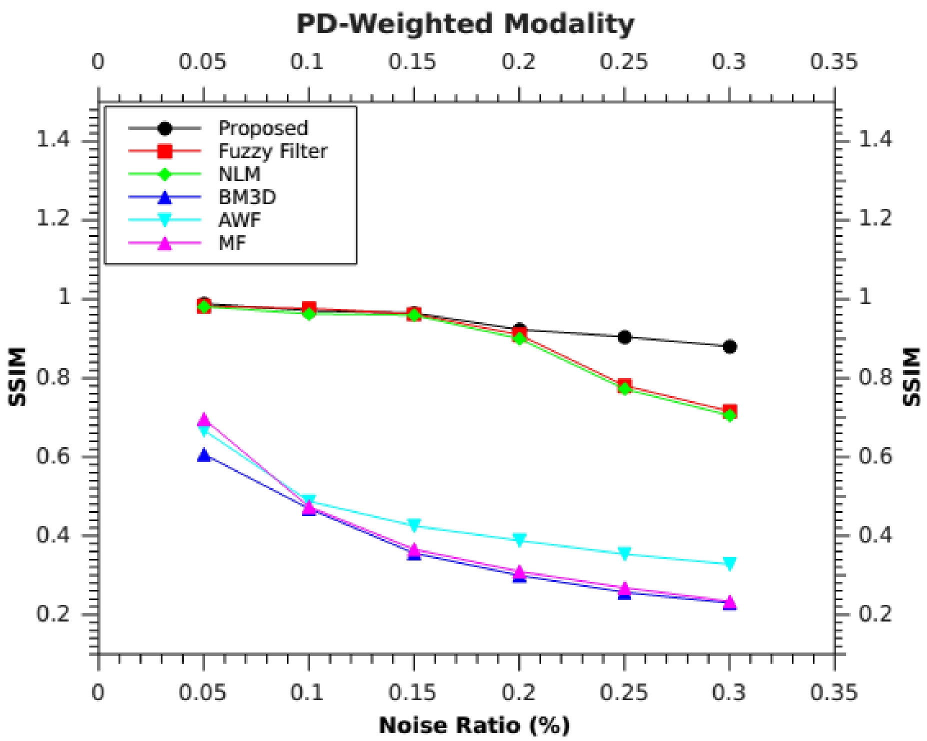
(c)


T1-Weighted Modality
SSIM
Noise Ratio (%)
Proposed
Fuzzy Filter
NLM
BM3D
AWF
MF

(d)



(e)



(f)

Figure 3.2: PSNR based comparison of various methods: (a) T1-weighted modality (b) T2-weighted modality (c) PD-weighted modality and SSIM based comparison of various methods: (d) T1-weighted modality (e) T2-weighted modality (f) PD-weighted modality.

3.4. Modified complex diffusion based nonlinear filter for restoration and enhancement of magnetic resonance images

In this chapter, a PDE based modified complex diffusion based nonlinear filter is proposed, for removal of Rician noise from MRI. The proposed method is casted into a variational framework. The introduced filter consists of two terms wherein the first term is a data likelihood term and the second term is a prior function. The first term is obtained by minimizing the negative log likelihood of Rician pdf. The mathematical simplification has also been introduced to compute the first term. Due to ill-posedness of the likelihood term, a prior function is introduced which is a nonlinear complex diffusion based filter. A regularization parameter is used to balance the trade off between data fidelity term and prior. The finite difference scheme is used for the discretization of the proposed method. The performance analysis and comparative study of the proposed method with other standard methods is presented for Brain Web dataset. The values of performance measures such as PSNR, RMSE, structure similarity index map (SSIM), and correlation parameter (CP) are presented for various noise levels. From the simulation results, it is observed that the proposed method is performing better in comparison to other methods in consideration.

3.4.1. Methods and Models

In this section we proposed complex diffusion based prior for restoration and enhancement of MRI. Complex diffusion based filter is suitable choice for the energy term $\phi(\|\nabla I\|)$ based on the concept of energy function.

$$\phi(\|\nabla I\|) = div(c(\mathrm{Im}(I))\nabla I) \tag{3.14}$$

Substituting the value of $\phi(\|\nabla I\|)$ in equation (3.4) nonlinear energy functional equation reads:

$$E(I) = \arg_{\min}\left\{\int_{\Omega}\left[L(p(I/M)) + \lambda.div(c(\mathrm{Im}(I))\nabla I)\right]d\Omega\right\} \tag{3.15}$$

The log likelihood term in the equation (3.15) of Rician PDF that describes the Rician noise in the MRI. To make the PDE defined by equation (3.15) adapted to Rician noise, the data attachment term plays a vital role. The complex diffusion and the final model for the restoration of MR image, corrupted from Rician noise can replace the second term in equation (3.15), given as:

$$\frac{\partial I}{\partial t} = L^{'}\{p(I/M)\} + \lambda.div(c(\mathrm{Im}(I))\nabla I) \tag{3.16a}$$

with initial condition

$$I_{t=0} = I_0 \tag{3.16b}$$

The initial condition as the noisy image I_0 of the PDE equation (3.16b), creates after certain iterations till its convergence the filtered de-noised Rician image. A PDE based nonlinear complex diffusion based filter adapted to Rician's noise in MR images is represented by equation (3.16). Where term $c(\mathrm{Im}(I))$ in equation (3.16a) is known as diffusion coefficient and given as:

$$c(\mathrm{Im}(I)) = \frac{e^{i\theta}}{1 + \left(\frac{\mathrm{Im}(I)}{k\theta}\right)^2} \tag{3.17}$$

Here, k is edge threshold parameter and the value of k ranges from 1 to 1.5 for digital images [85]. The evolution of real part of the image is controlled by the linear forward diffusion. The evolution of imaginary part of the image is controlled by both the real and imaginary equations for nonlinear complex diffusion process defined by Eqs. (3.16a), (3.16b) and (3.17). A qualitative property of edge detection i.e. second smoothed derivative is described by the imaginary part of the image for small value of θ. Where as real values depict the properties of ordinary Gaussian scale -space. For large values of θ, the imaginary part feeds back into the real part creating the wave like ringing effect which is an undesirable property. Here, for experimentation purposes the value of θ is chosen to be $\pi/30$ .The adaptive value of edge threshold parameter is used in Eq.(3.17). It is defined as negative exponential distribution [85]:

$$kt \approx k_0 \exp(-\alpha t) \tag{3.18}$$

where α and k_0 are constants, usually 1.

Therefore, the proposed complex diffusion based model adapted to Rician's distributed noise reads:

$$\frac{\partial I}{\partial t} = -(\frac{I}{\sigma^2} - \frac{2k_1}{I}) + \lambda \nabla.(c(\mathrm{Im}(I))\nabla I) \tag{3.19a}$$

with initial condition

$$I_{t=0} = I_0 \tag{3.19b}$$

Discretization of the proposed model:

The equation (3.19a) can be discretized using finite difference schemes [109] is the proposed model. The proposed PDE based model is represented in the discretized form given as:

$$I^{n+1}(x,y) = I^n(x,y) + \Delta t.[-(\frac{I^n(x,y)}{\sigma^2} - \frac{2k_1}{I^n(x,y)}) + \lambda.div(c(\mathrm{Im}(I^n(x,y)))\nabla I^n(x,y))] \tag{3.20}$$

The von-Neumann analysis [109], shows that condition require $\frac{\Delta t}{(\Delta x)^2} < 1/4$, for the numerical scheme, given by equation (3.20) to become stable. If the size of the grid is set to be Δx=1, after that Δt < ¼ i.e. Δt < 0.25. Hence, for the stability of equation (3.20), the value of Δt is set to be 0.24.

3.4.2. Results and discussions

Brain Web database [1] is used for simulated (synthetic) and real (clinical) data sets of normal brain MR images, to compare the effectiveness of the proposed technique. There are three modalities (pulse sequences) dataset present in the Brain Web databases [1] which are T1, T2 and PD weighted. The proposed method and other standard methods used for comparison purposes were implemented using MATLAB R2014.

The performance of restoration results are analyzed for images artificially degraded by Rician noise. NLM filter [19], adaptive Wiener filter (AWF) [110], Fuzzy-based hybrid filter [105], MF [110] and BM3D de-noising filter [111] are familiar existing techniques used for comparing the proposed method. The best setups as proposed by the authors and the free parameters of these methods are used during experimentation. To obtain the best results the relevant values of the parameters are given below:

- BM3D [111]: for computing the noise level, actual noise variance is taken and other parameter adjusted according to the authors in the article [111].
- MF [110]: window of size 3×3.
- AWF [110]: window using a 5×5 neighborhood. The noise variance is manually located to the real value to achieve the best performance.
- Fuzzy based hybrid filter [105]: the parameters are adjusted according to the authors in the article.
- NLM filter [19]: the noise level is computed by using the actual noise variance, radius of the searching area=5, radius of the local area=3, the correction constant=0.15.

The parameters are adjusted empirically for de-noising MR images and the setup of all the parameters using the proposed scheme is shown in the Table 3.3. The ground truth MR data are artificially contaminated with a noise variance having the range 5–30 % to evaluate the quantitative metrics. Based on PSNR, RMSE, SSIM and CP average restoration results over 4-50 iterations or till the convergence of all these de-noising methods are computed.

The performance analysis and comparative study of the proposed method with other standard methods are represented in Table 3.4 and Table 3.5. On the basis of quantitative results PSNR (RMSE) and SSIM (CP) represented in Table 3.4 and Table 3.5 respectively for different levels of Rician noise. The PSNR RMSE SSIM and CP values show that at low as well as at high rates of Rician noise, the proposed technique has much better restoration results than existing methods.

Retaining the important structural information, such as texture and edges, is considered as an important task in image restoration during noise smoothing process. The detailed information, present in the image do not quantify PSNR and RMSE. A well-known quantitative measure SSIM and CP is used for measuring the detail preservation performance of the proposed filter shown in the Table 3.5. The proposed technique is superior in terms of retaining structural information at all noise levels clearly shown in Fig. 3.4(d), 3.4(e), 3.4(f).

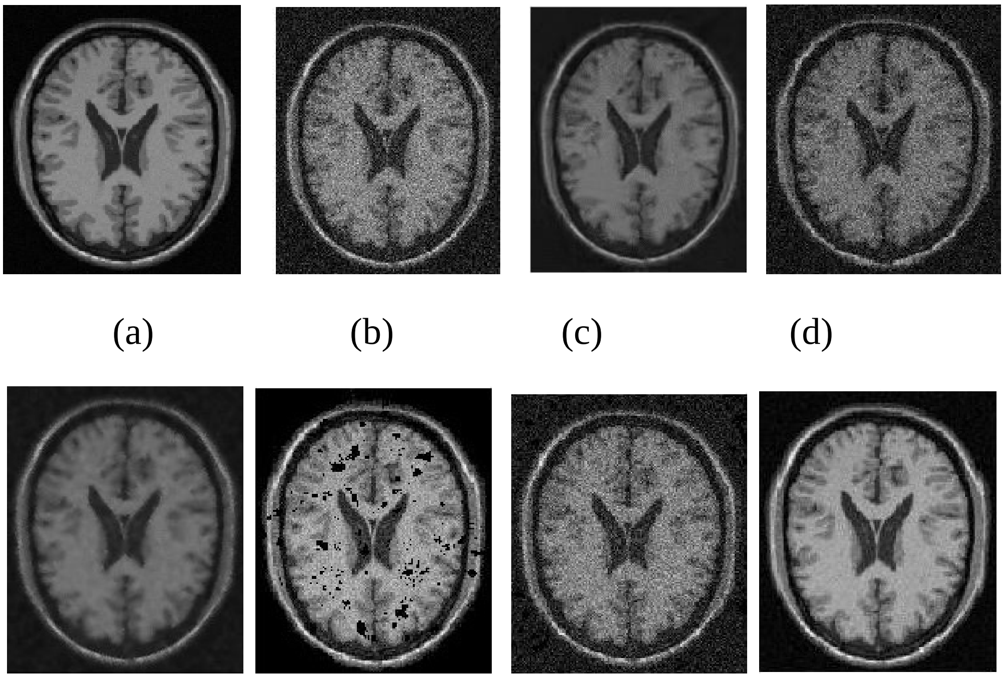
The non-local filter [19] gives good results at low-noise-corrupted detailed regions and the local filter [110] performs better at smooth regions degraded with high noise. Whereas, the proposed technique performs better in both situations i.e. at low as well as high noise variance in image data. The fuzzy hybrid filter [105] is based on noise contamination and its region characteristic, which are adaptively assigned appropriate fuzzy weights to local and non-local filters and the restoration results are better than NLM [19] based method.

The proposed technique is an improvement in comparison with the best performing NLM filter and fuzzy based hybrid filter [105].Improvement in the Table 3 shows at a low noise rate (5 %), 6.5 dB for T1, 6.5 dB for T2, and 6.6 dB for PD modality. Results, shows that the efficiency of proposed filter also increases as the noise rate increases. At high noise rates, the proposed technique accurately differentiates the low and high noise regions, hence the better result obtained. Similarly, in the case of PSNR and RMSE, Table 3.4 shows that the proposed scheme outperforms the existing techniques. Comparison of the proposed filter using PSNR values are shown in Fig. 3.4(a), 3.4(b), 3.4(c) respectively using simulated data sets. The above figure clearly indicates that the proposed technique is superior at all noise levels.

Fig.3.3 illustrates detailed results, obtained with the close up view of the restored images for better inspection. In order to compare the visual performance, existing and proposed approaches, incorporate real image, noisy image and the restored image. The visual results for simulated MR slice is corrupted with 10 % level of Rician noise, in Figure3.3. The BM3D and AWF are unable to smooth out noise completely and provide a blurred output as it can be observed from the Fig. 3.3.

Median filter is good at preserving some details at the cost of some noisy spots. NLM filter removes the noise completely, but taking much more time and most of the image structural information has been lost. Similarly, fuzzy based hybrid filter provides better results with noisy spots than NLM filter. To overcome these limitations, the proposed approach produces better results.

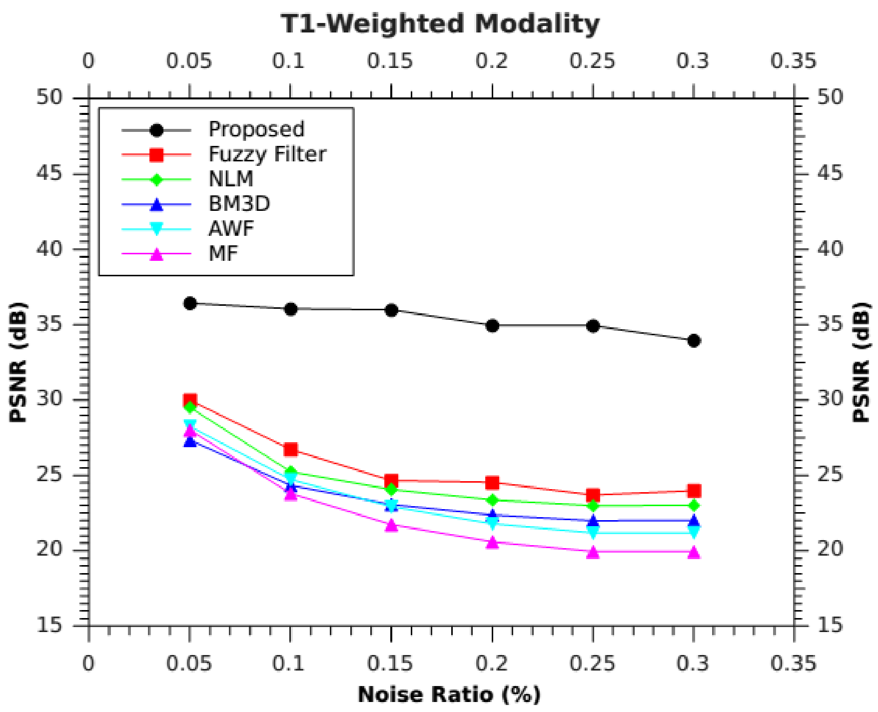
On the basis of quantitative and visual results it is apparent that the proposed approach has produced more accurate results. Such as more noise removing ability, and preservation of edges and structural information, at all levels of Rician noise.



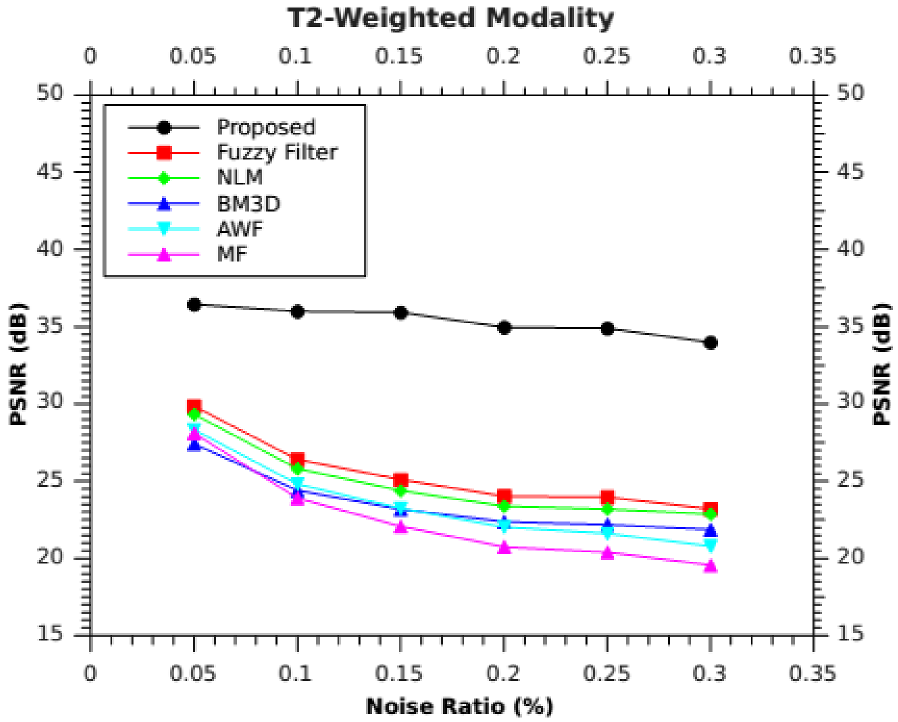
(a) (b) (c) (d)

(e) (f) (g) (h)

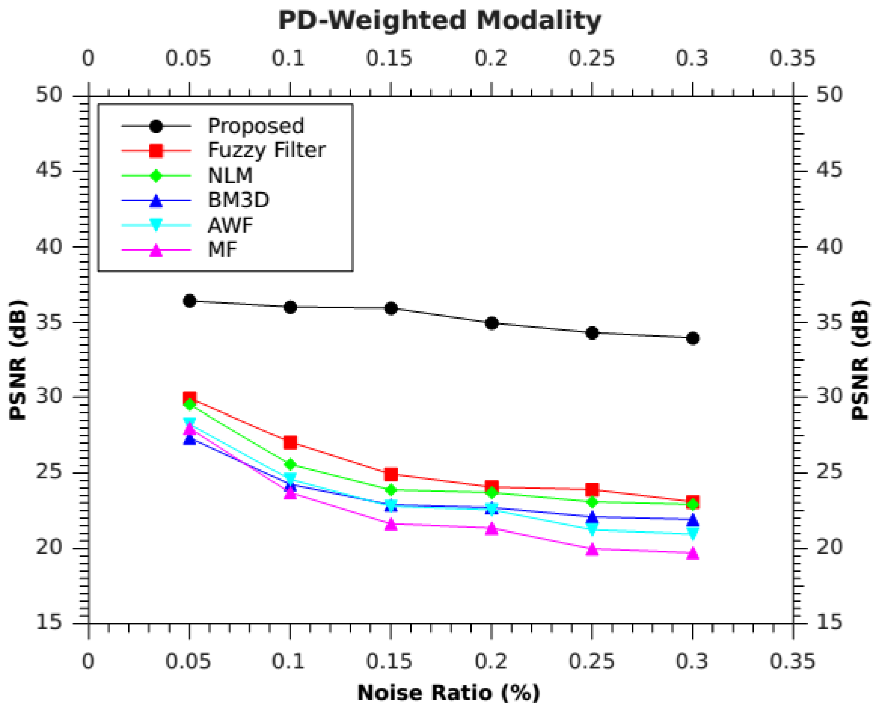
Figure 3.3: Simulated T1 weighted MR image with Rician noise (a) Original image (b) 10% noisy image (c) BM3D (d) MF (e) AWF (f) NLM (g) Fuzzy filter (h) Proposed.



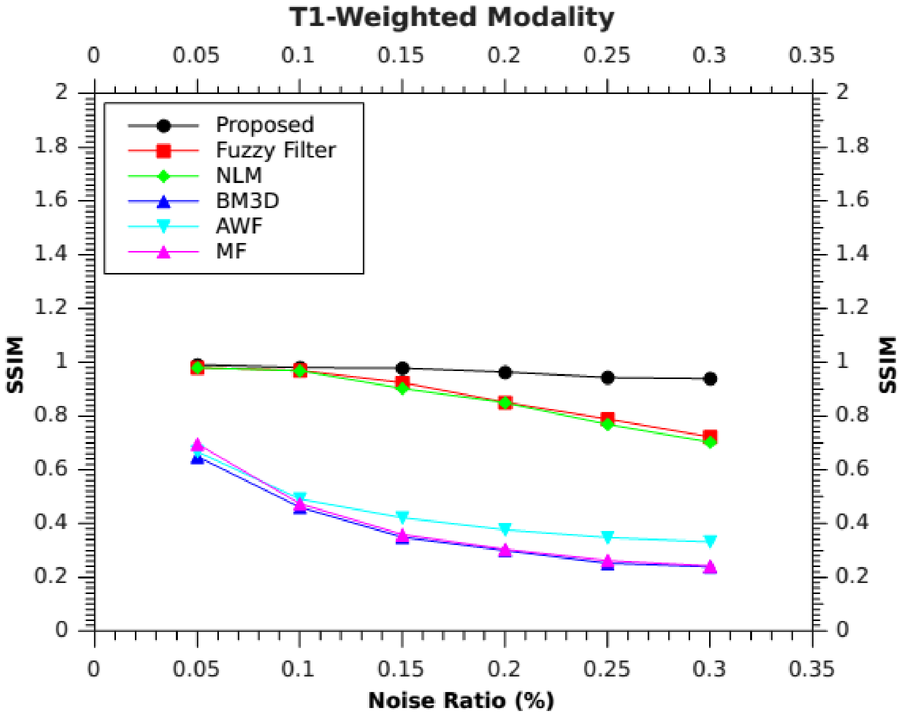
(a)


T2-Weighted Modality
Proposed
Fuzzy Filter
NLM
BM3D
AWF
MF
PSNR (dB)
Noise Ratio (%)

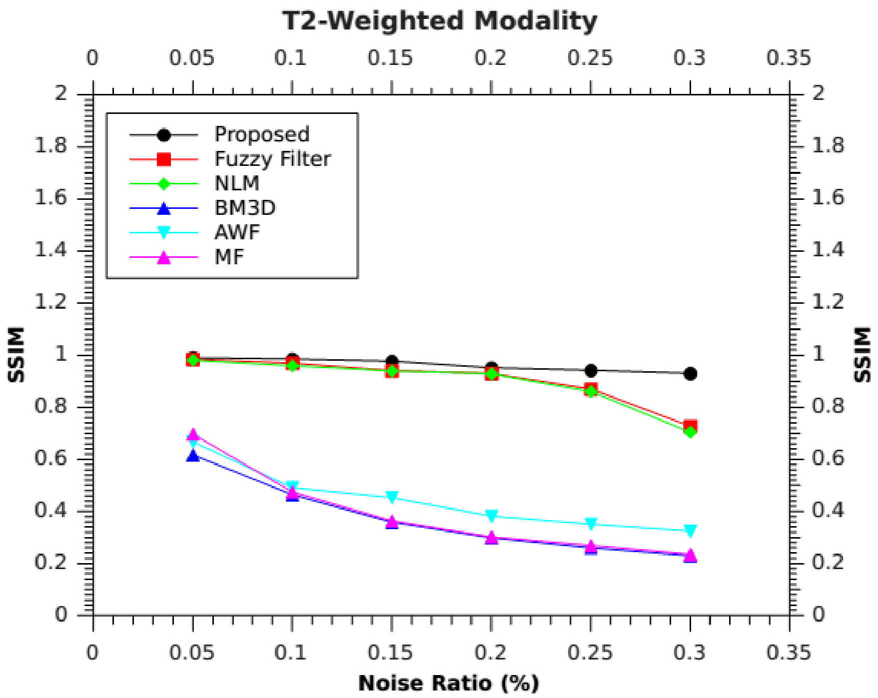
(b)


PD-Weighted Modality
Proposed
Fuzzy Filter
NLM
BM3D
AWF
MF
PSNR (dB)
Noise Ratio (%)

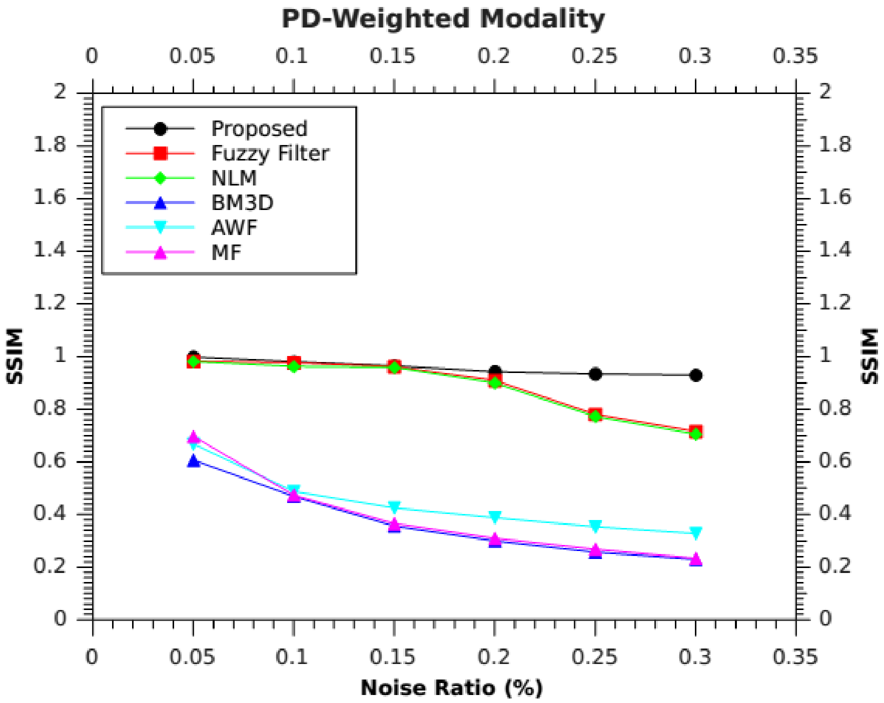
(c)


T1-Weighted Modality
Proposed
Fuzzy Filter
NLM
BM3D
AWF
MF
SSIM
Noise Ratio (%)

(d)


T2-Weighted Modality
Proposed
Fuzzy Filter
NLM
BM3D
AWF
MF
SSIM
Noise Ratio (%)

(e)



(f)

Figure 3.4: PSNR based comparison of various methods: (a) T1-weighted modality (b) T2-weighted modality (c) PD-weighted modality and SSIM based comparison of various methods: (d) T1-weighted modality (e) T2-weighted modality (f) PD-weighted modality.

Table 3.3: Parameters setup of the proposed method for de-noising MR images

Parameter	Description	Value
Num_Iter	Number of iterations used as a parameter to getting desired output at four in the proposed method.	4.0
Δt	Integration constant which is used as a parameter to calculate the desired output at zero point one in the proposed method.	0.10
k	Edge threshold parameter used to controls the diffusion, getting desired output at one point four in the proposed method.	1.4
θ	Used as a parameter in the diffusion coefficient, getting desired output at $\pi/30$ in the proposed method.	$\frac{\pi}{30}$
λ	Regularization parameter used for making balance between likelihood term and regularization function, getting desired output at zero point nine in the proposed method.	0.9

k_1	Positive number used to calculate the Rician noise, getting desired output at one in the proposed method.	1

Table 3.4: Quantitative comparison on Simulated MR data (Brain Web) using PSNR (RMSE)

Modality (slice)	Noise ratio (%)	MF [110]	AWF [110]	BM3D [111]	NLM method [19]	Fuzzy-filter [105]	Proposed Method
T1-weighted (slice 70)	5	28.0(10.1)	28.2(9.9)	27.3(11.9)	29.5(9.2)	29.9(8.9)	36.4(2.8)
	10	23.7(16.5)	24.7(14.8)	24.3(15.8)	25.2(14.4)	26.7(13.7)	36.0(3.0)
	15	21.7(20.9)	22.9(18.2)	23.0(18.6)	24.0(18.1)	24.6(16.6)	35.9(3.6)
	20	20.5(23.9)	21.7(20.8)	22.3(20.1)	23.3(20.1)	24.5(17.5)	34.9(3.7)
	25	19.9(25.7)	21.1(22.3)	21.9(29.8)	22.9(20.4)	23.6(18.1)	34.9(3.8)
	30	19.9(25.6)	21.1(22.1)	22.0(29.9)	23.0(21.3)	23.9(18.9)	33.9(3.9)
	Mean	22.3(20.5)	23.3(18.0)	23.5(21.0)	24.7(17.3)	25.5(15.6)	35.3(3.4)
T2-weighted (slice 70)	5	28.1(10.0)	28.3(9.8)	27.4(10.3)	29.3(7.8)	29.8(7.4)	36.3(2.7)
	10	23.8(16.3)	24.8(14.7)	24.4(18.4)	25.8(11.9)	26.4(10.4)	36.2(3.0)
	15	22.0(20.1)	23.2(17.6)	23.1(23.5)	24.3(16.6)	25.1(15.1)	35.9(3.1)
	20	20.7(23.4)	22.0(20.2)	22.3(26.7)	23.3(19.5)	24.0(17.0)	34.9(3.3)
	25	20.3(24.4)	21.6(21.2)	22.1(28.1)	23.1(21.9)	23.9(18.2)	34.9(3.3)
	30	19.5(26.9)	20.7(23.3)	21.8(28.8)	22.8(22.4)	23.2(18.9)	33.9(3.4)
	Mean	22.4(20.2)	23.4(17.8)	23.5(22.7)	24.8(16.7)	25.4(14.5)	35.3(3.1)
PD-weighted (slice 50)	5	27.9(10.2)	28.2(9.8)	27.3(12.0)	29.5(9.0)	29.9(8.6)	36.5(2.8)
	10	23.6(16.7)	24.5(15.1)	24.2(20.6)	25.5(13.2)	27.0(12.0)	36.1(3.0)
	15	21.6(21.2)	22.7(18.5)	22.8(24.8)	23.8(16.9)	24.9(15.9)	35.9(3.0)
	20	21.3(21.9)	22.5(19.0)	22.6(26.2)	23.6(17.2)	24.0(16.0)	34.9(3.2)
	25	19.9(25.6)	21.2(22.2)	22.0(28.3)	23.0(19.1)	23.8(17.8)	34.3(3.3)
	30	19.7(26.4)	20.9(22.9)	21.9(29.6)	22.9(41.4)	23.0(18.0)	33.9(3.4)
	Mean	22.3(20.3)	23.3(17.9)	23.5(23.6)	24.7(16.1)	25.5(14.7)	35.2(3.1)
Overall Mean		22.3(20.3)	23.4(17.9)	23.5(22.4)	24.7(16.7)	25.5(14.9)	35.3(3.2)

Table 3.5: Quantitative comparison on Simulated MR data (Brain Web) using SSIM (CP)

Modality (slice)	Noise ratio (%)	MF [110]	AWF [110]	BM3D [111]	NLM method [19]	Fuzzy-filter [105]	Proposed Method
T1-weighted (slice 70)	5	0.69 (0.17)	0.66 (0.36)	0.64(0.47)	0.97(0.98)	0.98(0.98)	0.99(0.98)
	10	0.47 (0.17)	0.49(0.33)	0.46(0.47)	0.96(0.96)	0.96(0.97)	0.98(0.98)
	15	0.35 (0.16)	0.42 (0.32)	0.34(0.46)	0.90(0.94)	0.92(0.95)	0.97(0.98)
	20	0.30 (0.16)	0.37 (0.32)	0.29(0.46)	0.84(0.92)	0.85(0.92)	0.96(0.98)
	25	0.26 (0.15)	0.34 (0.32)	0.25(0.45)	0.76(0.90)	0.78(0.91)	0.94(0.97)
	30	0.24 (0.12)	0.33(0.31)	0.23(0.42)	0.70(0.84)	0.72(0.85)	0.93(0.97)
	Mean	0.38 (0.16)	0.43(0.33)	0.37(0.46)	0.86(0.92)	0.87(0.93)	0.96(0.98)
T2-weighted (slice 70)	5	0.69(0.19)	0.66(0.41)	0.61(0.49)	0.98(0.98)	0.98(0.98)	0.99(0.98)
	10	0.47(0.17)	0.49(0.34)	0.46(0.47)	0.95(0.97)	0.96(0.97)	0.98(0.98)
	15	0.36(0.16)	0.45(0.33)	0.35(0.46)	0.93(0.95)	0.94(0.96)	0.97(0.98)
	20	0.30(0.16)	0.38(0.32)	0.29(0.46)	0.92(0.93)	0.93(0.94)	0.95(0.98)
	25	0.26(0.16)	0.35(0.32)	0.25(0.46)	0.86(0.91)	0.87(0.92)	0.94(0.97)

	30	0.23(0.16)	0.32(0.31)	0.23(0.46)	0.70(0.84)	0.72(0.86)	0.93(0.97)
	Mean	0.39(0.17)	0.44(0.34)	0.37(0.47)	0.89(0.93)	0.90(0.94)	0.96(0.98)
PD-weighted (slice 50)	5	0.69(0.17)	0.66(0.39)	0.60(0.47)	0.98(0.98)	0.98(0.98)	0.99(0.98)
	10	0.47(0.17)	0.48(0.33)	0.46(0.47)	0.96(0.97)	0.97(0.97)	0.98(0.98)
	15	0.36(0.16)	0.42(0.32)	0.35(0.46)	0.95(0.96)	0.96(0.96)	0.96(0.98)
	20	0.30(0.16)	0.38(0.32)	0.29(0.46)	0.89(0.92)	0.90(0.93)	0.94(0.98)
	25	0.26(0.14)	0.35(0.32)	0.25(0.44)	0.77(0.90)	0.78(0.91)	0.93(0.97)
	30	0.23(0.12)	0.32(0.31)	0.22(0.42)	0.70(0.86)	0.71(0.87)	0.93(0.97)
	Mean	0.39(0.15)	0.44(0.33)	0.36(0.45)	0.88(0.93)	0.88(0.94)	0.95(0.98)
Overall Mean		0.39(0.16)	0.44(0.33)	0.37(0.45)	0.87(0.93)	0.88(0.93)	0.96(0.98)

3.5. Conclusions

This chapter presented two proposed method for restoration and enhancement of magnetic resonance images using experimental study of various recent denoising methods. First method, an efficient PDE-based nonlinear filter adapted to Rician noise for restoration and enhancement of magnetic resonance images has been proposed in this chapter. In the proposed scheme likelihood term, regularization parameter and regularization function have been observed in an innovative manner. The mathematical calculation of log likelihood of Rician noise pdf has been simplified. On the basis of obtained qualitative results in terms PSNR and SSIM. the proposed method gives better performance (denoising) than existing techniques over simulated MR brain images. Further, visual results clearly indicate that the proposed technique has the capability of better noise removal. Second method, modified nonlinear complex diffusion based filter adapted to Rician noise was proposed for restoration and enhancement of magnetic resonance images. The proposed filter consists of two terms namely data fidelity and prior. The data fidelity term i.e. likelihood term is derived from Rician pdf and a nonlinear complex diffusion based filter is used as a prior. Further, mathematical simplifications have been introduced for likelihood term for efficient implementation of the algorithm. The proposed method was tested on Brain Web data set for varying noise levels and performance was evaluated in terms of PSNR, RMSE, SSIM and CP. From obtained results and comparative analysis with other standard methods, it is observed that the proposed method is performing better. Further, visual results clearly indicate that the proposed technique has the capability of better noise removal. In future research, it would be useful to estimate the combined model for Rayleigh, Gaussian and Rician noise removal purpose for medical images. From practical point of view, the proposed method can be further modified by taking into account both Rayleigh distribution and Gaussian distribution. In addition, experimental results show that proposed method suffers from intrinsic limitation of some computational cost in practical cases. Nevertheless, the proposed model is still worthy of consideration since it generates denoising results which are quantitatively and qualitatively better with some current state-of–the-art methods.