Chapter 2 : THEORETICAL BACKGROUND

This chapter discusses about theoretical background for restoration and enhancement of MRI. In this chapter, we have also given an overview of magnetic resonance images and its properties which provide basis for medical image applications discussed in subsequent chapters of the thesis. Further, in this chapter literature survey of prominent approaches for magnetic resonance images are given. Section 2.1 presents the Introduction for MR images. 2.2 presents the detailed noise patterns in MR images. Section 2.3 presents the literature survey of MRI restoration and enhancement Methods. Section 2.4 presents the dataset description. Section 2.5 presents the detailed discussion about various performance measures used for qualitative and quantitative analysis.

2.1. Introduction

Magnetic resonance imaging (MRI) is a notable imaging technique to provide highly detailed images of tissues and organs in the human body. MRI is primarily used to demonstrate the pathological or other physiological alterations of living tissues [30,31]. It provides information that differs from other imaging modalities such as ultrasound and computed tomography (CT). Its major technological advantage is that it can characterize and discriminate among tissues using their physical and biochemical properties. MRI produces sectional images of equivalent resolution in any projection without moving the patient, and it is limited only by its spatial resolution and long imaging times. The ability to obtain images in multiple planes adds to its versatility and diagnostic utility and offers special advantages for radiation and/or surgical treatment planning. Further, the inherent flexibility of MRI also permits its application in many clinical tasks other than imaging static anatomy.

The visual quality of magnetic resonance images plays an important role in accuracy of clinical diagnosis which can be seriously degraded by existing noise during acquisition process. In a single channel signal acquisition, the MR image is commonly reconstructed by computing the inverse discrete Fourier transform of the raw data. The signal component of the measurement is present in both real and imaginary channels. Each of the orthogonal channels is affected by additive white Gaussian noise [32]. Most commonly, the magnitude of the reconstructed MRI image is used for visual inspection and automatic computer analysis. Since the magnitude of the MRI signal is the square root of the sum of the squares of two independent Gaussian variables, it follows Rician distribution [33]. In multichannel signal acquisition, the MR image is reconstructed by combining complex images, and the noise distribution is described by noncentral Chi distribution [34, 35]. Moreover, in case of parallel imaging, the noise amplitude varies according to the spatial location of the image and can follow Rician or Chi distribution according to the reconstruction technique [36].

In general, there are two typical ways to reduce the noise in the images. One way is to acquire the data several times and average them. However, it increases the acquisition time. Another way is to de-noise the images by using the post processing methods. In the literatures, numerous approaches to denoising MR images have been proposed including the classic spatial and temporal filters [30], approaches based on anisotropic diffusion filter [37,38], the nonlocal means algorithm [39,40], bilateral and trilateral filters [41,42], the wavelet transform [43,44], the curvelet and the contourlet transforms [45,46], maximum likelihood approach [47,48], linear minimum mean square error estimation [49,50], nonparametric neighborhood statistics/estimation [51,52] and singularity function analysis [53,54]. The aim of this chapter is to summarize these literatures for MRI denoising.

2.2. Noise patterns in MR images

In this section, we have discussed various noise patterns in MR images, and its properties which provide basis for computer vision applications discussed in subsequent chapters of the thesis. MRI system is working on the principles of nuclear magnetic resonance (NMR), to map the spatial location and associated properties of specific nuclei or protons in a subject using the interaction between an electromagnetic field and nuclear spin [30,31].

MRI, even if the scanner technology has undergone tremendous improvements in spatial resolution, acquisition speed and signal-to-noise ratio (SNR), the diagnostic and visual quality of MR images are still affected by the noise in acquisition. However, MRIs contain varying amount of noise of diverse origins, including noise from stochastic variation, numerous physiological processes, eddy currents, artifacts from the magnetic susceptibilities between neighboring tissues, rigid body motion, non-rigid motion and other sources [55,56]. Identifying and reducing these noise components in MR images is necessary to improve the validity and accuracy of studies designed to map the structure and function of the human body.

The main noise in MRI is due to thermal noise that is from the scanned object. The variance of thermal noise can be described as the sum of noise variances from independent stochastic processes representing the body, the coil and the electronics [57]. Such a noise degrades the acquisition of any quantitative measurements from the data. The signal-to-noise ratio depends on static field intensity, pulse sequence design, tissue characteristics, RF coil and sequence parameters, such as voxel size (limiting spatial resolution), number of averages in the image acquisition and receiver bandwidth. In this section, the noise distribution in MRI for both single coil and multiple coils acquisition are explained.

The raw data obtained during MRI scanning are complex values that represent the Fourier transform of a magnetization distribution of a volume of tissue. An inverse Fourier transform converts these raw data into magnitude, frequency and phase components that more directly represent the physiological and morphological features of interest in the person being scanned. Therefore, noise in the k-space in MR data from each coil is assumed to be a zero mean uncorrelated Gaussian process with equal variance in both real and imaginary parts because of the linearity and orthogonality of the Fourier transform [32,33]. However, it is common practice to transform the complex valued images into magnitude and phase images. Since computation of a magnitude (or phase) image is a non-linear operation, the probability density function (PDF) of the MR data changes. In single coil MRI systems, magnitude data in spatial domain is modeled as the Rician distribution and the so-called Rician noise (the error between the underlying image intensities and the measurement data) is locally signal dependent [33].

$$p(I/M) = \frac{M}{\sigma_n^2}\exp\left(-\frac{M^2+I^2}{2\sigma_n^2}\right)J_0\left(\frac{IM}{\sigma_n^2}\right)\in\left(M\right) \tag{2.1}$$

Where I denotes amplitude of a noise-free image, σ_n^2 denotes the Gaussian noise variance, $J_0(.)$ is the modified zero order Bessel function. $\in(.)$ is the unit step

Heaviside function, and M is the magnitude variable of MR image. In high SNR i.e., high intensity (bright) regions of the magnitude image, the Rician distribution tends to a Gaussian distribution with mean $\sqrt{I^2+\sigma_n^2}$ and variance σ_n^2 given as

$$p(I/M)=\frac{1}{\sqrt{2\pi\sigma_n^2}}\exp(-\frac{M^2-\sqrt{I^2+\sigma_n^2}}{2\sigma_n^2})\in(M) \tag{2.2}$$

In the image background, where SNR is zero due to the lack of water proton density in the air, the Rician PDF simplifies to a Rayleigh distribution with PDF

$$p(I/M)=\frac{M}{\sigma_n^2}\exp\left(-\frac{I^2}{2\sigma_n^2}\right)\in\left(M\right) \tag{2.3}$$

The MR images acquired using parallel imaging with multiple coil system, noise is highly inhomogeneous. The acquired signal in the complex spatial domain in each coil may also be modeled as the original signal corrupted with complex additive Gaussian noise, with zero mean and equal variance σ_n^2. If no sub-sampling is done in the k-space, the composite magnitude image may be obtained using methods such as the sum of squares (SoS) [58]. Assuming the noise components are independent and identically distributed (IID), the envelope of the magnitude signal $M_L(x)$ will follow a noncentral Chi distribution with PDF [104]

$$p_{M_L}\left(\frac{M_L}{I_L},\sigma_n,L\right)=\frac{I_L^{1-L}}{\sigma_n^2}M_L^{\,L}e^{-\left(M_L^2+I_L^2\right)/2\sigma_n^2}\,J_{L-1}\left(\frac{I_L M_L}{\sigma_n^2}\right)\in\left(M_L\right) \tag{2.4}$$

where L is the number of coils. Eq. (4) reduces to the Rician distribution for L = 1. In the background, this PDF reduces to a central Chi distribution with PDF [58]

$$p_{M_L}\left(\frac{M_L}{\sigma_n},L\right)=\frac{2^{1-L}}{\Gamma\left(L\right)}e^{-M_L^{2L-1}/2\sigma_n^2}\in(M_L) \tag{2.5}$$

and Eq. (5) will become to Rayleigh when $L = 1$.

2.3. Literature Survey of MRI restoration and enhancement methods

In MRI, there is an intrinsic trade-off between the signal-to-noise ratio (SNR) and resolution i.e., high resolution, low SNR and low resolution, high SNR. Human visual system is highly effective in recognizing structures even in the presence of a considerable amount of noise. When the SNR is too small or the contrast too low, it becomes very difficult for human visual system to detect anatomical structures. Noise reduction can be categorized in two groups: acquisition based noise reduction methods and post-acquisition image denoising. The method for improving the SNR during the acquisition of an image is either increasing acquisition time (i.e., time averaging over repeated measurements) or decreasing spatial resolution (i.e., enlarging voxel volume). However, the acquisition time is limited in practice due to constraints such as patient comfort and system throughput and by physical limitations arising in dynamic applications such as cardiac imaging and functional MRI. Therefore, there is a practical limit on the SNR of the acquired MRI data in most applications. Hence, post-acquisition image denoising is an inexpensive and effective alternative.

The aim of a post processing MRI de-noising algorithm is that of reducing the noise power while maintaining the original resolution of the useful features. In fact, in a diagnostic image, edge preservation is important in maintaining the original clinical significance. It is also important to reduce noise without introducing artifacts. This chapter focuses on the post-acquisition image de-noising methods, and reviews the various methods for de-noising MR images presented in the literature. The de-noising methods can be grouped based on filtering approach, transform approach and statistical approach. In filtering approach, the linear or non-linear filters are used to de-noise the MRI. In transform approach, the transforms such as wavelet transform, curvelet transform are used for de-noising MRI. In statistical approach, the estimation of noise based on Maximum likelihood approach, linear minimum mean square error (LMMSE) estimation, Markov random process, Empirical Bayes approaches are used for denoising MRI.

2.3.1. Filtering approach

2.3.1.1. Spatial and temporal filter

McVeigh *et al.* [59] proposed the spatial filter and temporal filter for reducing Gaussian noise in MR images. Spatial filter is one method by convolving an image with a filter in spatial domain. This technique reduces the variance in the image but blurs sharp edges by an amount related to the shape of the function used in the convolution. This process is equivalent to reduce high spatial frequencies in the image. MR imaging is particularly amenable to convolution filtering, since the data obtained in frequency domain, can simply be multiplied by a filter function which reduces higher spatial frequencies. This type of filter smoothen the final image, but the signal-to-noise ratio as a function of frequency is unaffected because both the noise and the signal are reduced by the same factor. With this type of image smoothening, there is a compromise between the reduction of noise and artifact, and the loss of spatial resolution. Temporal filter must be chosen in appropriate relation to the sampling interval in order to avoid the aliasing artifacts. This filter at best would work only on spin echo images. A temporal filter with too narrow a frequency response diminishes the signal at the edges of the image, too broad a frequency response introduces additional noise through aliasing.

2.3.1.2. Anisotropic diffusion filter

Perona and Malik *et al.* [37] developed a multiscale smoothing and edge detection scheme called anisotropic diffusion filter. This would overcome the drawback of spatial filtering and significantly improve the image quality by preserving object boundaries, efficiently removing noise in homogeneous regions and edge sharpening. This filter based on casting the problem in terms of a heat equation which is based on second order partial differential equation (PDE) in an anisotropic medium. Smoothening is formulated as a diffusive process, which is suppressed or stopped at boundaries by selecting the local gradient strengths in different directions. In this approach the image I is only convolved in the direction orthogonal to the gradient of the image which ensures the preservation of edges. The iterative de-noising process of initial image I_0 can be expressed as

$$\begin{cases} \dfrac{\partial I(x,t)}{\partial t} = div\big(c(x,t)\nabla I(x,t)\big) \\ I(x,0) = I_0(x) \end{cases} \tag{2.6}$$

where $\nabla I(x,t)$ is the image gradient at voxel x and iteration t, $\frac{\partial I(x,t)}{\partial t}$ is the partial temporal derivation of $I(x,t)$ and

$$c(x,t) = g\|\nabla I(x,t)\| = e^{-\|\nabla I(x,t)\|/k^2} \tag{2.7}$$

where k is the diffusivity parameter. This filtering technique is successfully applied to 2D and 3D MR images denoising by Gerig *et al.* [60]. Even though the performance of the noise filter is excellent, the underlying image model is piecewise constant or slowly varying. As a result, the edge sharpening causes a region with a constant gray value slope. Here, image noise is assumed to be zero mean and Gaussian distributed.

2.3.1.3. Nonlocal means filter

Buades *et al.* [39] proposed the nonlocal means (NLM) filter. Most of the existing denoising methods mainly rely on local pixels within a small neighbor to remove the noise. As a result, large scale structures are preserved while small structures are considered as noise and are removed. The NLM filter exploits the redundancy of information contained within the images to remove the noise. The restored intensity value of the voxel is calculated as the weighted average of all the voxel intensities within the image. In the nonlocal means Buades *et al.* [39], Given a discrete noisy image $u = \{u(i) \| i \in I\}$, the estimated value $NL[u](i)$, for a pixel i, is computed as a weighted average of all the pixels in the image,

$$NL[u](i) = \sum_{j \in I} w(i, j)u(j) \tag{2.8}$$

where the family of weights $\{w(i, j)\}_j$ depend on the similarity between the pixels i and j, and satisfy the usual conditions $0 \le w(i, j) \le 1$ and $\sum_j w(i, j) = 1$.

The similarity between two pixels i and j depends on the similarity of the intensity gray level vectors $u(N_i)$ and $u(N_j)$, where N_k denotes a square neighborhood of fixed size and centered at a pixel k. This similarity is measured as a decreasing function of the weighted Euclidean distance, $\left\| u(N_i) - u(N_j) \right\|_{2,a}^2$ where $a > 0$ is the standard deviation of the Gaussian kernel. The application of the Euclidean distance to the noisy neighborhoods raises the following equality:

$$E\left\| u(N_i) - u(N_j) \right\|_{2,q}^2 = \left\| u(N_i) - u(N_j) \right\|_{2,q}^2 + 2a^2 \tag{2.9}$$

This equality shows the robustness of the algorithm since in expectation the Euclidean distance conserves the order of similarity between pixels. The pixels with a similar gray level neighborhood to $u(N_i)$ have larger weights in the average. These weights are defined as,

$$w(i,j)=\frac{1}{Z(i)}e^{-\left(\left\|u(N_i)-u(N_j)\right\|_{2,a}^{2}\right)\Big/h^2} \tag{2.10}$$

where $Z(i)$ is the normalizing constant

$$Z(i)=\sum_j e^{-(\left\|u(N_i)-u(N_j)\right\|_{2,a}^{2})\Big/h^2} \tag{2.11}$$

and the parameter h acts as a degree of filtering. It controls the decay of the exponential function and therefore the decay of the weights as a function of the Euclidean distances.

2.3.1.4. Combination of domain and range filtering techniques

Tomasi and Manduchi *et al.* [41] proposed the bilateral filter as a non-iterative alternative to anisotropic diffusion filter. In both these approaches, images are smoothed while edges are preserved. Unlike anisotropic diffusion, bilateral filtering does not involve the solution of PDE and can be implemented in a single iteration. This filter is a combination of two Gaussian filters i.e., domain and range filters. Walker *et al.* [61] and Xie *et al.* [62] applied this filter for MRI and Hamarneh and Hradsky *et al.* [63] used this filter for reducing noise in Diffusion Tensor MRI.

Wong *et al.* [64,42] proposed the trilateral filter for reduction of noise in medical images which works along similar lines to bilateral filtering. It is not only taking the geometric and photometric similarities into account, but also makes use of the local structural similarity to smooth the images. By using the local structural information, the non-homogeneous regions in the images have been identified. In the homogeneous region, only low pass filtering is performed, where as in the non-homogeneous region, the intensity value at each pixel is replaced with an average value weighted by the geometric, photometric and local structural similarities between the neighboring pixels within a spatial window. It uses narrow spatial window to smooth an image while preserving the edges.

2.3.2. Transform domain approach

2.3.2.1. Wavelet Transform

Wavelets are mathematical functions that decompose data into different frequency components that can be studied with a resolution matched to their scale. Wavelet transforms are multiresolution representations of signals and images. They decompose a signal into a hierarchy of scales ranging from the coarsest scale to the finest one. Wavelet coefficients of a signal are the projections of the signal onto the multiresolution subspaces. Since the work of Donoho and Johnstone *et al.* [43,65], over the last two decades, there are many wavelet based noise reduction schemes for MRI [66,67]. The main advantage of the discrete wavelet transform (DWT) is that it can describe local features either spatially or spectrally, which makes it to filter out most of noise while at the same time preserving the edges and fine details.

Typically, a wavelet based denoising technique includes the following steps:

1. Transform the original image into wavelet domain and acquire the wavelet coefficients.

2. Process the wavelet coefficients. This step typically involves thresholding the wavelet coefficients to minimize the contribution of noise in the wavelet domain.

3. Take inverse wavelet transform on the processed coefficients to produce the denoised image.

The above steps can be repeated for number of wavelet transform scales, each representing different degrees of wavelet decomposition.

2.3.2.2. Curvelet transform

The denoising methods based on wavelet transform, are not suitable for describing the signals which have high dimensional singularities such as edges. To overcome the shortcomings of the wavelet transform, and to detect, represent and process high dimensional data, Candès and Donoho *et al.* [45] proposed the concept of curvelet transform based on the theory of multiscale geometric analysis. Curvelets are based on multiscale ridgelets combined with a spatial band pass filtering operation to isolate different scales. Starck *et al.* [68] firstly used the curvelet transform for image denoising.

Following steps are involved in the denoising algorithm of curvelet transform:

1. Compute all thresholds for curvelets.
2. Compute norm of curvelets.
3. Apply curvelet transform to noisy image.
4. Apply hard thresholding to the curvelet coefficients.
5. Apply inverse curvelet transform to the result of step 4.

2.3.2.3. Contourlet transform

The wavelet transform is powerful in representing images containing smooth areas separated with edges. However, it cannot perform well when the edges are smooth curves. The contourlets have the property of capturing contours and fine details in images. The Contourlet transform by Do and Vetterli *et al.* [69] is a geometrical image transform, which represents images containing contours and textures, and provides sparse representation at both spatial and directional resolutions.

The steps involved in the denoising algorithm of contourlet transform are described below:

1. Perform multiscale decomposition of the image using contourlet transform and determine the number of scales and directions.

2. Apply thresholding at each direction in each scale of contourlet coefficients.

3. Reconstruct the denoised image from the modified contourlet coefficients by applying inverse contourlet transform.

For MR images denoising, Latha and Subramanian [70] used this contourlet transform based image denoising method.

2.3.3. Statistical approach

A few works have been reported in the literature for MR images denoising based on statistics/estimation methods. Estimation of the noise variance on MR images is a necessary step in noise removal for several reasons. statistics/estimation methods Firstly, it gives a measure of the quality of the MR data and used to measure the SNR. Furthermore, knowledge of this noise variance is useful in the analysis of MRI system. Finally, it is a crucial parameter in image denoising, image segmentation and image registration. One of the first attempts proposed to estimate the magnitude MR image from a noisy image is due to Henkleman *et al.* [32] who investigated the effect of the noise on MR magnitude images. He showed that the noise leads to an overestimation of the signal amplitude and proposed a correction scheme based on image intensities. The conventional approach (CA) was proposed by McGibney and Smith *et al.* [71] utilizing the noise properties of the second order moment.

2.3.3.1. Maximum likelihood approach

Sijbers *et al.* [47,72] estimated the Rician noise level and performed signal reconstruction using maximum likelihood (ML) approach for reducing bias that appears in the conventional approach. A similar method is used by Jiang and Wang *et al.* [73]. Sijbers *et al.* [74] used this approach to estimate the image noise variance from the background mode of the histogram of MR image which is known to be Rayleigh distributed. He and Greenshields *et al.* [75] used the nonlocal maximum likelihood (NLML) estimation method for Rician noise. This method is based on deploying maximum likelihood estimator on the nonlocal neighborhood in order to predict the underlying noise. Rajan *et al.* [76] used the ML based estimation of the local variance for each pixel of the image using a local neighborhood when no background information is available, like cardiac and lung images. Rajan *et al.* [77] proposed the MRI denoising for spatially varying noise levels based on ML estimation using restricted local neighborhoods. Recently, Rajan *et al.* [48] presented the nonlocal maximum likelihood estimation for denoising multiple coil MR images.

2.3.3.2. Linear minimum mean square error estimation

Aja-Fernandez *et al.* [49,78] used the linear minimum mean square error (LMMSE) estimator for Rician noise. This method uses information of the sample distribution of local statistics of the image such as the local variance, the local mean and the local mean square value. That is, in this method, the true value for each noisy pixel is estimated by a set of pixels selected from a local neighborhood. Golshan and Hasanzadeh *et al.* [79] have developed a nonlocal processing of the LMMSE method for Rician noise removal in the 3D MRI which is using the hard threshold value of control parameters. And later, Golshan *et al.* [50] modified this by changing the control parameters according to the noise level. This method takes the advantage of the high degree of redundancy in the contents of 3D MR images and provides a suitable similarity measure to find the similar patterns within the given MR data and the required parameters are automatically chosen with respect to the estimated local SNR values.

2.3.3.3. Phase error estimation

Tisdall and Atkins *et al.* [80] proposed the phase error estimation scheme for MRI denoising. While denoising MR images using wavelet thresholding or anisotropic diffusion, there is the risk of over smoothing fine details particularly in images with lower SNR. The phase-corrected real reconstruction relies on an estimate of the phase error to correct the phase of each pixel so the imaginary component contains only noise and can be discarded. This approach offers the potential for image denoising without the risk of over-smoothening. The phase error estimation scheme is based on iteratively applying a series of non-linear filters, each used to modify the phase estimate into greater agreement with one piece of knowledge about the noise corrupted image, until it converges at the desired phase error estimate. This method is useful for displaying inversion recovery MR images, spin echo images and in partial k-space imaging MR images.

2.3.3.4. Non-parametric estimation method

The nonparametric neighborhood statistics method for denoising MR images was proposed by Awate and Whitaker *et al.* [51]. This method model images as random fields and uses reduction coupled with the Rician noise model as a means to recover higher-order statistics of image neighborhoods from noisy image Nonparametric neighborhood density estimation is used for characterizing neighborhood structure. It exploits such statistics for optimal Bayesian denoising of

MR images. Awate and Whitaker *et al.* [81] extended their work on nonparametric empirical Bayes approach for feature preserving denoising of MR images that bootstraps itself by inferring the prior, i.e., the uncorrupted image statistics, from the corrupted input data and the knowledge of the Rician noise model. This method preserves most of the important features in the brain MR images.

2.3.3.5. Singularity function analysis

Luo *et al.* [53,54] proposed the singularity function analysis (SFA) denoising method of MR images, which is based on the spectrum of the images. The wavelet de-noising methods, discard least significant wavelet coefficients (hard thresholding) or shrink less significant wavelet coefficients more than significant coefficients (soft thresholding) to achieve noise reduction [44,66]. All these methods, however, exhibit the same shortcoming of losing significant high-frequency components contained in the original noise-free image.

So far, the MRI de-noising techniques are discussed in detail and the summary of the advantages and limitations of MRI de-noising methods for spatially uniform and non-uniform noise distributions are tabulated in Table 2.1.

Table 2.1: Review of restoration and enhancement of MRI methods

Category of methods	**Paper**	**Brief descriptions**	**Advantage**	**Limitations**
Fuzzy based hybrid filter	(Sharif *et al.*, 2015)	This method uses estimated noise variance along with local and global statistics for the construction of a robust fuzzy membership function.	Appropriate construction of fuzzy membership parameters combines the advantages of local and non local estimates in an innovative manner.	Fuzzy based hybrid filter provides good results with noisy spots because unbiased estimation not computed for accurate results.
Adaptive nonlocal means	(Manjón *et al.,* 2010)	In the adaptive NLM filter for denoising MR images with spatially varying noise levels, such as those	This method does not require the prior knowledge of the coil sensitivity profiles	Computational burden of algorithmic complexity. So, neglecting the voxels /blocks with small

		obtained by parallel imaging and surface coil acquisitions. The information regarding the local image noise level is used to adjust the amount of denoising strength of the filter.	in the MRI scanner.	weights (i.e., most dissimilar patches to the current one) for speeding up the filter.
Singularity function analysis	(Luo *et al.*, 2010)	In this paper author proposed the singularity function analysis (SFA) denoising method of MR images, which is based on the spectrum of the images	This approach can efficiently denoise while maintaining high quality image.	The efficiency of this method depends on the selection of the frequency response, determination of the singular points and threshold
Noise adaptive nonlinear diffusion filter	(Krissian *et al.*, 2009)	This filter relies on a robust estimation of the standard deviation of the noise and combines local linear minimum mean square error filters and partial differential equations for MRI.	This filter combines volumetric, planar, and linear components of the local image structure, which can improve the filtering.	Requires the calculation of the noise map from the receiver coil matrix. This may be implemented by MRI scanner's reconstruction software.
Contourlet transform	(Do *et al.*, 2005)	The Contourlet transform is a geometrical image transform, which represents images containing contours and textures, and provides sparse representation at both spatial and directional resolutions	The contourlet have the property of capturing contours and fine details in images.	High computational complexity because of capturing the smooth contours in the image.
Nonlocal mean	(Buades *et al.*, 2005)	Most of the existing denoising methods mainly rely on local pixels within a small neighbor to remove the noise. As a result, large scale structures are preserved while small structures are considered as noise and are removed.	The NLM filter exploits the redundancy of information contained within the images to remove the noise.	Computational burden due to its complexity of calculating the weight of the pixel/voxel.
Bilateral and	(Wong *et al.*, 2004)	Unlike anisotropic diffusion, bilateral	This method was proposed as a non-	Use the narrow spatial window for

trilateral filter		filtering does not involve the solution of PDE and can be implemented in a single iteration. This filter is a combination of two Gaussian filters i.e., domain and range filters.	iterative alternative to anisotropic diffusion filter. In both these approaches, images are smoothed while edges are preserved.	the selection of the neighborhood pixels in order to calculate the new value of that pixel. So, large scale structures are preserved, while small structures are considered as noise and are removed.
Phase error estimation	(Sijbers *et al.*, 2004)	In this method the phase-corrected real reconstruction relies on an estimate of the phase error to correct the phase of each pixel so the imaginary component contains only noise and can be discarded.	This approach offers the potential for image denoising without the risk of over-smoothening.	In phase error estimation denoising, the convergence of the phase error estimate is a difficult task.
Curvelet transform	(Candés *et al.*, 1999)	The denoising methods based on wavelet transform, are not suitable for describing the signals which have high dimensional singularities such as edges.	Overcome the short comings of the wavelet transform,	Does not work well in smooth areas, produce curvelet-like artifacts.
Wavelet transform	(Nowak *et al.*, 1999)	Wavelet transforms are multiresolution representations of signals and images. They decompose a signal into a hierarchy of scales ranging from the coarsest scale to the finest one. Wavelet coefficients of a signal are the projections of the signal onto the multiresolution subspaces.	The main advantage of the discrete wavelet transform is that it can describe, local features either spatially or spectrally, which makes it to filter out most of noise while at the same time preserving the edges and fine details.	May introduce characteristic artifacts that can be quiet problematic and also difficult to confirm the scale and threshold of the wavelet.
Anisotropic diffusion filter	(Perona *et al.*, 1990)	This filter based on casting the problem in terms of a heat equation which is based on second order partial differential equation (PDE) in an	This would overcome the drawback of spatial filtering and significantly improve the image quality by	Usually erases small feature and transforms image statistics due to its edge enchantment causes blocky (staircase) effect in

		anisotropic medium.	preserving object boundaries, efficiently removing noise in homogeneous regions and edge sharpening.	the image.
Spatial filter	(McVeigh *et al.*, 1985)	This method was proposed for reducing Gaussian noise in MR images.	Spatial filter is one method by convolving an image with a filter in spatial domain. This technique reduces the variance in the image.	Blurring edges by averaging pixels with non similar patterns (suitable only for Gaussian noise).
Temporal filter	(McVeigh *et al.*, 1985)	Temporal filter must be chosen in appropriate relation to the sampling interval in order to avoid the aliasing artifacts.	Proper selection of frequency response for the filter is important in order to avoid the aliasing.	Temporal filter must be chosen in appropriate relation to the sampling interval in order to avoid the aliasing artifacts.

2.4. Dataset Description

In this section, the brief descriptions of dataset provided which are used in this thesis to do various operations and evaluations of the performance metrics using MATLAB of the restoration and enhancement of magnetic resonance images. The increased importance of automated computer techniques for anatomical brain mapping from MR images and quantitative brain image analysis methods leads to an increased need for validation and evaluation of the effect of image acquisition parameters on performance of these procedures. Validation of analysis techniques of in vivo acquired images is complicated due to the lack of reference data (“ground truth”). Also, optimal selection of the MR imaging parameters is difficult due to the large parameter space.

BrainWeb makes available to the neuroimaging community, online on "http://www.bic.mni.mcgill.ca/ brainweb/", a set of realistic simulated brain MR image volumes (simulated brain database, SBD) that allows the above issues to be examined in a controlled, system way. There are three modalities (pulse sequences) dataset present in the Brain Web data base [1] which are T1, T2 and PD weighted. Here in this thesis, downloaded nearly 1000 MR images from Brain Web dataset and used 70 to 80 images for experimental purpose.

2.5. Performance Measures

In this section, the brief descriptions of parameters used to evaluate the performance measures of restoration and enhancement of magnetic resonance images discussed as follows:

2.5.1. Root mean square error

The mean square error (MSE) and the root mean square error (RMSE) are the two error metrics used to compare image compression quality [81]. Lower value of RMSE means good segmentation (i.e. noise is minimum) while high value of RMSE indicates poor segmentation (i.e. noise is maximum).

$$MSE = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=}^{n} \left[I^{'}(i,j) - I(i,j) \right]^2$$

$$RMSE = \sqrt{MSE} \qquad (2.12)$$

where I is the original image without noise, $I^{'}$ is the filtered noise reduced image, $m \times n$ is the size of the image and $i = 1...m,\ j = 1....n$.

2.5.2. Peak Signal-to-Noise Ratio (PSNR)

The peak signal-to-noise ratio (PSNR) is the error metrics used to compare image compression quality [81]. Higher value of PSNR [81] means good segmentation (i.e.

noise is minimum) while low value of PSNR indicates poor segmentation (i.e. noise is maximum).

$$PSNR = 20\log_{10}\left[\frac{255}{RMSE}\right] \tag{2.13}$$

for optimal performance, measured values of PSNR should be large.

2.5.3. Correlation Parameter (CP)

The Correlation parameter (CP) is a qualitative measure for edge preservation. If one is interested in suppressing noise while at the same time preserving the edges of the original image then this parameter proposed in paper [82] can be used. To evaluate the performance of the edge preservation or sharpness, the correlation parameter is defined as follows

$$CP = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}(\Delta I - \Delta \bar{I}) \times (\Delta \hat{I} - \Delta \bar{\hat{I}})}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}(\Delta I - \Delta \bar{I})^2 \times \sum_{i=1}^{m}\sum_{j=1}^{n}(\Delta \hat{I} - \Delta \bar{\hat{I}})^2}} \tag{2.14}$$

where ΔI and $\Delta \hat{I}$ are high pass filtered versions of original image I and filtered image $\hat{I}$ obtained via a 3x3 pixel standard approximation of the Laplacian operator. The ΔI and $\Delta \hat{I}$ are the mean values of I and $\hat{I}$ respectively. The correlation parameter should be closer to unity for an optimal effect of edge preservation.

2.5.4. Structure similarity index map (SSIM)

The SSIM [83] is used to compare luminance, contrast and structure of two different images. It can be treated as a similarity measure of two different images. This similarity measure is a function of luminance, contrast and structure. The SSIM of two images X and Y be calculated as

$$SSIM(X,Y) = \frac{(2\mu_x\mu_y + C_1)\times(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)\times(\sigma_x^2 + \sigma_y^2 + C_2)} \tag{2.15}$$

where μ_i $(i = X \, or \, Y)$ is the mean intensity, $\sigma_i (i = X \, or \, Y)$ is the standard deviation, $\sigma_{xy} = \sigma_x.\sigma_y$ and C_i $(i = 1 \; or \; 2)$ is the constant to avoid instability when $\mu_x^2 + \mu_y^2$ is very close to zero and is defined as $C_i = (k_i L)^2$ in which $k_i << 1$ and L is the dynamic range of pixel values e.g. L=255 for 8-bit gray scale image. In order to have an overall quality measurement of the entire image, mean SSIM is defined as

$$MSSIM(X,Y) = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} SSIM(X_{ij}, Y_{ij}) \tag{2.16}$$

The MSSIM value should be closer to unity for optimal measure of similarity.

2.6. Conclusions

This chapter presented the theoretical background for restoration and enhancement of MRI. In this chapter, an overview of magnetic resonance images and its properties was given which provided basis for medical image processing. Further, in this chapter a literature survey of prominent approaches for restoration and enhancement of MR images was presented with pros ans cons.