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Evolution of C^1-wave and its collision with the blast wave in one-dimensional non-ideal gas dynamics

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Abstract

The aim of the present paper is to analyze the collision of discontinuity wave with blast wave in non-ideal gas flow through Lie group approach. The particular solution of the system of quasilinear hyperbolic PDEs, governing one-dimensional unsteady cylindrically symmetric motion in non-ideal gas, is obtained. The equation governing the evolutionary process of weak discontinuity in non-ideal gas is derived which is utilized to discuss the growth and decay process of the weak discontinuity. Furthermore, the collision of discontinuity wave with the blast wave is examined inside the state described by the exact solution of the blast wave. Also, the existence and uniqueness properties of reflection and transmission coefficients of the waves along with the discontinuity in the shock acceleration are discussed.

Keywords Group-theoretic method · Wave interaction · Non-ideal gas · Blast wave

Mathematics Subject Classification 76M60 · 76N15 · 76N30 · 76L05

1 Introduction

We study the problem of collision of the waves within the context of the system of quasilinear hyperbolic PDEs. An interesting remark in the context of such system of PDEs is that its solution admits discontinuity waves. Discontinuity waves are the special class of the solution of the non-linear hyperbolic system of PDEs which are characterized by a discontinuity in the flow variable and its normal derivative. For a non-linear system, to find analytical solution and discussion of its physics in gasdynamics is of great interest from both physical and mathematical point of view. One of the most interesting problems on the theory of non-linear waves in gasdynamics is the process of the formation of shock waves. Detailed investigation towards a better understanding of the wave interaction phenomenon has been

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studied by the authors Brun (1975), Morro (1978, 1980) and Jeffrey (1976). The authors Ruggeri (1980), Virgopia and Ferraioli (1994), Radha et al. (1993), Pandey and Sharma (2007), and Pandey (2010) have discussed the process of the interaction of waves in various fields like magnetogasdynamics, non-ideal gas, shallow water, elastic solids, relaxing gas, etc. Many authors Virgopia and Ferraioli (1994), Chaturvedi et al. (2019) studied the evolution of weak discontinuity in different material media. In past decades, Pandey et al. (2008), Arora et al. (2013), and Bira et al. (2016) have applied the group-theoretic method to obtain the self-similar solution of the system of PDEs. The blast waves are the strong shock waves which are produced when a large amount of energy is released suddenly. The mathematical analysis of exact solution of the strong shock problem is analyzed by Rogers (1957), Sakurai (1954, 1953), Murata (2006), Singh et al. (2011, 2012), and Ram et al. (2013). Lie group analysis is utilized to convert the system of PDEs to the system of ODEs, and after that, the system of ODEs is solved by change of variables to yield the exact particular solution of the blast wave. This solution is utilized to investigate the behavior of amplitude of discontinuity wave and its collision with the blast wave in the presence of non-idealness parameter. Also, Jena and Sharma (1999) and Jena (2007) have discussed the evolution of a discontinuity waves arising out of the system of hyperbolic PDEs governed by the Bernoulli-type ordinary differential equation.

In the present analysis, Lie group transformation method is used to determine an approximate analytical solution of the system of PDEs governing unsteady one-dimensional planer and non-planar flows of an inviscid non-ideal gas. In the recent years, Lie group theory (Baikov et al. 1988; Bluman and Anco 2008; Bluman and Kumei 2013; Donato and Oliveri 1994, 1995; O'Hara et al. 2013) provided extremely general and powerful methods to determine an analytical solution of the system of partial differential equations. The basic tool in the study is the use of the corresponding infinitesimal representation of Lie-algebras. The first approach to potential symmetries of a system of partial differential equations was proposed by Bluman and Anco (2008) and Bluman and Kumei (2013) to introduce an algorithm which yielded a new class of symmetries of given partial differential equations. With the help of symmetry generators, one can construct similarity variables which can reduce the system of partial differential equations (PDEs) to a system of ordinary differential equations (ODEs). Also, the Lie group analysis is systematic technique leading to the determination of invariant solutions of the initial and boundary value problems, as well as to the transformation of differential equations into equivalent forms in a easier way to handle (see Donato and Oliveri 1994, 1995; Ovsiannikov 2014). Recently, authors in Raja Sekhar (2019) and Zeidan and Sekhar (2018) considered the drift-flux modal of two-phase flow and discussed the Riemann problem and interaction of weak shocks using similar mathematical approach. The authors in Romenski et al. (2003), Zeidan et al. (2007, 2019, 2020), Zeidan and Touma (2014), Goncalves and Zeidan (2018), and Goncalves et al. (2019) have studied numerical and theoretical problem related to the non-linear wave propagation phenomenon in two-phase flow. In particular, Lie group analysis method is used to solve the Riemann problem for hyperbolic system like author in Conforto et al. (2012). The authors in Baikov et al. (1988), Ibragimov and Kovalev (2009) have used either the classical Lie symmetries or the first-order approximate Lie symmetries with the first approach. Moreover, in Mentrelli et al. (2008), some numerical simulations are performed to study the interaction of a weak discontinuity wave with elementary waves (shocks and characteristic shock) of the Riemann problem for the one-dimensional Euler equations governing the flow of ideal polytropic gases, and investigate the effects of shock strength, and the initial states on the jump in acceleration and the amplitude of reflected and transmitted waves. The system of transport equations for C^1 waves is much more general and it leads to the Bernoulli-type system when eigenvalues are simple. To study the effect of shock strength on the jump in shock acceleration and amplitude of reflected and transmitted waves, we have focused our attention to study the interaction between C^1 wave and the blast wave solutions for the Euler equations. Also, Jena (2005) has studied the interaction of non-linear waves with a bore in shallow water wave using Lie group transformation. Later on, Singh and Jena (2013) have studied the interaction of weak shock waves in polytropic reacting gas flow.

In the present work, one-dimensional unsteady cylindrically symmetric flow of an inviscid gas in the presence of the non-ideal parameter is studied. The main target of the paper is to get the similarity solution through Lie group analysis and discuss the collision of discontinuity wave with blast wave inside the state described by the blast wave in a non-ideal flow. The non-ideal parameter effects on the discontinuity wave, and reflected and transmitted wave have been examined. The motive of this paper is to give a contribution to the understanding of development of the shock waves and structure of wave in a non-ideal gas through the group-theoretic method. We employ group-theoretic method to obtain the exact invariant solution of the one-dimensional gasdynamics equations for the non-ideal gas.

The rest of the paper is structured as: Sect. 2 consists of Lie symmetric method for the basic equations of motion and jump condition for the blast wave problem in a non-ideal gas. Using Lie point transformation, we reduce the system of PDEs into system of ODEs and obtain the exact solution of the system. Section 3 describes the evolution of C^1-wave and derives the transport equation for the discontinuity wave. In Sect. 4, the collision of discontinuity wave with the blast wave is discussed. Section 5 is the result and discussions of this work in which we discuss the evolutionary behavior of discontinuity wave and the jump in shock acceleration affected by the amplitude of incident wave after wave collision. The influence of non-ideal parameter on the reflected and transmitted wave is also discussed. Ultimately, the conclusions of this study are provided in the last section.

2 Governing equations and Lie group approach

The governing equations for one-dimensional unsteady cylindrical flow in non-ideal gas may be written as:

$$\left.\begin{aligned} &\rho_t + \rho v_x + v\rho_x + \rho v x^{-1} = 0, \\ &\rho\left(v_t + v v_x\right) + p_x = 0, \\ &p_t + v p_x + \Gamma p\left(1 + b\rho\right)\left(v_x + v x^{-1}\right) = 0, \end{aligned}\right\} \tag{1}$$

where x and t are the independent variables which correspond to the space coordinate and time, respectively. ρ, p, v, and Γ are the density, pressure, gas velocity, and specific heats ratio of the gas, respectively. b is the parameter of non-idealness which is constant with $b\rho << 1$ in high temperature gases. The non-numeric subscripts represent the partial derivative with respect to indicated independent variables. The system of governing equations (1) is supplemented with the equation of state satisfied by the non-ideal gas can be written as $p = \rho(1 + b\rho)RT$, where T and R are the temperature and gas constant, respectively. Suppose the shock front, $x = \chi(t)$, is propagating with speed $\mathrm{d}\chi/\mathrm{d}t = V$ into the medium which is specified by:

$$\rho \equiv \rho_0,\ v \equiv 0,\ p \equiv \text{constant}, \tag{2}$$

The jump conditions for the blast wave at the shock front are:

$$\left.\begin{aligned} \rho\left(\chi(t), t\right) &= \frac{(\Gamma+1)}{\left(\Gamma-1+2\bar{b}\right)}\rho_0\left(\chi(t)\right), \\ v\left(\chi(t), t\right) &= \frac{2\left(1-\bar{b}\right)}{(\Gamma+1)}V, \\ p\left(\chi(t), t\right) &= \frac{2(1-\bar{b})}{(\Gamma+1)}\rho_0\left(\chi(t), t\right)V^2, \end{aligned}\right\} \tag{3}$$

where $\bar{b} = b\rho_0$.

Let us consider that the system of governing equations (1) has solution subject to jump condition (3) along a family of curves; this family of curves is known as similarity curves for which the system (1) reduces to a system of ODEs; these type of solutions are called similarity solutions. To obtain the similarity solutions and similarity curves, we consider the following transformations given by:

$$\left.\begin{aligned} &x^* = x + \varepsilon X(x, t, \rho, v, p), && t^* = t + \varepsilon\tau(x, t, \rho, v, p), \\ &\rho^* = \rho + \varepsilon D(x, t, \rho, v, p), && v^* = v + \varepsilon U(x, t, \rho, v, p), \\ &p^* = p + \varepsilon P(x, t, \rho, v, p), && \end{aligned}\right\} \tag{4}$$

where D, U, P, X, and τ are the generators which can be obtained in such way that the system (1) is constantly conformally invariant with respect to the Lie group of transformations (4). Following the straight forward procedure outlined in the literatures Bluman and Anco (2008); Bluman and Kumei (2013), we obtain the following generators:

$$\left.\begin{aligned} &X = (\alpha_{22} + 2a_1)x, \quad \tau = a_1 t + d, \quad D = 0, \\ &U = (\alpha_{22} + a_1)v, \quad\ \ P = 2(\alpha_{22} + a_1)p, \end{aligned}\right\} \tag{5}$$

where α_{22}, a_1, and d are the arbitrary constants.

Let us consider the case when $a_1 \neq 0$ and $\alpha_{22} + 2a_1 \neq 0$, and then, the form of similarity solutions and the similarity variables follow from the invariant surface condition which yields:

$$\tau\rho_t + X\rho_x = D, \tau v_t + Xv_x = U, \tau p_t + Xp_x = P. \tag{6}$$

On solving (6) with the help of (5), we obtain:

$$\rho = \hat{D}(\xi), p = t^{2(\delta-1)}\hat{P}(\xi), v = t^{\delta-1}\hat{U}(\xi), \tag{7}$$

where $\delta = \frac{(\alpha_{22}+2a_1)}{a_1}$, $\hat{D}(\xi)$, $\hat{U}(\xi)$, and $\hat{P}(\xi)$ are dimensionless quantities and are function of the similarity variable ξ, defined as:

$$\xi = \frac{x}{At^\delta}, \tag{8}$$

where the parameter A is constant which can be determined with the help of similarity exponent δ. At $\xi = 1$, the shock path $x = \chi(t)$ and the shock velocity V are given by:

$$\chi(t) = At^\delta, \qquad V = \frac{\delta\chi(t)}{t}. \tag{9}$$

Then, we have the following conditions for flow variables at the shock $\xi = 1$:

$$\rho|_{\xi=1} = \hat{D}(1), v|_{\xi=1} = t^{\delta-1}\hat{U}(1), p|_{\xi=1} = t^{2(\delta-1)}\hat{P}(1). \tag{10}$$

In perspective of the invariance of jump condition (3), Eq. (2.10) prompts to the form of $\rho_0(x)$ as $\rho_0(x) = \rho_c \left(\frac{x}{x_0}\right)^{\Theta}$, and following conditions on the functions $\hat{D}$, $\hat{U}$, and $\hat{P}$ at the shock:

$$\left.\begin{aligned} \hat{D}(1) &= \frac{(1+\Gamma)A^{\Theta}\rho_c}{(2\bar{b}+\Gamma-1)x_0^{\Theta}}, \\ \hat{U}(1) &= \frac{2(1-\bar{b})A\delta}{(1+\Gamma)}, \\ \hat{P}(1) &= \frac{2(1-\bar{b})A^{\Theta+2}\delta^2\rho_c}{(1+\Gamma)x_0^{\Theta}}, \end{aligned}\right\} \tag{11}$$

where ρ_c, x_0, and $\Theta = (1+a_1)/\delta a_1$ are some reference constants related to the medium.

Now, we rewrite Eqs. (7–9) with the help of (11) as:

$$\begin{aligned} &\rho = \rho_0 D^*(\xi), \qquad v = VU^*(\xi), \qquad p = \rho_0 V^2 P^*(\xi), \\ &\xi = \frac{x}{At^{\delta}}, \qquad X = At^{\delta}, \qquad V = \frac{X\delta}{t}, \end{aligned} \tag{12}$$

where $D^*(\xi) = \frac{x_0^{\Theta}\hat{D}(\xi)}{\rho_c A^{\Theta}}$, $U^*(\xi) = \frac{\hat{U}(\xi)}{A\delta}$, $P^*(\xi) = \frac{x_0^{\Theta}\hat{P}(\xi)}{A^{\Theta+2}\rho_c\delta^2}$.

Using (12) in (1) yields the system of ODEs in D^*, P^*, U^*, and on suppressing, the asterisk sign reduced system is:

$$\left.\begin{aligned} &(U-\xi)D' + DU' + \Theta D + DU\xi^{-1} = 0, \\ &(U-\xi)U'D + (\delta-1)\delta^{-1}UD + P' = 0, \\ &(U-\xi)P' + 2(\delta-1)\delta^{-1}P + \Theta P + \Gamma P(1+b\rho_c D)(U' + U\xi^{-1}) = 0, \end{aligned}\right\} \tag{13}$$

where the prime denotes the derivative of related function with respect to the independent variable ξ. Also, the jump conditions are given by:

$$\begin{aligned} D(1) &= \frac{(1+\Gamma)}{(2\bar{b}+\Gamma-1)}, \\ U(1) &= \frac{2(1-\bar{b})}{(1+\Gamma)}, \\ P(1) &= \frac{2(1-\bar{b})}{(1+\Gamma)}. \end{aligned} \tag{14}$$

Now, along with the jump conditions (14), the system (13) satisfies the following particular solutions:

$$\left.\begin{aligned} D &= \frac{(1+\Gamma)}{(2\bar{b}+\Gamma-1)}, \\ U &= \frac{2(1-\bar{b})}{(1+\Gamma)}\xi, \\ P &= \frac{2(1-\bar{b})}{(1+\Gamma)}\xi^2, \end{aligned}\right\} \tag{15}$$

with $\delta = (1+\Gamma)/2(\Gamma+\bar{b})$ and $\Theta = 4(\bar{b}-1)/(1+\Gamma)$, then the solution of the system (1) subject to the boundary conditions (3) is:

$$\left.\begin{aligned} \rho &= \rho_c^* \frac{(1+\Gamma)}{(2\bar{b}+\Gamma-1)} t^{\Theta\delta}, \qquad & v &= \frac{(1-\bar{b})}{(\Gamma+\bar{b})}\frac{x}{t}, \\ p &= \rho_c^* \frac{(1+\Gamma)}{2(\Gamma+\bar{b})^2} x^2 t^{\Theta\delta-2}, \qquad & \rho_0(\chi(t)) &= \rho_c^* t^{\Theta\delta}, \end{aligned}\right\} \tag{16}$$

where $\rho_c^* = \rho_c \frac{A^\Theta}{x_0^\Theta}$ and $\chi(t) = At^\delta$.

The dimensionless form of the solution of system (1) obtained in (16) can be written as:

$$\left.\begin{aligned} \rho &= \frac{(1+\Gamma)}{(2\bar{b}+\Gamma-1)} t^{\Theta\delta}, \qquad & v &= \frac{(1-\bar{b})}{(\Gamma+\bar{b})}\frac{x}{t}, \\ p &= \frac{(1+\Gamma)}{2(\Gamma+\bar{b})^2} x^2 t^{\Theta\delta-2}, \qquad & \rho_* &= x^\Theta, \\ v_* &= 0, \qquad p_* = 1, & & \end{aligned}\right\} \tag{17}$$

where the subscript $'*'$ denotes the value of flow variables ahead of the discontinuity curve $\chi = \chi(t)$ and V is the velocity of the discontinuity curve defined by:

$$V(t) = \frac{(1+\Gamma)}{2(\Gamma+\bar{b})} t^{-\frac{(2\bar{b}+\Gamma-1)}{2(\Gamma+\bar{b})}}. \tag{18}$$

3 Evolution of C^1-discontinuity

The system (1) can be written in the matrix form:

$$f_t + Mf_x = g, \tag{19}$$

where $f = (\rho, v, p)^{Tr}$, $g = (-\rho v x^{-1}, 0, -vC^2x^{-1})^{Tr}$ and $M = \begin{bmatrix} v & \rho & 0 \\ 0 & v & \rho^{-1} \\ 0 & \Gamma C^2 & v \end{bmatrix}$.

Here, f and g are column vectors. $M_{3\times3}$ is the matrix whose eigenvalues are:

$$\mu^{(1)} = (v+C), \qquad \mu^{(2)} = v, \qquad \mu^{(3)} = (v-C), \tag{20}$$

with left and right eigenvectors corresponding to each eigenvalues:

$$\left.\begin{aligned} E^{(1)} &= (\rho/C, 1, \rho C)^{Tr}, \qquad & L^{(1)} &= (0, 1, 1/\rho C), \\ E^{(2)} &= (1, 0, 0)^{Tr}, \qquad & L^{(2)} &= (1, 0, -1/C^2), \\ E^{(3)} &= (-\rho/C, 1, -\rho C)^{Tr}, \qquad & L^{(3)} &= (0, 1, -1/\rho C), \end{aligned}\right\} \tag{21}$$

where the parameter $C = \left(\frac{\Gamma p}{\rho(1-b\rho)}\right)^{1/2}$ is the speed of sound in non-ideal gas.

Now, we consider that the first derivative of the column vector $f(x, t_0)$ has discontinuity at $x_0 < x_1$; this amounts to assume that at $t = t_0$, there are both a shock wave at x_1 and C^1-discontinuity at x_0. Supposing C^1-discontinuity curve, originating from the point (x_0, t_0) in the column vector $f = (\rho, v, p)^{Tr}$, is moving along the curve obtained by $\mathrm{d}x/\mathrm{d}t = \mu^{(1)}$ swept by the shock. Then, the transport equation for the C^1-discontinuity across the characteristics of system (1) in terms of the independent variables x and t is given by Radha et al. (1993):

$$L^{(1)}\frac{d\Omega}{\mathrm{d}t}+L^{(1)}(f_x+\Omega)(\nabla\mu^{(1)})\Omega+((\nabla L^{(1)})\Omega)^{Tr}\frac{\mathrm{d}f}{\mathrm{d}t}+(L^{(1)}\Omega)((\nabla\mu^{(1)})f_x+\mu_x^{(1)}) - (\nabla(L^{(1)}g))\Omega=0, \tag{22}$$

where $\Omega = \pi E^{(1)}$, denotes the discontinuity in f_x across the C^1-discontinuity. π is the amplitude of the C^1-wave. Substituting Eqs. (20) and (21) in (22) yields the transport equation for the wave amplitude:

$$\frac{\mathrm{d}\pi}{\mathrm{d}t}+\theta_1\pi^2+\theta_2\pi=0, \tag{23}$$

where $\theta_1 = \frac{(1+\Gamma)t^\alpha}{2(t^\alpha-\hat{b})}$ with $\hat{b} = \frac{\bar{b}(1+\Gamma)}{(2\bar{b}+\Gamma-1)}$, $\alpha = \frac{2(1-\bar{b})}{(\Gamma+\bar{b})}$
and $\theta_2 = \frac{1}{4(t^\alpha-\hat{b})(\Gamma+\bar{b})t}\left\{6(t^\alpha-\hat{b})+\sqrt{2\Gamma(1-\bar{b})(2\bar{b}+\Gamma-1)}\left(\frac{(1+4\Gamma)}{\Gamma}t^\alpha+\hat{b}\right)\right\}$.
On integrating (23) yields:

$$\pi(t)=\frac{\pi_0\phi_1(t)}{1+\pi_0\phi_2(t)}, \tag{24}$$

where $\pi(1)=\pi_0$, $\phi_1(t)=\exp(-\int\theta_2\mathrm{d}t)$, and $\phi_2(t)=\int\theta_1\phi_1(t)\mathrm{d}t$.

In the analysis of the effect of non-ideal parameter, present in (24), on the evolutionary process of the C^1-wave, we observe the evolution in discontinuity wave while fixing the various parameters involved in (24). We consider the value of specific heat ratio as $\Gamma = 1.67$. We have used the MATHEMATICA software to generate the profile of amplitude of the C^1-wave. In view of (24), we observe that $\pi_0 > 0$ corresponds to the expansive wave. $\pi(t)$ is non-zero finite and continuous function on the interval $[1, t)$ and $\pi(t) \longrightarrow 0$ as $t \longrightarrow \infty$ which implies that C^1-wave decays. However, $\pi_0 < 0$ corresponds to the compressive wave. When $\pi_0 < 0$, there exist a critical value of $\pi(t)$, such that $|\pi_0| \le \pi_c$, where $\pi_c = 1/\phi_2(\infty)$. Then, $\pi(t)$ is non-zero finite and continuous function on the interval $[1, \infty)$ and $\pi(t) \longrightarrow 0$ as $t \longrightarrow \infty$. Then, we observe the decay in C^1-wave, which is shown in Fig. 2. If $|\pi_0| > \pi_c$, then there exist t_c belonging to $(1, \infty)$, such that $\phi_2(t_c) = 1/|\pi_0|$, and then $\pi(t)$ is non-zero finite and continuous on the interval $[1, t_c)$, and it tends to ∞ as t tends to t_c. This implies that there is growth in C^1-wave which is shown in Fig. 3 and these compressive waves terminate into shock waves after a finite time. The effect of non-idealness on the decay and growth of expansive and compressive waves, respectively, is shown by the curves in Figs. 1, 2, and 3. From Figs. 1, 2, and 3, we observe that on increasing the value of non-ideal parameter causes to decay the wave more rapidly in case of expansive wave and also an increase in the value of non-ideal parameter causes to slow down the growth of compressive wave.

4 Collision of the C^1-wave with blast wave

In this section, we analyze the interaction of the C^1-wave with the blast wave. The conservation system which is direct consequence of the governing system of PDEs (1) to obtain the amplitude of the reflected and transmitted C^1-wave, when the incident discontinuity comes in contact with the blast wave, and have the following forms in the regions to the left and right of the discontinuity curve $\mathrm{d}x/\mathrm{d}t = V$, where V is the propagation velocity of discontinuity curve:

$$
\left.\begin{aligned}
F_t(x,t,f)+G_x(x,t,f)=H(x,t,f),\\
F_t(x,t,f_*)+G_x(x,t,f_*)=H(x,t,f_*),
\end{aligned}\right\} \tag{25}
$$

where $f=(\rho, v, p)^{Tr}$ and $f_*=(\rho_*, v_*, p_*)^{Tr}$, are the solution vectors to the left and right of the discontinuity curve and F, G, H are given as:

$$
\left.\begin{aligned}
F&=\left(\rho, \rho v, \frac{(1-b\rho)}{(\Gamma-1)}+\frac{\rho v^2}{2}\right)^{Tr},\\
G&=\left(\rho v, \rho v^2+p, v\left(\frac{(1-b\rho)p}{(\Gamma-1)}+\frac{\rho v^2}{2}+p\right)\right)^{Tr},\\
H&=\left(-\frac{\rho v}{x}, -\frac{\rho v^2}{x}, -\frac{v(\Gamma-b\rho)p}{x(\Gamma-1)}-\frac{\rho v^3}{2x}\right)^{Tr}.
\end{aligned}\right\} \tag{26}
$$

Now, we consider the situation when the C^1-discontinuity wave intersects the blast wave at the time $t=t_p$. Let the fastest C^1-discontinuity of (25) originating from the point (x_0, t_0), propagating along the characteristic $\mathrm{d}\chi/\mathrm{d}t=\mu^{(1)}$, encounters the discontinuity curve $\mathrm{d}x/\mathrm{d}t=V$, originating from (x_1, t_0), at the point $P(x_p, t_p)$. At $t=t_p$, the eigenvalues of (25) on both sides of the discontinuity curve are:

$$
\left.\begin{aligned}
\mu^{(1)}&=\frac{2(1-\bar{b})+\sqrt{2\Gamma(\Gamma-1+2\bar{b})(1-\bar{b})(t_p^{\alpha}+\hat{b})t_p^{\alpha}}}{2(\Gamma+\bar{b})}t_p^{-\frac{\Gamma-1+2\bar{b}}{2(\Gamma+\bar{b})}},\\
\mu^{(2)}&=\frac{(1-\bar{b})}{(\Gamma+\bar{b})}t_p^{-\frac{\Gamma-1+2\bar{b}}{2(\Gamma+\bar{b})}},\\
\mu^{(3)}&=\frac{2(1-\bar{b})-\sqrt{2\Gamma(\Gamma-1+2\bar{b})(1-\bar{b})(t_p^{\alpha}+\hat{b})t_p^{\alpha}}}{2(\Gamma+\bar{b})}t_p^{-\frac{\Gamma-1+2\bar{b}}{2(\Gamma+\bar{b})}},\\
\mu_*^{(1)}&=\sqrt{\Gamma(t_p^{\alpha}+\bar{b})},\qquad \mu_*^{(2)}=0,\qquad \mu_*^{(3)}=-\sqrt{\Gamma(t_p^{\alpha}+\bar{b})}.
\end{aligned}\right\} \tag{27}
$$

From the above-computed values, we can see that:

$\mu_*^{(1)}(t_p), \mu_*^{(2)}(t_p), \mu_*^{(3)}(t_p) < V(t_p)$ and $\mu^{(2)}(t_p), \mu^{(3)}(t_p) < V(t_p), \mu^{(1)}(t_p) > V(t_p)$.

Thus, Lax evolutionary conditions must hold (See Ref. Boillatt and Ruggeri (1979)), i.e., $\mu_*^{(3)}(t_p) < \mu_*^{(2)}(t_p) < \mu_*^{(1)}(t_p) < V(t_p)$ and $\mu^{(1)}(t_p) > V(t_p) > \mu^{(2)}(t_p) > \mu^{(3)}(t_p)$. This ensures that when the incident wave (incident discontinuity) with speed $\mu^{(1)}$ interacts with the shock wave at the collision point $t=t_p$, it rises to two reflected waves only with speed $\mu^{(2)}$ and $\mu^{(3)}$. Then, the amplitudes of the reflected waves α_1 and α_2 with jump in the shock acceleration $|[\dot{V}]|$ at the collision point $t=t_p$ can be obtained from the following algebraic system of equations:

$$
\begin{aligned}
(F-F_*)_p|[\dot{V}]|+(\nabla F)_p E_p^{(2)}(V-\mu_p^{(2)})^2\alpha_1+(\nabla F)_p E_p^{(3)}(V-\mu_p^{(3)})^2\alpha_2\\
=-(\nabla F)_p E_p^{(1)}(V-\mu_p^{(1)})^2\pi.
\end{aligned} \tag{28}
$$

Using Eqs. (20), (21), and (26) in Eq. (28), we obtain the following system in the unknowns α_1, α_2, and $|[\dot{V}]|$:

$$\begin{aligned}
&(\rho-\rho_*)|[\dot{V}]| + (V-\mu^{(2)})^2\alpha_1 - \frac{\rho}{C}(V-\mu^{(3)})^2\alpha_2 \\
&\qquad = -\frac{\rho}{C}(V-\mu^{(1)})^2\pi, \\
&\rho v|[\dot{V}]| + v(V-\mu^{(2)})^2\alpha_1 - \frac{\rho}{C}(v-C)(V-\mu^{(3)})^2\alpha_2 \\
&\qquad = -\frac{\rho}{C}(v+C)(V-\mu^{(1)})^2\pi, \\
&\left(\frac{\rho v^2}{2} - \frac{b(\rho-\rho_*)}{(\Gamma-1)}\right)|[\dot{V}]| + \left(\frac{v^2}{2} - \frac{bp}{(\Gamma-1)}\right)(V-\mu^{(2)})^2\alpha_1 \\
&\qquad - \left(\frac{v^2}{2} - \frac{bp}{(\Gamma-1)}\frac{\rho}{C} - \rho v + \frac{(1-b\rho)}{(\Gamma-1)}\rho C\right)(V-\mu^{(3)})^2\alpha_2 \\
&\qquad = -\left(\frac{v^2}{2} - \frac{bp}{(\Gamma-1)}\frac{\rho}{C} + \rho v + \frac{(1-b\rho)}{(\Gamma-1)}\rho C\right)(V-\mu^{(1)})^2\pi.
\end{aligned} \tag{29}$$

On solving the system (29), we obtain:

$$\left.\begin{aligned}
|[\dot{V}]| &= -\left(\frac{A_1}{A_2}+1\right)\frac{\rho}{\rho_* v}(V-\mu^{(1)})^2\pi(t_p), \\
\alpha_1 &= \left\{\left(\frac{A_1}{A_2}-1\right)\frac{\rho}{C} + \left(\frac{A_1}{A_2}+1\right)\frac{\rho(\rho-\rho_*)}{\rho_* v}\right\}\frac{(V-\mu^{(1)})^2}{(V-\mu^{(2)})^2}\pi(t_p), \\
\alpha_2 &= \frac{A_1(V-\mu^{(1)})^2}{A_2(V-\mu^{(3)})^2}\pi(t_p),
\end{aligned}\right\} \tag{30}$$

where $A_1 = \left\{\frac{1}{(\rho-\rho_*)}\left(\frac{v^2}{2}-1\right) - \frac{b}{(\Gamma-1)}\right\} - \left(\frac{v^2}{2} - \frac{bp}{(\Gamma-1)} - \frac{1}{(\rho-\rho_*)}\right)\frac{\rho(\rho-\rho_*)}{\rho_* v} - \rho v + \frac{(1-b\rho)}{(\Gamma-1)}\rho C$
and $A_2 = \left\{\frac{1}{(\rho-\rho_*)}\left(\frac{v^2}{2}-1\right) - \frac{b}{(\Gamma-1)}\right\} + \left(\frac{v^2}{2} - \frac{bp}{(\Gamma-1)} - \frac{1}{(\rho-\rho_*)}\right)\frac{\rho(\rho-\rho_*)}{\rho_* v} + \rho v - \frac{(1-b\rho)}{(\Gamma-1)}\rho C.$

Substituting (17) and (18) in (30), we obtain the amplitude of reflected waves, inside the state described by the blast wave, at the collision time. Now, from the above-computed value of α_1 and α_2, we can obtain the amplitude vectors $\Omega_2 = \alpha_1 E^2(t_p)$ and $\Omega_3 = \alpha_2 E^3(t_p)$ of the reflected waves moving along the discontinuity curve with speeds $\mu^{(2)}$ and $\mu^{(3)}$, respectively. From Eq. (30), it is clear that there is no jump in the shock acceleration in the absence of incident wave (*i.e.*, $\pi(t_p) = 0$) which ensures that there are no reflected or transmitted waves at the point $t = t_p$. Also, from (30), we observe that the behavior of shock will depend on the incident discontinuity.

5 Results and discussion

In the present investigation, the group-theoretic method is utilized to analyze the collision of the weak discontinuity with blast wave in cylindrically symmetric flow of an inviscid non-ideal gas. We derive the particular solution of the system of governing equations of motion describing an unsteady cylindrically symmetric flows of a non-ideal gas, through Lie group analysis. The solution of blast wave obtained in the paper is in close agreement with the results reported by the many authors Murata (2006), Pandey et al. (2008). The system of

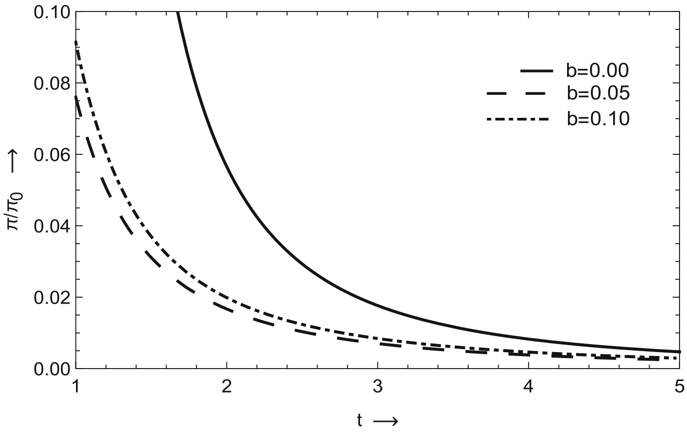

Fig. 1 The decay of C^1-wave ($\pi_0 > 0$) in non-ideal gas flow with $\Gamma = 1.67$

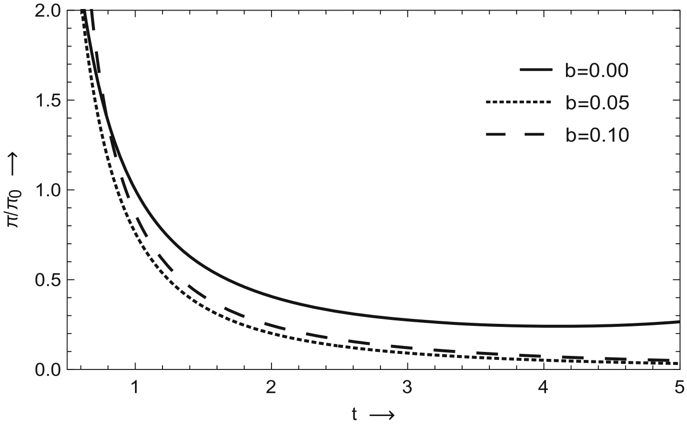

Fig. 2 The decay of C^1-wave ($-\pi_c \leq \pi_0 < 0$) in non-ideal gas flow with $\Gamma = 1.67$

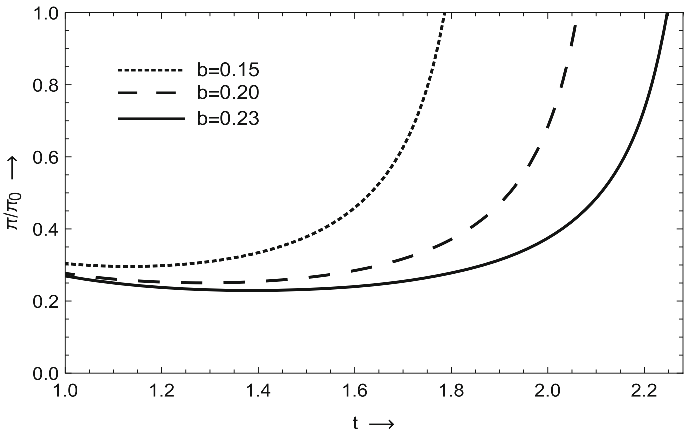

Fig. 3 The growth of C^1-wave ($\pi_0 < -\pi_c < 0$) wave in non-ideal gas flow with $\Gamma = 1.67$

transport equations for C^1 waves is much more general and it leads to the Bernoulli-type system when eigenvalues are simple. To study the amplitude of reflected and transmitted waves, we focused our attention to study the interaction between C^1 wave and the blast wave solutions for the Euler equations throughout the study. Also, the growth and decay process of weak discontinuity inside the case described by the blast wave is examined. We observe that if the value of the initial discontinuity π_0 related to the incident discontinuity increases, an increase in the value of the shock acceleration and amplitude is observed. It is noticeable that when the incident discontinuity with velocity μ^1 at $t = t_p$ impinges on the blast wave, it gives rise to two reflected waves with amplitudes α_1 and α_2 along the characteristics issuing from the collision point $t = t_p$, but there are no transmitted waves. These amplitudes of reflected waves, at the collision point $t = t_p$, are computed along with the discontinuity in the shock acceleration. We observe that if the incident discontinuity is expansive (compressive), then the reflected wave along the shock front is compressive (expansive), respectively. The behavior of incident wave is shown by the curves in Figs. 1, 2, and 3. From Fig. 3, we observe that after finite time, the compressive wave terminates into the shock wave, only if ($\pi_0 < -\pi_c < 0$). We noticed that the process of acceleration and deceleration of shock depends upon the behavior of incident wave (expansive or compressive). It is also noticeable from Fig.3 that an increase in the value of non-idealness parameter causes to delay the shock formation. This is in agreement with a recent work reported in Ref. Pandey (2010), Pandey and Sharma (2007), Jena (2007), Pandey and Sharma (2007), and Jena (2007).

6 Conclusion

In this study, we discussed the problem of interaction of C^1- wave with the blast wave in one-dimensional unsteady inviscid cylindrically symmetric flow of non-ideal gas using Lie group approach. The jump conditions for the blast wave in non-ideal gas flow are obtained. Through Lie group transformation, we obtained the solution of the hyperbolic system of PDEs describing the cylindrically symmetric flow of non-ideal gas. Also, we derived the transport equation for the C^1-wave and discussed its evolutionary process. Furthermore, the interaction phenomena of these non-linear waves are examined. It is also observed that the process of acceleration and deceleration of shock depends upon the behavior of incident wave. We obtained that after finite time, the compressive wave terminates into the shock wave, only if ($\pi_0 < -\pi_c < 0$). Also, it is observed that an increase in the value of non-ideal parameter causes to slow down the growth rate of compressive wave, i.e., there is delay in the process of shock formation in non-ideal gas flow.

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