Preface

Moving boundary problems attract the researchers very much since nineteen century due to its various practical applications in the different domains of science, engineering and industry such as the formation of ice, production of steel, vehicles design, preservation of foodstuffs, drug release processes, crystals growth, fluid flow in porous media, cryosurgery, sedimentation process near a shoreline, etc. Due to the presence of unknown domain/moving boundary and its nonlinear nature, the solutions to these problems are challenging and require a special treatment even in its simplest form. Moreover, there are many important physical processes associated to moving boundary problems which have not been effectively studied till now. To have better and suitable understanding of these processes, correct models and superb computations are needed. In the field of moving boundary problems, mathematical formulation corresponding to the real problem, solution of the problem and qualitative results such as existence and uniqueness are the three important directions of research.

In this thesis, the author presents some mathematical models allied with the moving boundary problems, and the approximate and analytical solutions of considered models are established. Moreover, existence and uniqueness of the analytical solutions are also discussed in some cases. The approximate techniques implemented here are simple, efficient and sufficiently accurate.

This thesis contains six chapters. Chapter 1 includes introduction of moving boundary problems, the literature survey and basic concepts of some techniques used in this thesis like shifted Chebyshev tau method, shifted Legendre collocation method and homotopy analysis method.

In Chapter 2, a Stefan problem including thermal conductivity and heat capacity as the functions of temperature is discussed. The exact solutions to the proposed problem are presented at $\alpha = \beta$ for two different specific cases, i.e. $m = n = 1$ and $m = n = 2$. For the general case, estimation of the solution to the problem is deliberated with the help of shifted Chebyshev tau method. To exhibit the accurateness of the obtained approximate solution, the comparison between exact and approximate solution are depicted through tables which shows that the approximate results are in good agreement with the exact solution. We also present the impact of parameters appeared in the considered problem on temperature profile and location of moving interface.

Chapter 3 involves the investigation of a one-dimensional Stefan problem by taking the variable specific heat and thermal conductivity which depend on temperature. Time-dependent heat flux is also assumed at the boundary $x = 0$ in the problem. The similarity solution for the problem is constructed when $\alpha = \beta$ and $p = q = 1$. The uniqueness and existence of the similarity solution to the problem have been also established. The problem is solved approximately for all α and β with the aid of the shifted Chebyshev tau method. To check the precision of the proposed approximate solution, we have made its comparisons with the obtained exact solution. The comparisons done are shown by tables which sufficiently agree with the exact solution. The effect of the parameters on the movement of the interface is also carried out.

In Chapter 4, a one phase Stefan problem with time and temperature dependent thermal conductivity is investigated. In this chapter, approximate solutions to the problem are obtained for the general case by Shifted Chebyshev tau method and shifted Legendre collocation method with the aid of similarity transformation. For a particular case, an exact solution of the proposed problem is also discussed and it is used to check the accuracy of the obtained approximate results. The effect of some parameters involved in the model on temperature distribution and movement of phase front is also analysed.

In Chapter 5, a mathematical model related to a problem of phase-change process with periodic surface heat flux and space-dependent latent heat is considered. We have used the homotopy analysis approach to acquire the solution to the problem. To show the correctness of the calculated result, the comparisons have been discussed with the existing exact solution at a particular case. For considered problem, this method performs well in terms of simplicity and accuracy. The effect of various parameters on the movement of the interface is also investigated in this chapter.

Chapter 6 presents a mathematical model of a two-phase Stefan problem which involves the varying thermal conductivity and specific heat with moving phase change material in a semi-infinite domain $[0,\infty)$. It is assumed that the material undergoing phase change is moving with unidirectional speed. In the model, the Dirichlet as well as convective boundary conditions have been considered at the left fixed boundary ($x = 0$). The analytical solution to the problem is established and dependence of tracking of moving interface on different parameters is also analysed.