Chapter 1

Introduction

1.1 Introduction

Today, fractional calculus and fractional controllers are highly accepted for modelling, quality control and process control operations in various process industries. The reason such application is to enhance the quality and quantity of supplies. This chapter provides a brief introduction and motivation for design of FOPID controllers and their parameter optimization. A significant literature review cum survey of fractional calculus and various existing optimization techniques for FOPID controller from the beginning till date is reported. Precisely, this chapter offers an insight into the organization of the work carried out in the thesis.

1.2 Motivation

The quantitative analysis facilitate the understanding of the large scale system and is natural phenomenon through mathematical modeling. The classical calculus is employed a simple tool to derive the mathematical representation of any natural process or large scale systems. In spite of having fewer calculations, the classical control theory has many limitations which fails to offer the exact model of natural phenomenon and real objects accurately. Hence, a precise mathematics is the prime desire of the mathematicians for accurate understanding of such systems. The aspiration to get the accurate model and complete understanding of physical phenomena of any system lead towards the non-integer order calculus also known as fractional calculus.

The fractional calculus describes the real objects and physical phenomena of any system more accurately than the “integer-order” calculus. Earlier, the main reason for engaging the integer-order models was the absence of solution methods for fractional differential equations. Since, last few decades the availability of fast computational tools and its generalized approach leads to the extensive use of fractional calculus. It is used in wide areas of applications like: system models, control theory, electrical circuit theory, capacitor theory, transmission lines etc. In the field of control theory different types of fractional controllers are successfully implemented for many practical systems like electrochemistry, biological systems, material science, viscoelasticity, mass transport, diffusion and other fields.

As conventional control theory study the different form of PID controllers, fractional calculus also demand an appropriate structure of controllers and is satisfactorily fulfilled. Among the various structures, the fractional order proportional integral and derivative (FOPID) controller is one. It is a generalized form of the conventional PID controller.

The fractional controller provides two additional tuning knobs which make the proper balance between the time domain specifications of the system and offer fast and robust controller for most of the systems. Along with the advantages few challenges also emerged. The prime complexity with the FOPID controller is to tune all the gain parameters and the fractional power of integral and derivative terms simultaneously. Hence, an efficient tool is desired for fast and optimum tuning of these controller parameters which inspires for optimization algorithms. Numerous optimization algorithms are successfully practiced in the literature for optimization of conventional PID controller parameters or for getting the optimum value of some specific function and in various other applications. Very few of them are in practice for design of FOPID controller. The availability of fewer algorithms for parameter optimization of FOPID controller seized to be a possible domain towards the motivation for the present work through the development of algorithms for *Design of Fractional Order Controller and their Parameter Optimization.*

1.3 Literature Survey

This section illustrates the history and development of FOPID controllers. It would be inappropriate to produce the complete literature of fractional theory thus, the literature playing significant role in the advancement of FOPID controllers and relevant to the present work is emphasized here.

1.3.1 Fractional calculus

The seed of fractional calculus was rooted by L'Hopital on Sept. 30th, 1695 by writing a letter to Leibniz about the notation for n^{th}-derivative of a linear function $f(x) = x, \frac{D^n x}{Dx^n}$, used in one of his publication. In the letter, L'Hopital asked a simple question to Leibniz that what will be the result of derivative of its order $n = 1/2$. Leibniz's response was: "An apparent paradox, from which one-day useful consequences will be drawn" [1]–[3]. This brief discussion guided the mathematicians to work for various possibilities in fractional calculus.

Oliver Heaviside was the first researcher who brought the attention of engineers to use the fractional calculus in engineering world in the 1890s [4]. Then Oldham and Spanier brought the first book on fractional calculus in 1974 [5]. Between the gap from 1890 to 1974 many researchers tremendously worked and presented their own notation and methodology for defining the fractional or non-integer order derivative and integral [1], [6]–[10]. Grunwald-Letnikov [11] and Riemann-Liouville [12] definitions are the most prominent in the history of fractional calculus.

Even though the fractional calculus attained an adequate mathematical background till early nineties the most significant part of the physical and geometrical realization was still missing. The very first and significant effort for the realization of the fractional operator area was done by Oldham and Spanier. They used a capacitor circuit to realize the classical integrator. The other generalization by Oldham and Spanier was a series resistor and a shunt capacitor connected to the previous network[4]. This circuit was also efficient for calculation of the fractional order integration[4]. Further, Oldham and Zoski presented a ladder network in the form of a chain of parallel combination of one resistor and one capacitor [4], [13]. They also suggested two different electronic circuits for calculating various orders of fractional operators. Further few other research articles were also published for physical and geometrical realization of fractional order calculus [5], [14]. Apart from this many research papers consisting the history and different aspects of fractional calculus are published [4]–[7], [9], [15]–[24]. In the present era, fractional calculus is extensively used for modeling and control purposes. Application of fractional calculus includes Physics [25], Continuum mechanics [26], Signal processing [27], [28], Electric transmission lines [29], Ultrasonic wave propagation in human cancellous bone [29], Fractional order control applications [30]–[37] and many more [38]–[41].

1.3.2 Fractional order controller

Manabe pioneered the fractional calculus for control application in 1961. He applied the fractional integrator as a substitute of a controller and presented the frequency and transient response of fractional-order integral [42], [43]. Barbosa discussed the same in 2003 [43]. The fractional-order algorithm is implemented for position/force hybrid control of robotic manipulators [44]. In addition, robustness and performance analysis of the proposed technique in time domain and its comparison with other control techniques is also available.

Oustaloup developed the FOPID controller in the form of CRONE (Commande Robuste d'Ordre Non-Entier, meaning Non-integer-order Robust Control) controller [45]. He applied the CRONE controller in different areas of control application and established the superior performance of FOPID controller over the conventional PID controller. In addition to this, he also presented three different generations of CRONE control techniques in 1993 [6], [7], [46].

Later in 1994, Podlubny proposed a generalized form of FOPID controller known as $PI^{\lambda}D^{\mu}$ controller [47]. In this sequence, Podlubny presented many research articles related to different applications of fractional order differentiation, integration and $PI^{\lambda}D^{\mu}$ controller [3], [48]–[51].

An analytical design technique for fractional order controller based on Ziegler–Nichols (ZN) technique and the Astrom–Hagglund techniques is presented in [52]. A different design approach by pole distribution of the characteristic equation in the complex plane is available in [53].

An FOPI controller is implemented to control a small fixed-wing unmanned aerial vehicle (UAV) in [54]. Implementation of the FOPID controllers for various real-world applications (like; velocity control of servo motor, control of DC-motor with elastic shaft, terminal voltage control of the automatic voltage regulator) as well as other field of engineering is also available [9], [50], [55]–[71]. Additionally, hardware implementation of fractional order controller is shown in [18], [72].

Table 1 presents the different form of FOPID controllers available in the literature shorted chronologically:

Table 1.1 Different forms of fractional order controllers [1], [47], [55], [73], [74]:

Types	**Controller Transfer Function**	**Author (s)**	**Year**
Bode	$G(s)=\dfrac{A_0}{\left(s/\omega_0\right)^k+1}$	H. Bode	1945
Tustin	$G(s)=A_0\left(\dfrac{w_c}{s}\right)^k$	A. Tustin	1958
CRONE First Generations	$\mathrm{C}(s)=C_0 s^{\alpha}$	Aiain Oustaloup and M. Bansard	1993
CRONE Second and third Generations	$F(s)=C(s)G(s)=\left(\dfrac{\omega_{cg}}{s}\right)^{\alpha}$	Aiain Oustaloup, Benoit Mathieu and Patrick Lanusse	1993
TID	$C(s)=\dfrac{T}{s^{-1/n}}+\dfrac{1}{s}+Ds$	B. J. Lurie	1994
Fractional Lead-Lag Compensator	$C_r(s)=C_0\left(\dfrac{1+s/\omega_b}{1+s/\omega_h}\right)^r$	H. F. Raynaud and A. ZergaInoh	2000
Modified FOPI	$C(s)=K_P\dfrac{1+T_D s}{s^{\alpha}}$	V. Feliu-Batlle et al.	2007 & 2009
Modified FOPI	$C(s)=K_P\left(\dfrac{1+T_D s}{s^{\alpha}}\right)\left(\dfrac{s+\eta}{s}\right)$	V. Feliu-Batlle et al.	2009
FO[PD]	$C(s)=K_P\left(1+K_D s\right)^{\mu}$	Y. Luo and Y. Chen	2009
FO[PI]	$C(s)=K_P\left(1+\dfrac{K_I}{s}\right)^{\lambda}$	Y. Luo et al.	2010
Non-Linear FPID	$\mathrm{u(t)=K_P}\left\|\mathrm{e(t)}\right\|^{\alpha}\mathrm{sign(e(t))}+$ $\mathrm{K}_D\left\|\mathrm{D}^{\mu}\mathrm{e(t)}\right\|^{\beta}\mathrm{sign(D}^{\mu}\mathrm{e(t))}+$ $\mathrm{K}_I\left\|\mathrm{D}^{\lambda}\mathrm{e(t)}\right\|^{\gamma}\mathrm{sign(D}^{\lambda}\mathrm{e(t))}$	F. Merrikh-Bayat and N. Mirebrahimi	2011
IMC Based	$C(s)=\mathrm{K}\left(1+\dfrac{1}{T_i s^{\alpha}}+\dfrac{T_d s^{\alpha}}{\gamma s^{\alpha}+1}\right)$	Tavakoli-Kakhki and M. Haeri	2011
Modified FPID	$C(s)=K_C\dfrac{(1+T_I s^{\mu})^2}{s^{\mu}}$	R. El-Khazali	2013

In addition to the tabulated forms, several other approaches is also adopted for design of fractional order controller are: Robust and optimal control based fractional controller [56], [75]–[78], Nonlinear form of the FOPID controller [79], Sliding mode based fractional controller [70], [80]–[83], Fuzzy based fractional controller [84], Neural network based fractional controller [8], Adaptive control based fractional controller [85], Application of fractional order PID controller for nonlinear systems [86], [87].

1.3.3 Tuning of fractional order PID controller

This subsection discusses the prevailing tuning algorithms for design of FOPID controller from its dawn. Tuning algorithms are always a simple and convenient tool for optimum design of conventional PID controller. However, optimization of parameters of FOPID controller is complicated because of two addition tuning knobs. This is the major concern due to which many optimization techniques used for optimization of conventional PID controller parameters are still untouched for fractional order controllers.

The idea for tuning of FOPID controller erupted by Caponetto and Fortuna in [88] (2002). He presented the non-integer form of classical PID controller and also elaborated the advantages of FOPID tuning over classical PID controller. The next effort is reported by Monje et al. [89]. They proposed an iterative optimization technique for optimization of FOPID controller parameters for fraction order system [89]. The effectiveness of the offered algorithm is verified by gain margin, phase margin, disturbance rejection, toughness to deviations in the gain of the plant and robustness in terms of high-frequency noise.

Cao et. al. [90] proposed genetic algorithm based tuning of FOPID controller. Integral of Time Absolute Error (ITAE) is chosen as the performance index for getting the optimum value of control parameters. Similarly, an adaptive genetic algorithm is performed in [91] for optimization of FOPID controller parameters for an active magnetic bearing system. The major difference in this algorithm is the different fitness function like rising time, settling time, maximum overshoot, and control input are considered for optimum tuning of control parameters.

Cao and Cao [92] presented an improved version of PSO algorithm for optimum design of FOPID controller. The researches guaranteed the particle position to be inside the defined search spaces. The tuning performance objective is the weighted mixture of ITAE and control input. Additionally, the PSO algorithm is used in [93] for optimization of FOPID controller for automatic voltage regulator (AVR) system. The major difference between [92] and [93] is the new fitness function defined in [93] which consist of time-domain as well as frequency-domain specifications. It is defined as:

$$J = w_1 M_P + w_2 t_r + w_3 t_s + w_4 e_{ss} + \int_0^T (w_5 |e(t)| + w_6 u^2(t))dt + \frac{w_7}{PM} + \frac{w_8}{GM} \tag{1.1}$$

where $w_n (n = 1, \ldots\ldots, 8)$ are the different weight on various parameters of the system. The fitness function also consist of rise-time (t_r), settling-time (t_s), study state error (e_{ss}), the integral of the absolute error (IAE), energy signal of the controller, gain margin (GM) and phase margin (PM) of the system. The fitness function is defined in such a way that if the closed-loop system with FOPID controller becomes unstable, then the fitness function will be penalized by a specific value. Robustness of the proposed FOPID controller is validated by considering uncertainties in the model parameters. Moreover, many works are published which discus the designing of FOPID controller using PSO algorithm [94]–[96]. Aghababa [94] proposed a FOPID controller using PSO algorithm for a five bar linkage robot. Jáuregui et al. [96] proposed a FOPID controller tuned using PSO algorithm for level control of a conical tank system. The controller is implemented on an enhanced mathematical model of the conical tank system. The simulation results are compared with the conventional PID controller tuned using the Root Locus Method (RLM) and PSO.

Pan and Das [95] proposed a fractional order automatic generation control (AGC) system for a hybrid power system. The hybrid system consisting of various independent power systems modules such as; diesel engine, wind turbine, fuel-cell, solar photovoltaic, and aqua electrolyzers. Other energy storage tools such as flywheel and the battery are also available in the system. The proposed controller sends and receives the signals over an unpredictable communication network with a delay from an isolated place. Particle swarm optimization algorithm is implemented for fine tuning of the controller parameters. Performance of the proposed controller is validated by comparing the results with conventional PID controller [92].

Brunno et al. [97] proposed an analytical tuning approach for parameter tuning of non-integer order PID controller. Tuning of controller parameter is performed on the basis of desired gain and phase margin for the system. Two examples are also provided in order to authenticate the strategy showing as fractional order PID beat the standard one. In addition to this, a digital implementation of fractional PID controller is shown by presenting the first step for a Field Programmable Analog Array (FPAA) implementation.

In this sequence another analytical approach based on Ziegler Nichols-type rules is presented by Valerio and Costa [98]. Similar assumption like conventional PID controller design using ZN-technique is made for FOPID tuning and implemented in a first order plant with a time-delay. It is also mentioned that the technique can be employed for tuning of the fractional order controller and selecting starting points for further tuning. In addition to this, implementation of Ziegler-Nichols method is also reported in [52],

[99]. In [99] the FOPID controller is optimized for velocity control of a servo system by Barbosa et al. The quality response of the system is observed according to the required specification by manual tuning of the parameter and varying parameters.

In [52] Yeroglu and Tan used the ZN-technique only for tuning the proportional and integral gain of the FOPID controller. The other two variables i.e the values of λ and μ are calculated by some nonlinear equations according to the desired gain margin and phase margin. Moreover, Astrom-Hagglund method is used for calculating the starting value of K_D for a particular phase margins and then it is further tuned using different tuning method.

Vinagre et al. [100] and Monje et al. [101] presented an auto-tuning approach for tuning of FOPID controller for industrial application. The technique permits the constraints of robustness for the FOPID system with an undemanding relation between its parameters. Additionally, the delay test on the controller and a proper relationship between delay and phase of the plant transfer function is also defined. Moreover, an identical technique based on bode shaping tuning approach is presented in [102] (2015) for an uncertain system.

Maione and Lino [103] proposed a novel tuning rule based on “symmetrical optimum (SOM)” for optimum design of FOPI controller. Based on the required specifications some tuning formulas are defined in the technique.

Chen et al. [104] proposed an Ms Constrained Integral Gain Optimization (MIGO) algorithm for optimization of fractional order controller for first order time-delay systems. The identical approach is followed by Ahn et al. in [105] for tracking the temperature profile of on heat flow tests. An improved version of the MIGO algorithm is proposed by [157] & [104].

Biswas et al. [107] proposed an improved differential evolution (DE) optimization technique for optimization of FOPID controller. The fitness function is defined in terms of the location of closed-loop dominant poles. These closed-loop poles are selected on the basis of maximum overshoot (MP) and rise time (t_r) given by the user specifications. The techniques is validated by comparing the response with other optimization technique like PSO and binary GA.

Kundu et al. [58] presented a modified invasive Weed Optimization algorithm for optimum design of FOPID controller. The algorithm is applied for designing FOPID controller for four different plants and results are validated with FOPID controller designed using existing PSO and GA techniques.

Batlle et al. [108] proposed genetic algorithm and a different forms of differential evolution based technique for optimum design of FOPID controller for unstable delayed process of water distribution in an irrigation main canal pool. The offered controller is robust for time delay variations and found to be more accurate and suitable for the delayed process as compared to the conventional PID controllers. Additionally, the controller drives a considerably small amount of current which saves energy of the actuator.

Luo et al. [109] proposed two different set of tuning technique for FOPI controller for the same fractional order system. The transfer function of the proposed FOPI controllers is given as:

$$G_{c_1}(s) = K_p\left(1 + \frac{K_i}{s^{\lambda}}\right) \tag{1.2}$$

and $$G_{c_2}(s) = K_p\left(1 + \frac{K_i}{s}\right)^{\lambda} \tag{1.3}$$

where, the value of λ is considered between (0, 2) and K_p, K_i is the proportional and integral gain constant respectively. Both of the two controllers work efficiently, with enhanced performance as compared to the conventional PID controller. The technique offers a practical and systematic way of the controller design for a specific class of fractional order plants.

Bouafoura and Braiek [110] proposed the design technique for FOPID controller using piecewise orthogonal functions. The design technique used piecewise orthogonal functions like Walsh, Block pulse, and Haar wavelets and for both integer and fractional order plants. Two different techniques are followed for designing the controller. Namely, the least square method and analytical approach.

Bayat and Ghartemani [111] proposed an analytical designing approach for FOPID controller for minimum-phase fractional order systems. In the technique root locus property of the fractional system is taken into consideration for designing of P, PI^{λ}, PD^{μ} and $PI^{\lambda}D^{\mu}$ controllers. Four different examples are explained and results are validated with pre-existing fractional order PID controllers.

Jesus et al. [112] proposed a fractional order nonlinear algorithm for controlling the heat diffusion systems. In the technique, combination of ISE and energy signal is used as the fitness function for minimization. The application of fractional calculus in the field of engineering and science is also discussed.

Lee and Chang [113] proposed an improved electromagnetism-like algorithm with genetic algorithm (IEMGA) for design and optimization of FOPID controller parameters. The IEMGA algorithm is a population based meta-heuristics algorithm and inspired from the electromagnetism theory. The technique does not require gradient calculations and capable of converging at a good result automatically. The ISE is chosen as the fitness function for optimization. Several illustrative examples are discussed to verify the performance and effectiveness of the algorithm and results are crosschecked against the other optimization techniques.

Tepljakov et al. [114] proposed a Fractional-order modeling and control (FOMCON) toolbox for MATLAB. This toolbox provides an easiest way to solve fractional order problems and a suitable platform for the beginners in fractional control. Different GUIs are also available for design and analysis of FOPID controllers.

Das et al. [115] proposed a fractional domain approach for tuning of FOPID controller for a higher order system. The controller obtained using this technique enhances the robustness and the ability of the system for high-frequency noise rejection.

Pan and Das [116] proposed a Chaotic multi-objective optimization technique for designing a FOPID controller for Automatic Voltage Regulator (AVR) systems. The performance of the controller is validated with conventional PID controller and found to be better in various aspects.

Das et al. [117] proposed a genetic programming based tuning method for FOPID controller. The algorithm is implemented on a higher order benchmark example. The tuning laws in the algorithm are derived using the controller effects and the error index. The genetic programming is used for rule extraction purposes only. Performance and robustness of the proposed controller are depicted in terms of time constant, system gain and the delay variation.

Maâmar et al. [118] proposed an Internal Model Control (IMC) based fractional PID controller. Here tuning is used to calculate the same controller of the IMC structure and daunting frequency domain specifications for the resultant system. This tuning technique is found very useful when the plant model is incomplete or inaccurate. An identical technique is practiced in [118]–[120].

Edet and Katebi [121] proposed a frequency domain based technique for fractional order PID controller design. The tuning method is similar to Ziegler-Nichol's rule for conventional PID controllers. The controller is simulated with a second order time delay system and achieves the desired robustness criterion.

Vali et al. [122] proposed an iterative learning scheme for designing the fractional order controller for a class of batch bioreactor. Fractional order iterative learning controller (FOILC) is used due to fractional dynamics present in the fed-batch systems. The controller performs very efficiently for the bioreactor system. Moreover, the simulation results prove the robustness and precision tracking property of the proposed controller.

Recently in 2016, many review works are published. Few among them are by Shah and Agasheon in [74], Soukkou and Belhour [123] and Li et al. [124]. In [74] and [123] different existing techniques for design and tuning of fractional order controller is reviewed. In [124] evaluation of existing numerical tools for fractional calculus and fractional order controls is discussed.

Various other rule based and analytical tuning approaches are also available in the literature for fine tuning of the FOPID controller parameters.

The above demonstrated literature is an evidence for the existence of fewer optimization techniques for FOPID controller. Thus, the present work lead towards the expansion of existing optimization algorithms and development of new algorithms for optimization of FOPID controller parameters.

1.4 Contribution of the Thesis

The major goal of the thesis is to offer fresh tuning algorithms for optimization of FOPID controller parameters. Expansion of the existing tuning algorithm for optimization of FOPID controller parameters is also presented. In addition, modified form of an existing tuning algorithm is proposed. A brief discussion about the limitations and disadvantages of the proposed algorithms is also available.

The proposed algorithms produces proficient and robust fractional controller for various benchmark and real-world problems. The performance of the proposed techniques are verified and validated with existing PID and FOPID controllers. Lastly, the probable future scopes of the work is discussed.

1.5 Thesis Outline

The thesis is divided in seven chapters to attain every aspect of the above discussed contribution. Organization of the thesis is drafted below:

Chapter 1 establish a brief review of existing literature and motivation for the present work. It also gives an insight into the organization of the work carried out in the thesis.

Chapter 2 contributes the necessary preliminaries required for this work. In addition, stability of the fractional order system, approximation of fractional order operators, and an overview of FOMCON toolbox are discussed. Moreover, this chapter also enlightens the fractional order controllers in different forms and advantages of using fractional order controllers.

Chapter 3 presents a brief description about the NM-algorithm. Optimization of FOPID controller parameters is also performed using NM-algorithm for a variety of system.

Chapter 4 discusses the detail strategy and operation of GWO technique. The optimization of FOPID controller parameter is performed using this technique for same systems practiced in the previous chapter.

In **Chapter 5** MGWO technique is presented. Further, parameters of FOPID controller are optimized using MGWO for the same systems practiced in chapter 3.

In **Chapter 6** a comparative analysis of all proposed algorithms is carried out for variety of problems.

Lastly**, Chapter 7** concludes the work with significant and acceptable development of the efficient tuning algorithm for design of FOPID controller. The chapter also enlightens and recommends the future scope for the further exploration of the research area.

1.6 Summery

The main purpose of optimization is to find the most appropriate values of the controller parameters. The emphasis of the present work is to develop new algorithms for optimization of fractional order controller parameters. The succeeding chapter will discuss about the essential preliminaries for the thesis.